On the Drive Train Design of Gear Coupled Manipulators

Dar-Zen Chen* and Shinn-Chang Wang
Department of Mechanical Engineering
National Taiwan University
Taipei, Taiwan
e-mail: dzchen@ccms.ntu.edu.tw

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A systematic methodology for the drive train design of gear coupled manipulators is developed. The approach is based on the idea that the generalized inertia forces contribution to the system's dynamic equations can be divided into the contribution of the virtual open-loop chain and that of the drive train. It is shown that the contribution of the drive train to the system's dynamic equations is a function of the orientations of its rotational axes and the orientations of joints. Conditions to determine these orientations such that the drive train can have configuration-invariant contribution to the system's dynamic equations are derived. From the design conditions, mechanical details of the drive train such as the choice of actuator locations, the arrangements of the drive train and joints, the compatibility between orientations of joints and rotational axes of the drive train can be determined accordingly. This approach, together with the developed design methods for the serial type manipulators, provides a complete design methodology in designing new robots or improving the dynamic performance of existing ones.

1. INTRODUCTION

The kinematic structure of a robot manipulator often takes the form of an open-loop configuration. An open-loop robot manipulator is mechanically simple and easy to construct. However, it does require the actuators to be located along the joints which, in turn, degrades the dynamic performance of the system. For this reason, many robot manipulators are constructed in a partially closed-loop configuration to ease the actuator design and to reduce the inertia loads on the actuators. For the case of gear coupled manipulators (GCMs), gear trains are used to permit the actuators to be located as close to the base link as possible.

*To whom all correspondence should be addressed.

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Various methods for the design of open-loop manipulators can be found in the literature. Yang and Tzeng,[1] based on eliminating coefficients of nonlinear terms in the system's kinetic and potential energy equation, proposed a method to design robots with better dynamic behavior. Youcef-Toumi and Asada[2] derived the necessary conditions of kinematic structure and mass distribution for robots to possess decoupled and/or configuration-invariant inertia. Park and Cho[3] derived general conditions on link parameters such as mass, location of mass center, and inertia distribution to simplify the complex robot dynamics. Soylu and Sarrafi[4] presented a methodology for linearization and optimization of robot dynamics by the design of inertial parameter. Although most robot manipulators are driven by motors through gear trains, most of the previous studies did not address the inertial effect of the drive train. Strictly speaking, these approaches are only applicable to design of direct-drive manipulators. Chen and Tsai[5] described a methodology for gear ratios determination of GCMs based on kinematic isotropy and optimum acceleration capacity. Chen[6] discussed the compatibility between structure matrix and Jacobian matrix and developed a methodology for the drive train configuration arrangement. However, these approaches did not elaborate on the mechanical details such as the location of actuators, the arrangement of the drive train, and the compatibility between orientations of joints and rotational axes of the drive train.

In what follows, a methodology for the arrangement of the drive train of GCMs will be presented. It will be shown that the drive train of a GCM can have configuration-invariant contribution to the system's dynamic equations through proper choice of its location and orientations of rotational axes. A set of design conditions regarding the arrangement of the drive train and compatibility between the orientations of its rotational axes and joint axes will be established. The approach, together with the developed methods for the open-loop manipulators, provides a complete design methodology in designing new GCMs or improving the dynamic performance of existing ones.

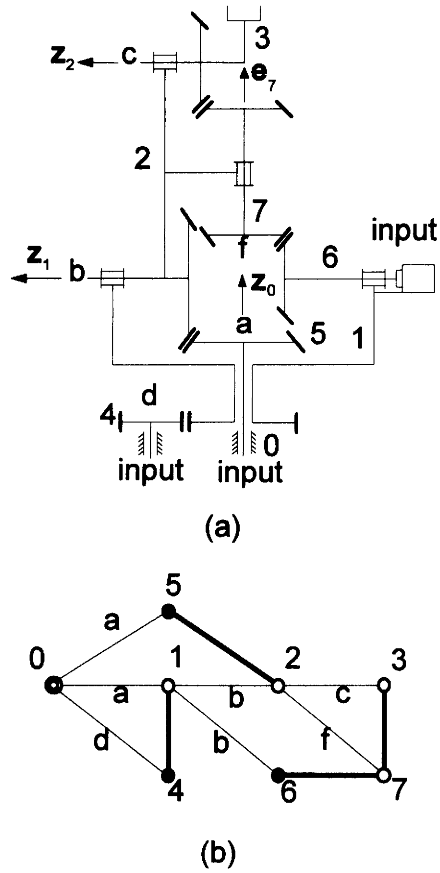

Figure 1. A 3-DOF GCM; (a) Functional representation, (b) Canonical graph representation.

