Topological Synthesis of Geared Robotic Mechanisms

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An efficient and systematic methodology for the topological synthesis of geared robotic mechanisms is developed. The approach is based on the idea that the kinematic structure of a geared robotic mechanism can be described by an equivalent open-loop chain (EOLC) and mechanical transmission lines (MTLs) which drive the joints of the EOLC. It is shown that an MTL can be decomposed as an input unit and several transmission units connected in series. The characteristics of the input and transmission units are laid out and a systematic methodology is developed to enumerate all admissible input and transmission units from the existing atlas of nonfractionated geared kinematic chains. The atlas of admissible MTLs are then generated accordingly. Thus, all admissible geared robotic mechanisms can be enumerated efficiently with the atlas of admissible MTLs and a preferred form of the EOLC. With this new methodology, some novel structures of geared robotic mechanisms are obtained and it is believed that new areas of application can be further explored.

1 Introduction

The kinematic structure of a robotic mechanism often takes the form of an open-loop configuration. An open-loop robotic mechanism is mechanically simple and easy to construct. However, it does require the actuators to be located along the joint axes which, in turn, degrades the dynamic performance of the system. For this reason, many robotic mechanisms are constructed in a partially closed-loop configuration to ease the actuator design and/or to reduce the inertia loads on the actuators. For the case of geared robotic mechanism, gear trains are used to permit the actuators to be located as close to the base as possible.

The major breakthrough in the pursuit of a systematic approach for the design and analysis of mechanisms appears to be the application of graph theory. The concept of separation of the kinematic structure of a mechanism from its function in the structural synthesis of mechanisms was first introduced by Buchsbaum and Freudenstein (1970). This approach is useful for conceptualizing the mechanical design, surveying the potentially useful classes, and clarifying the structural similarities of mechanisms. Applications of graph theories to the structural synthesis of nonfractionated geared kinematic chains (GKCs) were investigated by Freudenstein (1971), Tsai (1987), Ravisankar and Mruthyunjaya (1985), Kim and Kwak (1990), Tsai and Lin (1989), and Hsu (1992), etc. For the enumeration of geared robotic mechanisms, Lin and Tsai (1989) enumerated an atlas of bevel-gear-type spherical wrist mechanisms with up to eight links based on the results of two degree-of-freedom (dof) nonfractionated GKCs. Belfiore and Tsai (1991) used the concept of partial separation between structure and function to generate geared robotic wrists with all actuators connected to the ground. However, by comparing these graph theory based works of Lin and Tsai (1989) and Belfiore and Tsai (1991), inconsistent results are found. Chang and Tsai (1990) introduced the concept of mechanical transmission line (MTL) to describe how the resultant joint torques are affected by the input actuators. Based on the concept of MTL, admissible structure matrices, which describe the mechanical coupling of the mechanism, for three-dof articulated gear mechanisms were generated (Chang and Tsai, 1990). However, no information about the admissible geared robotic mechanisms with certain mechanical coupling was given. Also, it is found that the MTL defined by Chang and Tsai (1990) is only one of the many possible forms an MTL can have.

In this paper, a systematic methodology for the topological synthesis of geared robotic mechanisms is developed from both structural and mechanical coupling points of view. It will be shown that admissible MTLs can be enumerated systematically from the existing atlas of non-fractionated GKCs (Freudenstein, 1971; Tsai, 1987; Ravisankar and Mruthyunjaya, 1985; Kim and Kwak, 1990; Tsai and Lin, 1989; and Hsu, 1992). With the atlas of admissible MTLs, admissible geared robotic mechanisms of desired mechanical coupling can be efficiently and systematically enumerated.

2 Structural Characteristics

The functional representation of a wrist mechanism is shown in Fig. 1(*a*). It has three dof, links 1, 4 and 6 are the input links and link 3 is the output link, called end-effector. In the graph representation, links are represented by vertices, revolute joints by thin edges, gear pairs by heavy edges, and thin edges are labeled according to their axes location. By re-arranging the coaxial links, Tsai (1988) showed that a canonical graph can be uniquely defined. In the canonical graph, there are no repeated axis labels in any thin-edged path originated from the base link. Figure 1(*b*) shows the canonical graph of Fig. 1(*a*). Among these thin-edged paths, the linkage starts from the base link and ends at the output link is called the equivalent open-loop chain (EOLC). Each link in the EOLC is referred to as a primary link while all the other links which are not rigidly attached to the primary links, including gears and actuator rotors, are called the secondary links. For the wrist mechanism shown in Fig. 1(*a*), links 0, 1, 2, and 3 are the primary links and form the EOLC of the mechanism while links 4, 5, and 6 are the secondary links. The relative angular displacement between any two adjacent primary links is referred to as a joint angle for the EOLC. A geared robotic mechanism is said to be with independent joint motion if the number of dof of the mechanism is equal to the number of joints.

Let $\mathbf{\Theta}$ and $\mathbf{X}$ be the displacement vectors associated with the joint and end-effector spaces. Let $\dot{\mathbf{\Theta}}$ and $\dot{\mathbf{X}}$ be the time derivatives of $\mathbf{\Theta}$ and $\mathbf{X}$. And let τ and $\mathbf{F}$ be the generalized force vectors in the joint-space and end-effector-space, respectively. The velocity vectors at the joint-space and end-effector-space are related by the Jacobian matrix, $\mathbf{J}$, as

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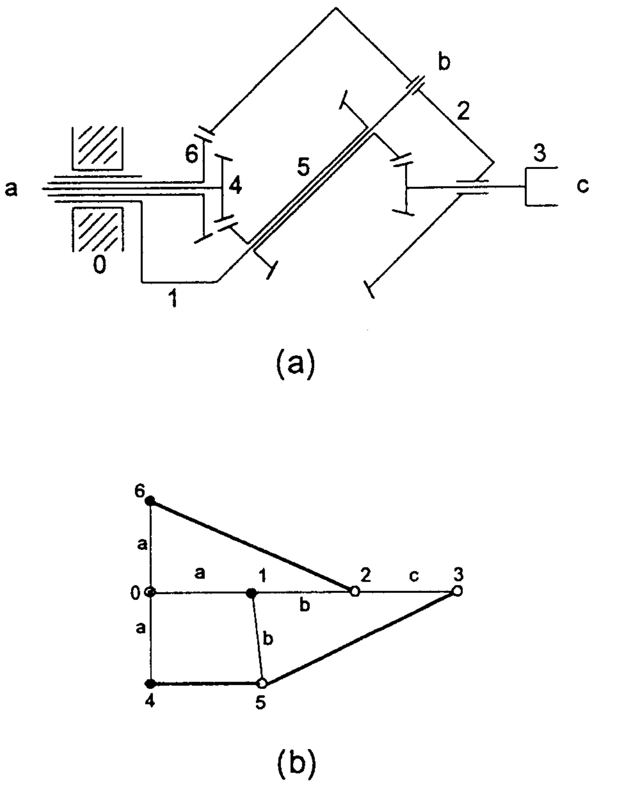