2. GEAR COUPLED MANIPULATORS (GCMs)

Figure 1 shows the functional representation and canonical graph representation (Tsai[7]) of a GCM. It has 3 degree-of-freedom (DOF). Links 4, 5, and 6 are input links and link 3 is the output link, called the end-effector. In Figure 1(b), the thin-edged path starting from the base link to the end-effector is defined as the equivalent open-loop chain (EOLC) (Tsai[7]). Each link in the EOLC is referred to as a primary link, and all other links are called the secondary links (Chen and Tsai[5]). A secondary link j is said to be carried by a primary link i if it is at one thin-edge away from primary link i in the canonical graph. By treating all secondary links carried by primary link i as being rigidly attached to itself, virtual link i^* (Chen and Wang[8]) is defined. Hence, virtual link i^* moves exactly the same as primary link i and has the mass–inertia properties of primary link i and all secondary links carried by it. A collection of virtual links forms the virtual open-loop chain (VOLC). Thus, kinematic properties of the VOLC can be computed in exactly the same way as those of the EOLC.

The arrangement of the secondary links, which describes where the input actuators are located and how the input torques are transmitted to various joints of the manipulator, forms the mechanical transmission lines (MTLs) (Chang and Tsai[9]). Through MTLs, which consist of spur or bevel gear trains, input torques are transmitted to the end-effector on which the payload can be attached. Hence, the collection of secondary links forms the drive train of a GCM.

Chen and Wang[8] showed that the contribution to the generalized inertia forces of a secondary link can be divided into two parts. The first part is due to the motion of its associated primary link and the second part is due to the relative motion of its associated primary link. The first part of the generalized inertia forces contribution can be incorporated with the generalized inertia forces contribution of its associated primary link. Thus, the generalized inertia forces contribution of a GCM can be decomposed into the contribution of the VOLC and the contribution due to the relative motion of the drive train with respect to their associated primary links. Hence, the design of a GCM can be treated as a problem of designing a VOLC and its drive train. Note that the design problem of a VOLC is the same problem as the design of open-loop manipulators. Methods such as the mass and inertia relocation approach (Yang and Tzeng[1]; Youcef-Toumi and Asada[2]), the linearization approach (Soylu and Sarrafi[4]) can be applied to the design of the VOLC. In what follows, we will concentrate on the design of the drive train with configuration-invariant contribution to the system's dynamic equations due to its relative motion with respect to the associated primary links.

3. DYNAMIC EQUATIONS

Beginning from the base link, primary links are numbered sequentially from 0 to n for an n-DOF GCM. Based on the Denavit and Hartenberg Convention (Denavit and Hartenberg[10]), a coordinate system (x_i, y_i, z_i) is attached to primary link i and shown as in Figure 2. Let $a_{i,i-1}$ be the offset distance between $\mathbf{z}_{i-1}$ and $\mathbf{z}_i$ axes, $\alpha_{i,i-1}$ be the twist angle between the $\mathbf{z}_{i-1}$ and $\mathbf{z}_i$ axes, $d_{i,i-1}$ be the translational distance measured from the $\mathbf{x}_{i-1}$-axis to the $\mathbf{x}_i$-axis along the $\mathbf{z}_{i-1}$-axis. The joint angle measured from the $\mathbf{x}_{i-1}$-axis to the $\mathbf{x}_i$-axis along the $\mathbf{z}_{i-1}$-axis, $q_{i,i-1}$, is referred to as the joint angle of joint i, θ_i. Joints 1, 2,..., and i, which precede primary link i, are defined as inner joints of primary link i while joints $i+1$, $i+2, \ldots$, and n, which succeed primary link i, are defined as outer

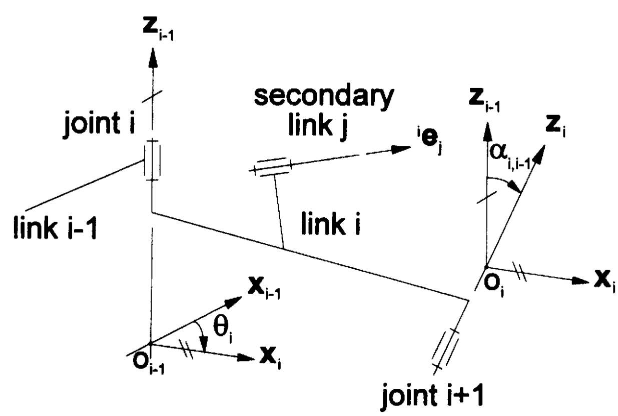

Figure 2. Coordinate system of primary link.

joints of primary link i (Chen and Wang[8]). The unit vector ${}^i\mathbf{z}_{k-1}$ denotes the orientation of the rotational axis of joint k referred in the $(\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i)$ coordinate system and can be represented as

$$ {}^i\mathbf{z}_{k-1} = {}^i\mathbf{R}_{i-1} \cdots {}^{k+1}\mathbf{R}_k \, {}^k\mathbf{z}_{k-1} \tag{1} $$

where rotation matrix ${}^i\mathbf{R}_{i-1}$ is written as

$$ {}^i\mathbf{R}_{i-1} = \begin{bmatrix} C\theta_i & S\theta_i & 0 \\ -C\alpha_{i,i-1}S\theta_i & C\alpha_{i,i-1}C\theta_i & S\alpha_{i,i-1} \\ S\alpha_{i,i-1}S\theta_i & -S\alpha_{i,i-1}C\theta_i & C\alpha_{i,i-1} \end{bmatrix} \tag{2} $$

and where $C\theta_{i,i-1} = \cos(\theta_{i,i-1})$, $S\theta_{i,i-1} = \sin(\theta_{i,i-1})$, $C\alpha_{i,i-1} = \cos(\alpha_{i,i-1})$, and $S\alpha_{i,i-1} = \sin(\alpha_{i,i-1})$.

Table I shows the unit vectors along the rotational axes of joints. It can be seen that ${}^i\mathbf{z}_{k-1}$ is a function of joint p ($p = k+1, \ldots, i$) and $\alpha_{q,q-1}$ ($q = k, \ldots, i$) and the unit vector along rotational axis of joint i, ${}^i\mathbf{z}_{i-1}$, is a function of $\alpha_{i,i-1}$ only, i.e.,

$$ {}^i\mathbf{z}_{i-1} = [0 \quad S\alpha_{i,i-1} \quad C\alpha_{i,i-1}]^T \tag{3} $$

With the joint angles as the generalized coordinates, Chen and Wang[8] showed that the equations of motion of GCMs can be written as

$$ \mathbf{G}_v + \mathbf{G}_s = \mathbf{A}\xi \tag{4} $$

where $\mathbf{G}_v$ is the generalized inertia force contribution of the VOLC, $\mathbf{G}_s$ is the generalized inertia force contribution due to relative motion of drive train, ξ is the input torque vector associated with the actuator-space, and $\mathbf{A}$ is the structure matrix (Chang and Tsai[9]).

Table I. Orientations of joints

$$
{}^1\mathbf{z}_0 = \begin{bmatrix} 0 \\ S\alpha_{1,0} \\ C\alpha_{1,0} \end{bmatrix} \qquad
{}^2\mathbf{z}_0 = \begin{bmatrix} S\alpha_{1,0}S\theta_2 \\ S\alpha_{1,0}C\alpha_{2,1}C\theta_2 + C\alpha_{1,0}S\alpha_{2,1} \\ -S\alpha_{1,0}S\alpha_{2,1}C\theta_2 + C\alpha_{1,0}C\alpha_{2,1} \end{bmatrix}
$$