Fig. 1 (*a*) Functional representation of a wrist mechanism (*b*) canonical graph representation

$$\dot{\mathbf{X}} = \mathbf{J}\dot{\Theta} \quad (1)$$

and the torque vectors are related by

$$\tau = \mathbf{J}^T\mathbf{F} \quad (2)$$

where $(\ )^T$ denotes the transpose of ( ).

From Eqs. (1) and (2), it can be seen that the geometry of the EOLC determines the kinematic properties of the geared robotic mechanism since the Jacobian matrix **J** is a function of the joint angles only. Hence, the kinematic behaviors such as the orientation, position, velocity, acceleration of the end-effector as well as the workspace and singularities problem of a geared robotic mechanism are determined once the geometrical parameters of the EOLC are specified.

Let Φ be the displacement vector associated with the actuator-space, $\dot{\Phi}$ be its time derivative and ξ be the generalized force vector in the actuator-space. The vectors at actuator-space and joint-space can be derived by using fundamental circuit theory and coaxial conditions (Tsai, 1988) and be written as

$$\dot{\Phi} = \mathbf{A}^T\dot{\Theta} \quad (3)$$

and the torque vectors at the joint-space and actuator-space are related by

$$\tau = \mathbf{A}\xi \quad (4)$$

where **A** is the structure matrix (Chang and Tsai, 1990) and the elements of **A** are functions of gear ratios.

Chang and Tsai (1990) showed that each column of the structure matrix **A** represents an MTL. From Eqs. (3) and (4), it is clear that the form of structure matrix **A** describes the mechanical coupling of the mechanism, i.e., where the input actuators are located and how the input torques are transmitted to various joints of the EOLC. Note that the number of dof for a geared robotic mechanism with independent joint motion is equal to the number of MTLs associated with it.

For the wrist mechanism shown in Fig. 1, Eq. (4) can be written as

$$\begin{bmatrix} \tau_{10} \\ \tau_{21} \\ \tau_{32} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & r_{2,6} & r_{5,4} \\ 0 & 0 & r_{5,4}r_{3,5'} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_4 \\ \xi_5 \end{bmatrix} \quad (5)$$

where τ_{10}, τ_{21}, and τ_{32} are the joint torques, ξ_1, ξ_4, and ξ_5 are the torques applied at input link 1, 4 and 5, respectively, $r_{i,j} = \pm N_i/N_j$ is the gear ratio for the gear pair mounted on links i and j, positive or negative according as a positive rotation of gear i results in a positive or negative rotation of gear j about their pre-defined axes of rotation, and where N_i denotes the number of teeth on gear i.

By collecting the vertices and edges associated with an MTL in the canonical graph representation of a geared robotic mechanism, a graph representation of the MTL can be identified. Two MTLs are said to be decoupled if they do not have any secondary link in common in their graph representation. Otherwise, these two MTLs are said to be coupled. In this paper, we shall concentrate on the geared robotic mechanisms of independent joint motion with decoupled MTLs. Figure 2(*a*) shows the graph representation of the MTLs of the mechanism shown in Fig. 1. It can be seen that there are three MTLs for the mechanism. For the MTL corresponds to the first column of **A** in Eq. (5), torque is exerted on primary link 1 directly. For the MTL corresponds to the second column of **A** in Eq. (5), torque exerted by primary link 6 is transmitted to link 2 by gear pair mounted on links 6 and 2. For the MTL corresponds to the third column of **A** in Eq. (5), torque exerted on primary link 4 is transmitted to primary link 3 by gear pairs mounted on links 4 and 5, and 5 and 3. The MTL with link 1 as the input link is called a direct drive MTL since link 1 is a primary link.

3 Structural Decomposition of MTL

Figure 3(*a*) shows the typical MTL defined by Chang and Tsai (1990) in simplified planar representation. In the simplified

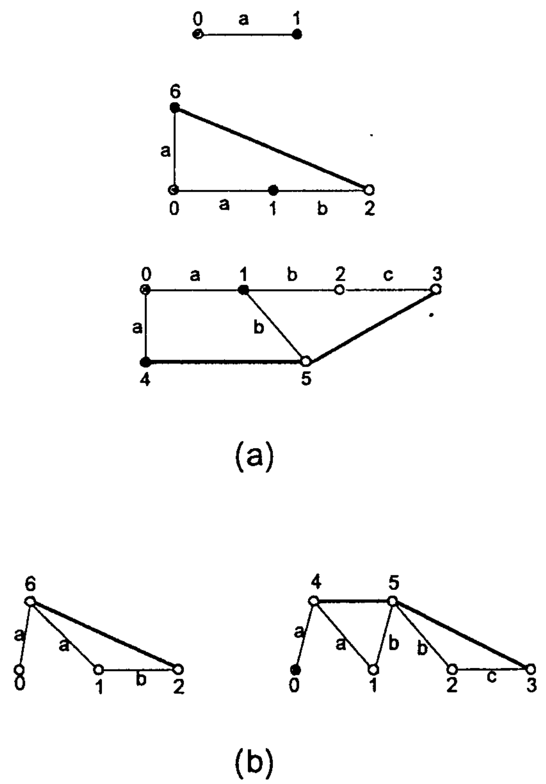

Fig. 2 (*a*) Graph representation of MTLs (*b*) pseudo-isomorphic graphs of MTLs

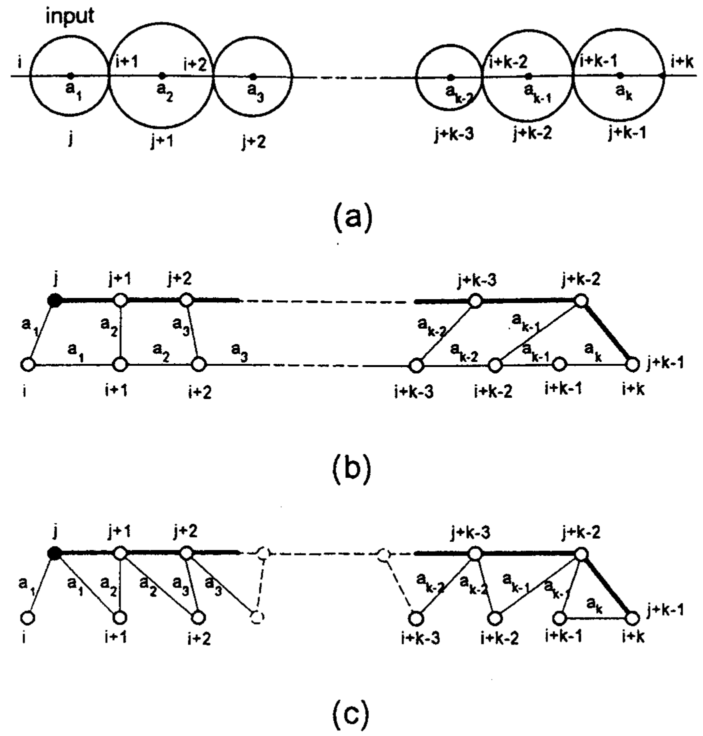

Fig. 3 (*a*) Planar representation (*b*) graph representation (*c*) pseudo-isomorphic graph of Chang and Tsai (1990)

planar representation, all joint axes are twisted until they are parallel to each other and point toward the same positive direction while all gear pairs are represented by equivalent spur gear pairs (Chang and Tsai, 1990). In the typical MTL, a series of primary links numbered i, $i + 1$, . . . , $i + k$, are connected together by revolute joints to form an open-loop chain, and