$$
{}^2\mathbf{z}_1 = \begin{bmatrix} 0 \\ S\alpha_{2,1} \\ C\alpha_{2,1} \end{bmatrix} \qquad
{}^3\mathbf{z}_1 = \begin{bmatrix} S\alpha_{2,1}S\theta_3 \\ S\alpha_{2,1}C\alpha_{3,2}C\theta_3 + C\alpha_{2,1}S\alpha_{3,2} \\ -S\alpha_{2,1}S\alpha_{3,2}C\theta_3 + C\alpha_{2,1}C\alpha_{3,2} \end{bmatrix}
$$

$$
{}^3\mathbf{z}_2 = \begin{bmatrix} 0 \\ S\alpha_{3,2} \\ C\alpha_{3,2} \end{bmatrix} \qquad
{}^3\mathbf{z}_0 = \begin{bmatrix} S\alpha_{1,0}(S\theta_2C\theta_3 + C\alpha_{2,1}C\theta_2S\theta_3) + C\alpha_{1,0}S\alpha_{2,1}S\theta_3 \\ C\alpha_{1,0}C\alpha_{2,1}S\alpha_{3,2} + S\alpha_{1,0}(-C\alpha_{3,2}S\theta_2S\theta_3 + C\alpha_{2,1}C\alpha_{3,2}C\theta_2C\theta_3) \\ +S\alpha_{2,1}(C\alpha_{1,0}C\alpha_{3,2}C\theta_3 - S\alpha_{1,0}S\alpha_{3,2}C\theta_2) \\ C\alpha_{1,0}C\alpha_{2,1}C\alpha_{3,2} + S\alpha_{1,0}(S\alpha_{3,2}S\theta_2S\theta_3 - C\alpha_{2,1}S\alpha_{3,2}C\theta_2C\theta_3) \\ +S\alpha_{2,1}(-C\alpha_{1,0}S\alpha_{3,2}C\theta_3 - S\alpha_{1,0}C\alpha_{3,2}C\theta_2) \end{bmatrix}
$$

With the concept of virtual links, $\mathbf{G}_v$ can be formulated by any of the methods used for open-loop manipulators (Hollerbach[11]; Lee et al.[12]; Thomas and Tesar[13]; and Walker and Orin,[14] etc.) as

$$\mathbf{G}_v = \mathbf{M}_v\ddot{\Theta} + \mathbf{C}_v(\Theta, \dot{\Theta}) \tag{5}$$

where $\mathbf{M}_v$ is an $(n \times n)$ inertia coefficient matrix and $\mathbf{C}_v$, an $(n \times 1)$ vector, represents the Coriolis and centrifugal effects associated with $\mathbf{G}_v$.

The generalized inertia force contribution due to the relative motion of drive train, $\mathbf{G}_s$, can be written as (Chen and Wang[8]),

$$\mathbf{G}_s = \sum_{i=0}^{n}\sum_{j}\mathbf{M}_{j/i}\ddot{\Theta} + \sum_{i=0}^{n}\sum_{j}\left\{\sum_{r=1}^{n}\left[\sum_{k=1}^{n}\sum_{h=1}^{n}\mathbf{C}_{j/i,r})_{h,k}\dot{\theta}_h\dot{\theta}_k\right]\right\} \tag{6}$$

where

$$\mathbf{M}_{j/i} = \sum_{r=1}^{n}\sum_{k=1}^{n}\mathbf{M}_{j/i})_{r,k} \tag{7}$$

and where $\mathbf{M}_{j/i}$ is and $(n \times n)$ inertia coefficient matrix due to a secondary link j carried by primary link i, $\mathbf{M}_{j/i})_{r,k}$ is the (r,k) element of $\mathbf{M}_{j/i}$, and $\mathbf{C}_{j/i,r}$ is an $(n \times n)$ coefficient matrix specifying Coriolis and centrifugal effects due to a secondary link j carried by primary link i associated with joint r, and $\mathbf{C}_{j/i,r})_{h,k}$ is the (h,k) element of $\mathbf{C}_{j/i,r}$.

Chen and Wang[8] showed that $\mathbf{M}_{j/i})_{r,k}$ can be expressed according to joints r and k being inner joints and/or outer joints of primary link i as follows.