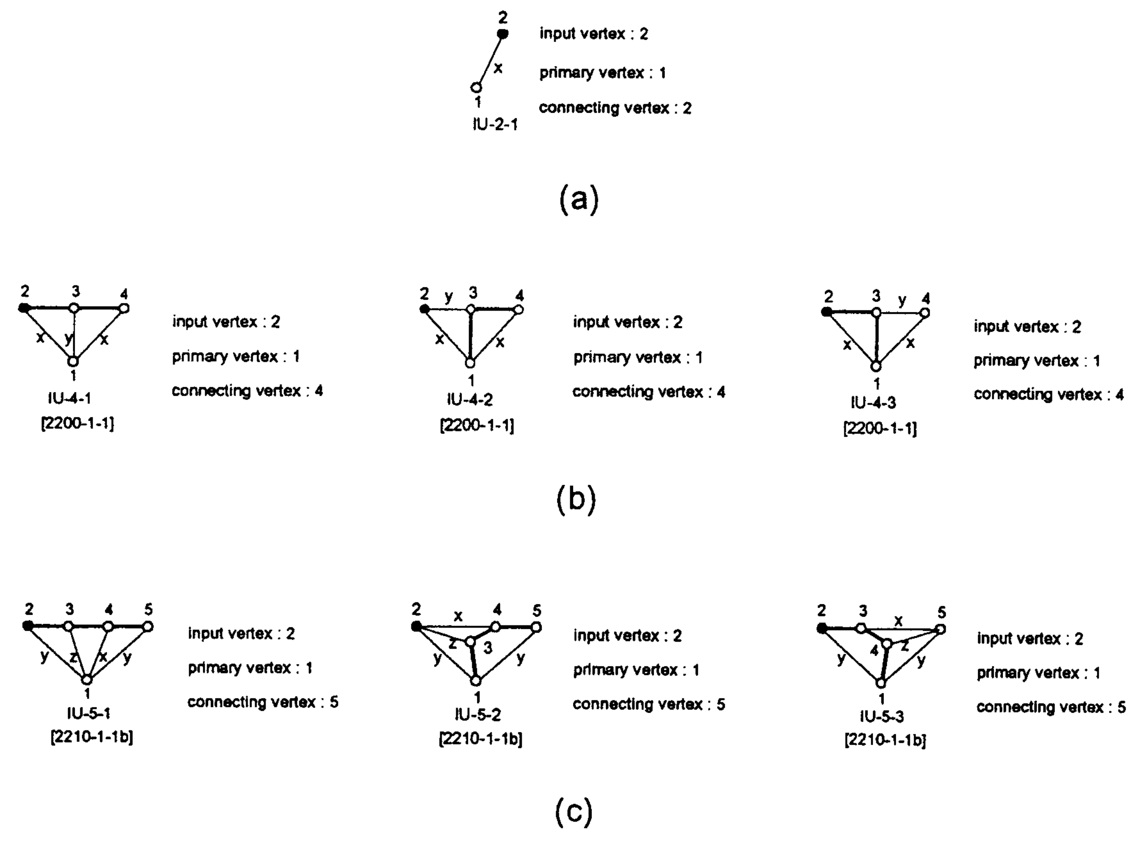

Fig. 4 Admissible input units (*a*) two-link (*b*) four-link (*c*) five-link

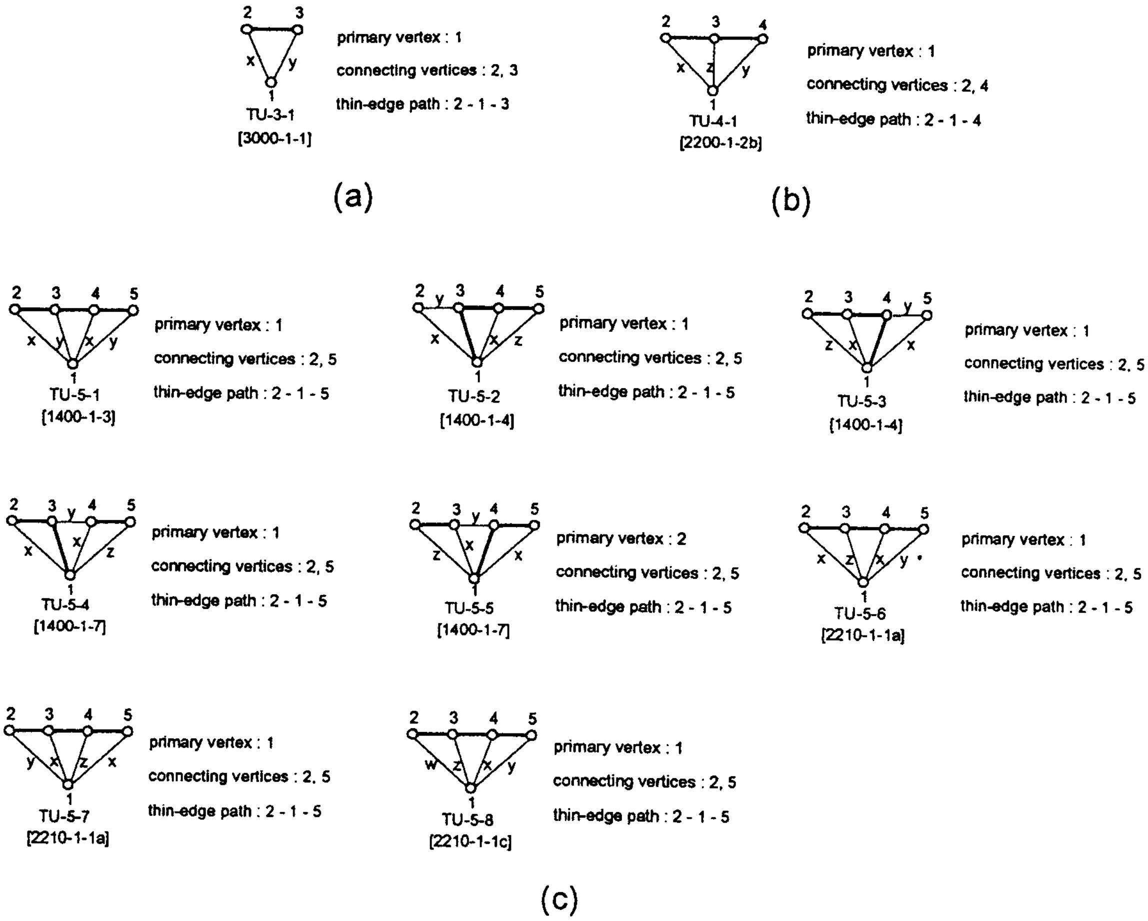

Fig. 5 Admissible transmission units (*a*) three-link (*b*) four-link (*c*) five-link

secondary links numbered $j, j+1, \ldots, j+k-2$ are pivoted about the $a_1, a_2, \ldots,$ and a_{k-1} joint axes, respectively. The last secondary link, $(j+k-1)$, is rigidly attached to primary link $(i+k)$, and the rotation of gear j with respect to link i is considered as the input. Figure 3(b) shows the graph representation of the typical MTL.

By re-arranging the coaxial links, a pseudo-isomorphic graph (Tsai and Lin, 1989) of Fig. 3(b) can be obtained and shown

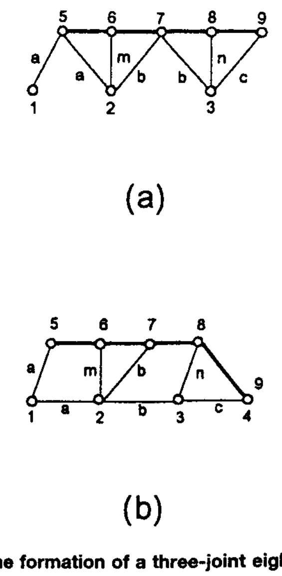

Fig. 6 The formation of a three-joint eight-link MTL

in Fig. 3(c). From Fig. 3(c), it is clear that the MTL defined by Chang and Tsai (1990) can be viewed as a collection of nonfractionated kinematic chains connected in series. The first unit, a two-link chain which contained the input link, is called the input unit. The remaining are a series of three-link GKCs, called the transmission units, connected together. Each unit is connected to its adjacent unit by sharing a common secondary link called the connecting link. Note that the joint between connecting link and primary link of an unit is coaxial with that of its associated adjacent unit. Hence, by re-arranging the coaxial conditions between the joints at the connecting links and primary links of each set of adjacent units, an articulated joint of the EOLC can be formed. Figure 2(b) shows the pseudo-isomorphic graphs of the non-direct drive MTLs of Fig. 2(a). It can be seen that, for the MTL corresponds to the second column of **A** in Eq. (5), the two-link chain ⟨0, 6⟩ acts as the input unit and the three-link chain ⟨1, 2, 6⟩ acts as the transmission unit while secondary link 6 acts as the connecting link between these two units. By re-arranging the coaxial links at axis-a, an articulated joint can be formed between primary links 0 and 1. Also, for the MTL corresponds to the third column of **A** in Eq. (5), secondary link 4 acting as a connecting link connects the input unit ⟨0, 4⟩ and transmission unit ⟨1, 4, 5⟩, while secondary link 5 acting as a connecting link connects transmission units ⟨1, 4, 5⟩ and ⟨2, 3, 5⟩. By re-arranging the coaxial links at axis-a and axis-b of the MTL, articulated joints can be formed between primary links 0 and 1 at axis-a and between primary links 1 and 2 at axis-b, respectively.

However, the two-link input unit and three-link transmission unit identified in the MTL defined by Chang and Tsai (1990) are only special cases among many possible forms. It is clear

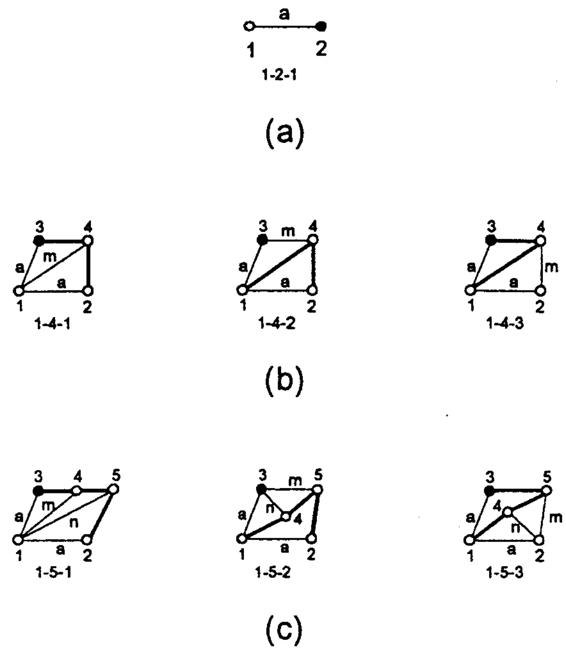

Fig. 7 Admissible one-joint MTLs up to five-link (*a*) two-link (*b*) four-link (*c*) five-link

that it is possible to have input and transmission units other than the two-link chain and three-link GKC. By comparing these with that defined by Chang and Tsai (1990), fundamental requirements for a kinematic chain and/or a GKC to be used as an input and/or transmission units can be set as follows:

For input unit:

Table 1 Number of admissible two-joint MTLs

Number	IU			TU			Number of MTLs	
of links	2	3	4	3	4	5	subtotal	total
4	1			1			1	1
5	1				1		1	1
6	1					8	8	11
			3	1			3	

R1. An input unit has an input link, a connecting link, and at least one primary link.
R2. For a two-link kinematic chain to be used as an input unit:

(a) One link of the two-link chain can be assigned as a primary link and the other link as a secondary link.
(b) The secondary link is the input link and is served as a connecting link.

Table 2 Number of admissible three-joint MTLs

Number	IU			TU1			TU2			No. of MTLs	
of links	2	3	4	3	4	5	3	4	5	subtotal	total
6	1			1			1			1	1
7	1			1				1		1	2
	1				1		1			1	
8	1			1					8	8	20
	1					8	1			8	
	1				1			1		1	
			3	1			1			3	

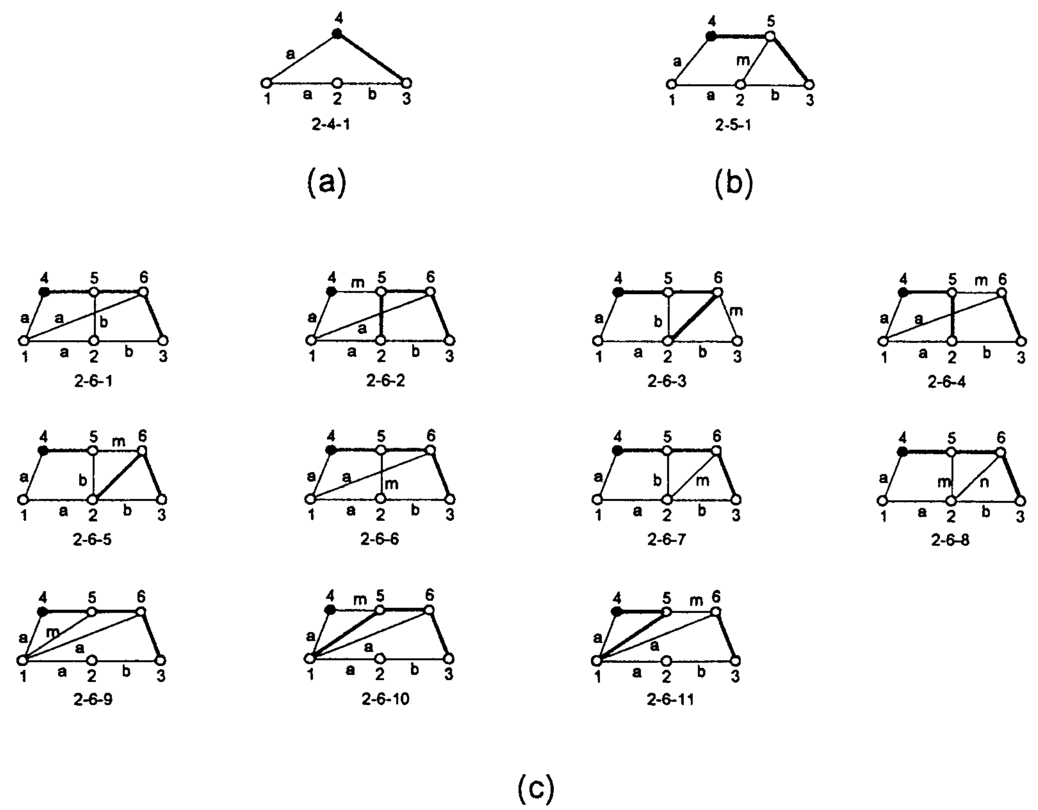

Fig. 8 Admissible two-joint MTLs up to six-link (*a*) four-link (*b*) five-link (*c*) six-link