Region 1: Joints r and k are inner joints of primary link i,

$$\mathbf{M}_{j/i})_{r,k} = 0 \tag{8}$$

Region 2: Joints r and k are outer joints of primary link i,

$$\mathbf{M}_{j/i})_{r,k} = I_j b_{j,r} b_{j,k} \tag{9}$$

where gear ratio $b_{j,k}$ is the coefficient of the relative angular velocity of secondary link j with respect to primary link i associated with joint k.

Region 3: Joint r is an inner joint and joint k is an outer joint of primary link i,

$$\mathbf{M}_{j/i})_{r,k} = I_j b_{j,k}\,{}^i\mathbf{e}_j \cdot {}^i\mathbf{z}_{r-1} \tag{10}$$

where ${}^i\mathbf{e}_j$ is the unit vector along the rotational axis of secondary link j represented in the coordinate system of primary link i.

From Eqs. (8) and (9), it can be concluded that joints r and k are both inner or outer joints of primary link i, $\mathbf{M}_{j/i})_{r,k}$ is configuration-invariant. While joint r is an inner joint and joint k is an outer joint of primary link i, from Eq. (10), $\mathbf{M}_{j/i})_{r,k}$ is configuration-variant and is a function of the projection of ${}^i\mathbf{e}_j$ onto ${}^i\mathbf{z}_{r-1}$, the unit vector along inner joint r.

Chen and Wang[8] also showed that $\mathbf{C}_{j/i,r})_{h,k}$ can be classified according to joint r being an inner or outer joint, and joints h and k being inner and/or outer joints of primary link i as follows.

Joint r is an inner joint of primary link i,

$$\mathbf{C}_{j/i,r})_{h,k}$$

$$= -\frac{1}{2} I_j b_{j,k} {}^i\mathbf{e}_j \cdot \frac{\partial {}^i\mathbf{z}_{h-1}}{\partial \theta_r} \quad \begin{cases} \text{joint } h\text{: a joint precedes joint } r \\ \text{joint } k\text{: an outer joint of primary link } i \end{cases} \tag{11a}$$

$$= \frac{1}{2} I_j b_{j,k} {}^i\mathbf{e}_j \cdot \frac{\partial {}^i\mathbf{z}_{r-1}}{\partial \theta_h} \quad \begin{cases} \text{joint } h\text{: an inner joint of primary link } i \text{ and succeeds joint } r \\ \text{joint } k\text{: an outer joint of primary link } i \end{cases} \tag{11b}$$

$$= -\frac{1}{2} I_j b_{j,h} {}^i\mathbf{e}_j \cdot \frac{\partial {}^i\mathbf{z}_{k-1}}{\partial \theta_r} \quad \begin{cases} \text{joint } h\text{: an outer joint of primary link } i \\ \text{joint } k\text{: a joint precedes joint } r \end{cases} \tag{11c}$$

$$= \frac{1}{2} I_j b_{j,h} {}^i\mathbf{e}_j \cdot \frac{\partial {}^i\mathbf{z}_{r-1}}{\partial \theta_h} \quad \begin{cases} \text{joint } h\text{: an outer joint of primary link } i \\ \text{joint } k\text{: an inner joint of primary link } i \text{ and succeeds joint } r \end{cases} \tag{11d}$$

$$= 0 \quad \text{otherwise} \tag{11e}$$

Joint r is an outer joint of primary link i,

$$\mathbf{C}_{j/i,r})_{h,k}$$

$$= \frac{1}{2} I_j b_{j,r} {}^i\mathbf{e}_j \cdot \frac{\partial {}^i\mathbf{z}_{h-1}}{\partial \theta_k} \quad \begin{cases} \text{joint } h\text{: an inner joint of primary link } i \text{ and precedes joint } k \\ \text{joint } k\text{: an inner joint of primary link } i \end{cases} \tag{12a}$$

$$= \frac{1}{2} I_j b_{j,r} {}^i\mathbf{e}_j \cdot \frac{\partial {}^i\mathbf{z}_{k-1}}{\partial \theta_h} \quad \begin{cases} \text{joint } h\text{: an inner point of primary link } i \\ \text{joint } k\text{: an inner joint of primary link } i \text{ and precedes joint } h \end{cases} \tag{12b}$$

$$= 0 \quad \text{otherwise} \tag{12c}$$

From Eqs. (11) and (12), it can be seen that $\mathbf{C}_{j/i,r})_{h,k}$ can be divided into two regions as follows.