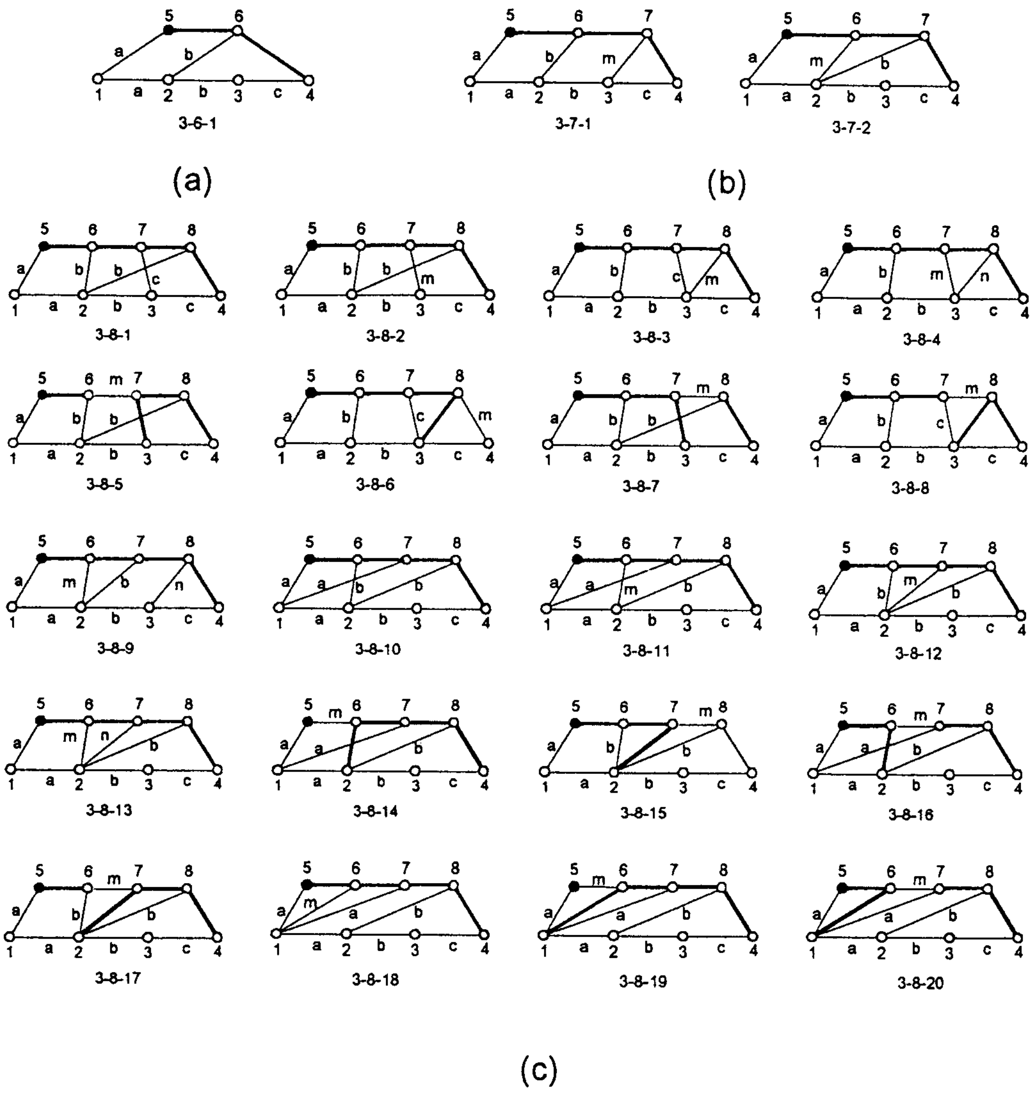

Fig. 9 Admissible three-joint MTLs up to eight-link (*a*) six-link (*b*) seven-link (*c*) eight-link

R3. For a one-dof GKC to be used as an input unit:

(a) There is one link in the GKC can be assigned as a primary link and the other links in the GKC are characterized as secondary links.
(b) The connecting link is a secondary link incident with the primary link.
(c) The input link is a secondary link incident with the primary link and is coaxial with the primary link.
(d) A link is considered as a redundant link if removal of the link and its associated joints from the GKC does not change the torque transmission between the input link and connecting link. We shall only consider those GKCs with no redundant link.

For transmission unit:

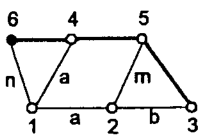

Fig. 10 A derived MTL

R4. A transmission unit has one primary link and two connecting links.
R5. For a one-dof GKC to be used as a transmission unit:

(a) There is one link in the GKC can be assigned as a primary link and the other links in the GKC are characterized as secondary links.
(b) The connecting link is a secondary link incident with the primary link.
(c) The connecting links and the primary link form a thin-edge path with 2 distinct edge labels.
(d) A link is considered as a redundant link if removal of the link and its associated joints from the GKC does not change the torque transmission between the connecting links. We shall only consider those GKCs with no redundant link.

With these fundamental requirements, admissible input and transmission units can be identified systematically from the atlas of nonisomorphic GKCs enumerated by Freudenstein (1971), Tsai (1987), Ravisankar and Mruthyunjaya (1985), Kim and Kwak (1990), Tsai and Lin (1989), and Hsu (1992), etc. Figures 4 and 5 show the graph representation of admissible input units and transmission units up to five links, respectively. In Figs. 4 and 5, IU stands for the input unit and TU stands for

Table 3 Structure matrices of geared robotic mechanisms with grounded actuators

$\begin{bmatrix} \# & \# & \# \\ \# & \# & \# \\ \# & \# & \# \end{bmatrix}$	$\begin{bmatrix} \# & \# & \# \\ \# & \# & \# \\ 0 & \# & \# \end{bmatrix}$	$\begin{bmatrix} \# & \# & \# \\ 0 & \# & \# \\ 0 & \# & \# \end{bmatrix}$	$\begin{bmatrix} \# & \# & \# \\ \# & \# & \# \\ 0 & 0 & \# \end{bmatrix}$	$\begin{bmatrix} \# & \# & \# \\ 0 & \# & \# \\ 0 & 0 & \# \end{bmatrix}$
g^3-1	g^3-2	g^3-3	g^3-4	g^3-5

the transmission unit, respectively, and the number between the brackets is the ID number used in the Freudenstein (1971).

4 Enumeration of Admissible Transmission Lines

The enumeration process of admissible transmission lines from the atlas of input and transmission units can be broadly classified into the following stages.

Step 1: Choose the number of components in the MTL and determine the number of links of each component.

In this step, the number of links and the number of joints influenced by the MTL are first specified. The number of components in an MTL is equal to the number of joints it influences, i.e., an n-joint MTL has n components. In these n components, there are one and only one input unit and $n - 1$ transmission units. Note that an MTL can have only the input unit as its component. Let m be the number of links of an n-joint MTL, m_{iu} the number of links of the input unit, and $m_{tu})_j$ the number of links of the j-th transmission unit, respectively. Since there are $n - 1$ connecting links for an n-joint MTL, the following equation must be satisfied.

$$m = m_{iu} + \sum_{j=1}^{n-1} m_{tu})_j - (n - 1) \tag{6}$$

or

$$m + n - 1 = m_{iu} + \sum_{j=1}^{n-1} m_{tu})_j \tag{7}$$

Hence, Eq. (7) can be used to choose the proper set(s) of combination of input and transmission units. For example, an eight-link three-joint MTL ($m = 8$ and $n = 3$), we have

$$\sum_{j=1}^{2} m_{tu})_j = 8 \quad \text{for } m_{iu} = 2 \tag{8}$$

or

$$\sum_{j=1}^{2} m_{tu})_j = 6 \quad \text{for } m_{iu} = 4 \tag{9}$$

From Fig. 4, the input units with $m_{iu} = 2$ and 4 are IU-2-1 and IU-4-series. From Fig. 5, the transmission units which satisfy Eq. (8) are {TU-3-1, TU-5-series} and {TU-4-1, TU-4-1} while those satisfy Eq. (9) is {TU-3-1, TU-3-1}, respectively.

Step 2: Determine the arrangement sequence of units.

At this point, the arrangement sequence of units will be determined. The input unit is the first component of the MTL while the sequence of transmission units is permutable. Thus, the following sequences of input and transmission units can be used to construct an eight-link three-joint MTL: {IU-2-1, TU-3-1, TU-5-series}, {IU-2-1, TU-5-series, TU-3-1}, {IU-2-1, TU-4-1 TU-4-1}, and {IU-4-series, TU-3-1, TU-3-1}.

Step 3: Connect the units and construct the joints by re-arranging the coaxial links.

At this step, units are connected together by sharing the common connecting links between adjacent units according to the arrangement sequence. By re-arranging the coaxial links between connecting links and primary links, articulated joints are formed.