Region 1: (1) joint r is an inner joint and joints h and k are both inner joints or outer joints of primary link i; and (2) joint r is an outer joint and either joint h or joint k is an outer joint of primary link i, $\mathbf{C}_{j/i,r})_{h,k}$ is equal to zero.

Region 2: (1) joint r is an inner joint and either joint h or joint k is an outer joint and the other is an inner joint of primary link i, (2) joint r is an outer joint and joints h and k are both inner joints of primary link i, $\mathbf{C}_{j/i,r})_{h,k}$ is configuration-variant and is a function of the projections of ${}^i\mathbf{e}_j$ onto the partial derivative of the unit vector along an inner joint with respect to the other inner joint which succeeds it.

4. DESIGN METHODOLOGY

For a secondary link j carried by a primary link i, assuming the projection of ${}^i\mathbf{e}_j$ onto the unit vector along an inner joint r is a constant, we have

$${}^i\mathbf{e}_j \cdot {}^i\mathbf{z}_r = \text{constant} \tag{13}$$

Since the orientation of secondary link j is a fixed unit vector, the partial derivative of Eq. (13) with respect to an inner joint h which succeeds inner joint r can be written as

$$\frac{\partial\left({}^i\mathbf{e}_j \cdot {}^i\mathbf{z}_r\right)}{\partial \theta_h} = \frac{\partial(\text{constant})}{\partial \theta_h} = {}^i\mathbf{e}_j \cdot \frac{\partial {}^i\mathbf{z}_r}{\partial \theta_h} = 0 \tag{14}$$

From Eqs. (13) and (14), it can be concluded that if the projection of ${}^i\mathbf{e}_j$ onto ${}^i\mathbf{z}_r$ is a constant, the projection of ${}^i\mathbf{e}_j$ and $\partial {}^i\mathbf{z}_r/\partial\theta_h$ is equal to zero. Hence, region 3 of $\mathbf{M}_{j/i}$ and region 2 of $\mathbf{C}_{j/i,r}$ can be made configuration-invariant if the projections of orientations of secondary links onto the orientations of their inner joints are made to be constants. This leads to the following proposition:

Proposition: *For a secondary link j carried by primary link i, the contribution to the system's dynamic equations due to the relative motion of the secondary link j with respect to primary link i can be made configuration-invariant through proper choices of*: (1) *the location of secondary link j*, (2) *the orientation of secondary link j, and* (3) *the compatibility between orientations of a secondary link j and inner joints of primary link i.*

5. DESIGN CONDITIONS

In this section, design conditions such as the orientation of a secondary link j and the compatibility between orientations of a secondary link j and inner joints of primary link i will be discussed according to the location of the secondary link j.

5.1. For the Secondary Link j Carried by Primary Link 0

For a secondary link j carried by primary link 0, the base link, there is no inner joint. Elements of region 3 of inertia coefficient matrix $\mathbf{M}_{j/0}$ are equal to zero. Hence, we have

Axiom 1: *The contribution to the system's dynamic equations of a secondary link j carried by primary link* 0 *is equal to zero. The orientation of the secondary link,* $\mathbf{e}_j$, *can be an arbitrary unit vector.*

5.2. For the Secondary Link j Carried by Primary Link 1

For a secondary link j carried by primary link 1, joint 1 is the inner joint of primary link 1. Let ${}^1\mathbf{e}_j = [x, y, z]^T$, from Table I and Eq. (10), the configuration-variant element of inertia coefficient matrix $\mathbf{M}_{j/1})_{i,k}$ for joint k as an outer joint of primary link 1 can be written as

$$\mathbf{M}_{j/1})_{1,k} = I_j b_{j,k} {}^1\mathbf{e}_j \cdot {}^1\mathbf{z}_0 = I_j b_{j,k} u_1 \tag{15}$$

where

$$u_1 = yS\alpha_{1,0} + zC_{1,0}$$

Equation (15) leads to a constant with a given twist angle $\alpha_{1,0}$. Thus, elements of region 3 in $\mathbf{M}_{j/1}$ are configuration-invariant. Hence, we have

Axiom 2: *The contribution to the system's dynamic equations of a secondary link j carried by primary link* 1 *is configuration-invariant. The orientation of the secondary link,* $\mathbf{e}_j$, *can be an arbitrary unit vector.*

Note that, from Eq. (15), elements of region 3 in $\mathbf{M}_{j/1}$ can be made to zero if ${}^1\mathbf{e}_j = [x, y, z]^T$ is arranged as $y{:}z = -C\alpha_{1,0}{:}S\alpha_{1,0}$.

5.3. For the Secondary Link j Carried by Primary Link 2

For a secondary link j carried by primary link 2, joints 1 and 2 are inner joints of primary link 2. Let ${}^2\mathbf{e}_j = [x, y, z]^T$, from Table I and Eq. (10), configuration-variant elements of inertia coefficient matrix $\mathbf{M}_{j/2})_{i,k}$ for joint k as an outer joint of primary link 2 can be written as

$$\mathbf{M}_{j/2})_{2,k} = I_j b_{j,k} {}^2\mathbf{e}_j \cdot {}^2z_1 = I_j b_{j,k} u_2 \tag{16a}$$

$$M_{j/2})_{1,k} = I_j b_{j,k} {}^2\mathbf{e}_j \cdot {}^2\mathbf{z}_0 = I_j b_{j,k}\{S\alpha_{1,0}[xS\theta_2 + v_2 C\theta_2] + u_2 C\alpha_{1,0}\} \tag{16b}$$

where

$$u_2 = yS\alpha_{2,1} + zC\alpha_{2,1} \tag{16c}$$

$$v_2 = yC\alpha_{2,1} - zS\alpha_{2,1} \tag{16d}$$

Equation (16a) leads to a constant with a given twist angle $\alpha_{2,1}$. Conditions for Eq. (16b) to be a constant are (1) $\alpha_{1,0}$ is equal to 0°, or (2) ${}^2\mathbf{e}_j$ is arranged as $\pm[0, S\alpha_{2,1}, C\alpha_{2,1}]^T$. Hence, for Eqs. (16a) and (16b) to be constants simultaneously, we have

1. Twist angle $\alpha_{1,0} = 0°$: With this condition and from Eqs. (16a) and (16b), elements of region 3 in $\mathbf{M}_{j/2}$ can be rewritten as

$$\mathbf{M}_{j/2})_{2,k} = \mathbf{M}_{j/2})_{1,k} = I_j b_{j,k} u_2 \tag{17}$$

Hence, we have

Axiom 3: *The contribution to the system's dynamic equations of a secondary link j carried by primary link* 2 *is configuration-invariant if joints* 1 *and* 2 *are parallel to* *each other, i.e., $\mathbf{z}_0 // \mathbf{z}_1$. The orientation of the secondary link, $\mathbf{e}_j$, can be an arbitrary unit vector.*

Note that, from Eq. (17), elements of region 3 in $\mathbf{M}_{j/2}$ can be made to zero if ${}^2\mathbf{e}_j = [x, y, z]^T$ is arranged as $y{:}z = -C\alpha_{2,1}{:}S\alpha_{2,1}$.

2. ${}^2\mathbf{e}_j = \pm[0, S\alpha_{2,1}, C\alpha_{2,1}]^T$: With this condition, elements of region 3 in $\mathbf{M}_{j/2}$ from equations (16a) and (16b) can be rewritten as

$$\mathbf{M}_{j/2})_{2,k} = \pm I_j b_{j,k} \tag{18a}$$

$$\mathbf{M}_{j/2})_{1,k} = \pm I_j C\alpha_{1,0} b_{j,k} \tag{18b}$$

Hence, from Eq. (3), we have

Axiom 4: *The contribution to the system's dynamic equations of a secondary link j carried by primary link 2 is configuration-invariant if its orientation is parallel to that of joint 2, i.e., $\mathbf{e}_j // \mathbf{z}_1$.*