Step 4: Assign the last primary link.

The formation of an MTL is completed by assigning the connecting link of the last unit in the arrangement sequence as a primary link. By assigning the connecting link of the input units as a primary link one-joint MTLs can be obtained. Note that since the connecting link of the two-link chain is also the input link, by assigning it as a primary link a direct drive MTL is obtained.

Step 5: Permute the sequence of transmission units, repeat steps 3 to 4 until all the possible permutations are accessed.

For example, an eight-link three-joint MTL with IU-2-1 as input unit, TU-4-1 as the first transmission unit followed by a TU-4-1 as the last transmission unit can be formed as shown in Fig. 6. Note that, in Fig. 6(a), link 9 is the connecting link of the last transmission unit and is assigned as primary link 4 in Fig. 6(b). Hence, with this systematic methodology, all admissible m-link n-joint MTLs can be efficiently enumerated with anticipated results with the atlas of admissible input and transmission units. Figure 7 shows the graphs of one-joint MTLs up to 5 links. Table 1 shows the combination of admissible two-joint MTLs with up to six links. Figure 8 shows the graphs of admissible two-joint MTLs with up to six links. Table 2 shows the combination of admissible three-dof MTLs with up to eight links while Figure 9 shows their graph representations. In Figs. 7, 8, and 9 the MTLs are classified according to the number of joints it influences and its number of links. For example, a MTL of 3-8-# indicates it is a three-joint MTL with eight-link, and # is its series number. The MTLs obtained above shall be called the basic MTLs. In order to change the direction of rotation, to achieve a higher gear reduction, and/or to extend the center distance, idler gear(s) can be added in a basic MTL to relocate the input actuator. For example, Figure 10 shows the addition of an idler gear 6 to the two-joint MTL 2-5-1. We

Table 4 Three-dof geared robotic mechanisms with grounded actuators

	m	m_i / No. of admissible MTLs			subtotal	total
		i = 1	i = 2	i = 3		
$\begin{bmatrix} \# & \# & \# \\ \# & \# & \# \\ 0 & \# & \# \end{bmatrix}$ g^3-2	9	4 / 1	6 / 1	6 / 1	1	1
$\begin{bmatrix} \# & \# & \# \\ 0 & \# & \# \\ 0 & \# & \# \end{bmatrix}$ g^3-3	8	2 / 1	6 / 1	6 / 1	1	1
	9	2 / 1	6 / 1	7 / 2	2	2
$\begin{bmatrix} \# & \# & \# \\ \# & \# & \# \\ 0 & 0 & \# \end{bmatrix}$ g^3-4	8	4 / 1	4 / 1	6 / 1	1	1
	9	4 / 1	4 / 1	7 / 2	2	3
		4 / 1	5 / 1	6 / 1	1	
$\begin{bmatrix} \# & \# & \# \\ 0 & \# & \# \\ 0 & 0 & \# \end{bmatrix}$ g^3-5	7	2 / 1	4 / 1	6 / 1	1	1
	8	2 / 1	4 / 1	7 / 2	2	3
		2 / 1	5 / 1	6 / 1	1	
	9	2 / 1	4 / 1	8 / 20	20	33
		2 / 1	5 / 1	7 / 2	2	
		2 / 1	6 / 11	6 / 1	11	

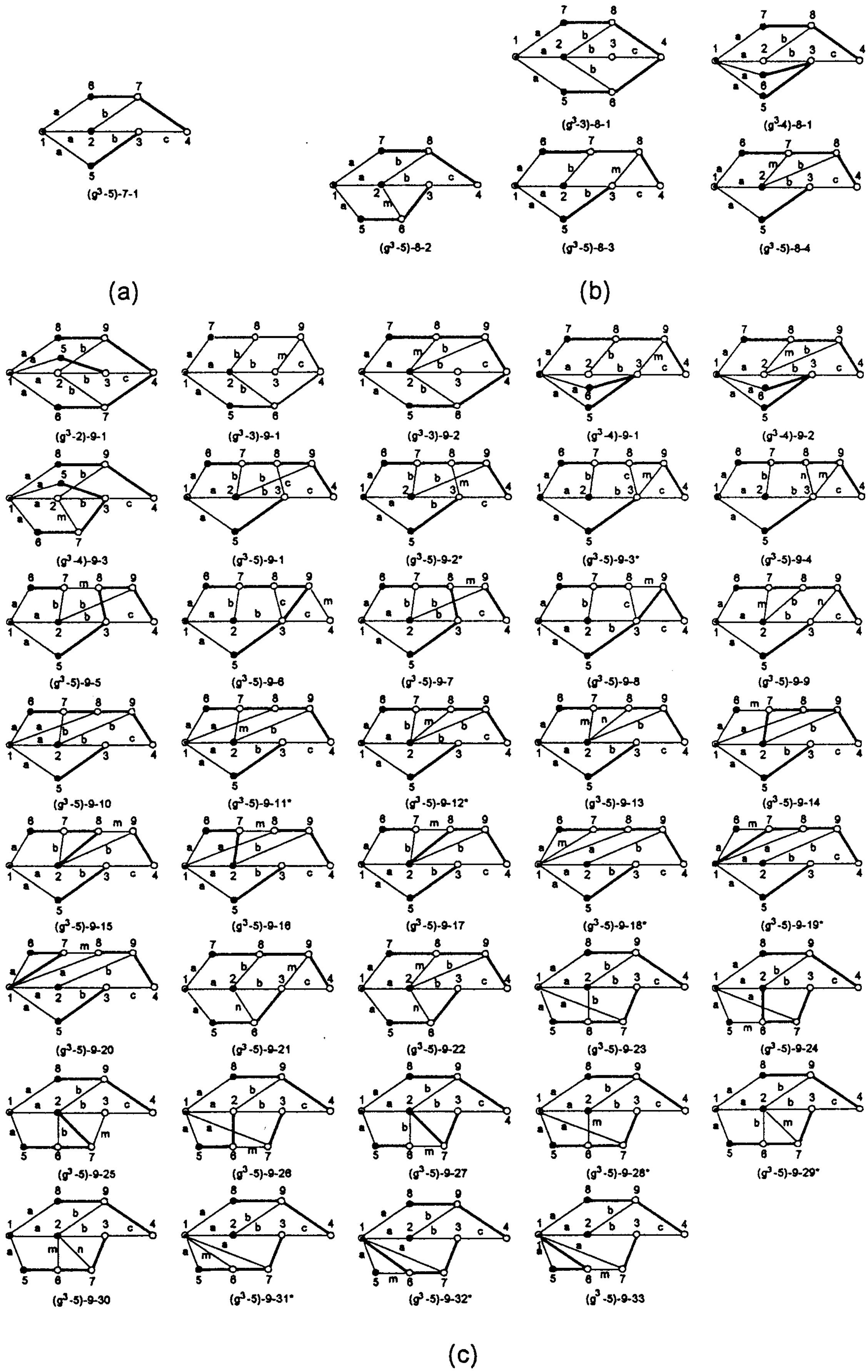

Fig. 11 Admissible 3-dof geared robotic mechanisms with grounded actuators (*a*) seven-link (*b*) eight-link (*c*) nine-link

shall call those MTLs with relocated input actuator the derived MTLs as opposed to the basic MTLs.

5 Synthesis of Geared Robotic Mechanisms

The characteristics of structure matrix **A** of an n-dof geared robotic mechanism with n articulated joints can be summarized as follows (Chang and Tsai, 1990).

(a) The matrix is an $n \times n$ square matrix with nonzero determinant.
(b) The elements of the i-th row of the matrix represent the influence coefficients for the i-th joint.
(c) The i-th column of the matrix represents the i-th MTL of the geared robotic mechanism and the nonzero elements in it must be consecutive.
(d) Switching the columns of the matrix results in renumbering the corresponding MTLs.

Applying the above rules, Chang and Tsai (1990) showed that there are four admissible structure matrices for two-dof geared robotic mechanisms and there are thirty admissible structure matrices for three-dof geared robotic mechanisms. Hence, the enumeration of admissible geared robotic mechanisms can be treated as the problem of choosing proper MTLs from the atlas of admissible MTLs according to the form of structure matrix **A**. This process can be classified into the following stages.

Step 1: Determine the joint number of EOLC and the number of MTLs.

At this step, the number of links and the dof of the geared robotic mechanism are specified first. For an n-dof m-link geared robotic mechanism with no redundant actuation, n MTLs are required to drive an EOLC with n independent joints. Thus, the link number of the EOLC is $n + 1$.

Step 2: Choose the form of structure matrix **A**.

The form of structure matrix **A** must be specified from the four and thirty admissible structure matrices for two-dof and three-dof geared robotic mechanisms since it describes the mechanical coupling of the MTLs.

Step 3: Determine the number of links of each MTL.

Once the form(s) of interested structure matrix is chosen, the types of MTLs can be determined accordingly. For an m-link n-dof geared robotic mechanism, there are $n + 1$ primary links with n MTLs. Let m_i be the number of links of the i-th n_i-joint MTL. For the n_i-joint MTL, there are $n_i + 1$ primary links. Thus, for an n-dof m-link geared robotic mechanism, the following equation must be satisfied.

$$m = (n + 1) + \sum_{i=1}^{n} [m_i - (n_i + 1)] \qquad (10)$$

or

$$m - 1 = \sum_{k=1}^{n} (m_i - n_i) \qquad (11)$$

Hence, Eq. (10) can be used to choose the proper set(s) of combination of MTLs. Table 3 shows the structure matrices of the robotic geared mechanisms with grounded actuators (Chang and Tsai, 1990). Assuming that structure matrix **A** of $g^3 - 4$ form is desired for a nine-link three-dof geared robotic mechanism ($m = 9$ and $n = 3$). Since a three-dof geared robotic mechanism is desired, an EOLC with four primary link is required. From the form of interested structure matrix, it can be seen that the first two MTLs of mechanism are two-joint MTLs and the third MTL is a three-joint MTL. From Eq. (11), we have

$$m_1 + m_2 + m_3 = 15 \qquad (12)$$

From Figs. 8 and 9, it can be shown that the link number of these MTLs have the following combinations: {4, 4, 7} and {4, 5, 6}. From Figs. 8 and 9, the four-link and five-link two-joint MTLs are 2-4-1 and 2-5-1, the six-link three-joint MTL is 3-6-1, while the seven-link three-joint MTLs are 3-7-1 and 3-7-2. Thus, there are three nine-link three-dof geared robotic mechanisms with the desired structure matrix, namely, {2-4-1, 2-4-1, 3-7-1}, {2-4-1, 2-4-1, 3-7-2}, and {2-4-1, 2-5-1, 3-6-1}. Note that the two-joint MTLs share the primary links with each other while the three-joint MTL shares its first two joints with the two-joint MTLs.

Step 4: Repeat steps 2 to 3, until all admissible structure matrices **A** are accessed.

Hence, admissible m-link n-dof geared robotic mechanisms of a desired structure matrix, i.e., a desired mechanical coupling, can be systematically and efficiently enumerated with anticipated results. For the purpose of demonstration, all admissible three-dof geared robotic mechanisms up to nine-link with grounded actuators are used as illustrative examples. Table 4 shows the combinations of basic MTLs of geared robotic mechanisms with grounded actuators up to nine links. From Eq. (11), it can be shown that there is no nine-link geared robotic mechanism with $g^3 - 1$ as its structure since the fewest link number of three-joint MTLs is equal to six. Figure 11 shows the canonical graph representations of geared robotic mechanisms with grounded actuators up to nine links. In Fig. 11, graphs marked with asterisk are believed to be new by comparing the results of Lin and Tsai (1990), Belfiore and Tsai (1991), and Belfiore (1993).

6 Conclusion

In this paper, kinematic structures of geared robotic mechanisms are investigated and classified. It is shown that the kinematic structure of a geared robotic mechanism can be treated as an EOLC with several MTLs which drive it. It is also shown that an MTL can be viewed as an input unit connected with a series of transmission units. Rules for the enumeration of input and transmission units are outlined and a systematic procedure for the construction of MTLs are developed. It is shown that, with the atlas of admissible MTLs, geared robotic mechanisms of desired mechanical couplings can be efficiently and systematically enumerated with anticipated results. The methodology has been demonstrated by the enumeration of three-dof geared wrist mechanisms with grounded actuators. Some of the mechanism configurations presented here are believed to be new and novel.

We believed that this method of enumeration, which use the atlas of geared kinematic chains developed by earlier investigators as foundation, is more straightforward, more efficient, and more reliable than those approaches start from scratch. Although we have used the enumeration of three-dof wrist mechanisms as an example, the methodology presented here is completely general and can be applied to the enumeration of n-dof geared robotic mechanisms.

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