5.4. For the Secondary Link *j* Carried by Primary Link 3

For a secondary link j carried by primary link 3, joints 1, 2, and 3 are inner joints of primary link 3. Let ${}^3\mathbf{e}_j = [x, y, z]^T$, from Table I and Eq. (10), configuration-variant elements in the inertia coefficient matrix $\mathbf{M}_{j/3})_{i,k}$ for joint k as an outer joint of primary link 3 can be rewritten as

$$\mathbf{M}_{j/3})_{3,k} = I_j b_{j,k} {}^3\mathbf{e}_j \cdot {}^3\mathbf{z}_2 = I_j b_{j,k} u_3 \tag{19a}$$

$$\mathbf{M}_{j/3})_{2,k} = I_j b_{j,k} {}^3\mathbf{e}_j \cdot {}^3\mathbf{z}_1 = I_j b_{j,k}[S\alpha_{2,1}(xS\theta_3 + v_3 C\theta_3) + u_3 C\alpha_{2,1}] \tag{19b}$$

and

$$\begin{aligned}\mathbf{M}_{j/3})_{1,k} &= I_j b_{j,k} {}^3\mathbf{e}_j \cdot {}^3\mathbf{z}_0 \\ &= I_j b_{j,k}\{S\alpha_{1,0}[C\alpha_{2,1}C\theta_2(xS\theta_3 + v_3 C\theta_3) \\ &\quad + S\theta_2(xC\theta_3 - v_3 S\theta_3)] \\ &\quad + S\alpha_{2,1}[C\alpha_{1,0}(xS\theta_3 + v_3 C\theta_3) - u_3 S\alpha_{1,0} C\theta_2] \\ &\quad + u_3 C\alpha_{1,0} C\alpha_{2,1}\}\end{aligned} \tag{19c}$$

where

$$u_3 = yS\alpha_{3,2} + zC\alpha_{3,2} \tag{19d}$$

$$v_3 = yC\alpha_{3,2} - zS\alpha_{3,2} \tag{19e}$$

Equation (19a) leads to a constant with a given twist angle $\alpha_{3,2}$. Conditions for Eq. (19b) to be a constant are: (1) $\alpha_{2,1}$ is equal to 0°, or (2) ${}^3\mathbf{e}_j$ is arranged as $\pm[0, S\alpha_{3,2}, C\alpha_{3,2}]^T$. Conditions for Eq. (19c) to be a constant are: (1) $\alpha_{1,0}$ and $\alpha_{2,1}$ are equal to 0°, or (2) $\alpha_{1,0}$ is equal to 0° and ${}^3\mathbf{e}_j$ is arranged as $\pm[0, S\alpha_{3,2}, C\alpha_{3,2}]^T$, or (3) $\alpha_{2,1}$ is equal to 0° and ${}^3\mathbf{e}_j$ is arranged as $\pm[0, S\alpha_{3,2}, C\alpha_{3,2}]^T$. Thus, for Eqs. (19a–c) to be constants simultaneously, we have

1. Twist angles $\alpha_{1,0}$ and $\alpha_{2,1}$ are equal to 0°: With the condition, elements of region 3 in $\mathbf{M}_{j/3}$, from Eqs. (19a–c) can be rewritten as

$$\mathbf{M}_{j/3})_{1,k} = \mathbf{M}_{j/3})_{2,k} = \mathbf{M}_{j/3})_{3,k} = \pm I_j b_{j,k} u_3 \tag{20}$$

Hence, we have

Axiom 5: *The contribution to the system's dynamic equations of a secondary link j carried by primary link 3 is configuration-invariant if joints 1, 2, and 3 are parallel to each other, i.e., $\mathbf{z}_0 // \mathbf{z}_1 // \mathbf{z}_2$. The orientation of the secondary link, $\mathbf{e}_j$, can be an arbitrary unit vector.*

Note that from Eqs. (19d) and (20), elements of region 3 in $\mathbf{M}_{j/3}$ can be made to zero if ${}^3\mathbf{e}_j = [x, y, z]^T$ is arranged as $y{:}z = -C\alpha_{3,2}{:}S\alpha_{3,2}$.

2. Twist angle $\alpha_{2,1} = 0°$ and ${}^3\mathbf{e}_j = \pm[0, S\alpha_{3,2}, C\alpha_{3,2}]^T$: With the condition, elements of region 3 in $\mathbf{M}_{j/3}$, from Eqs. (19a–c), can be rewritten as

$$\mathbf{M}_{j/3})_{1,k} = \pm I_j C\alpha_{1,0} b_{j,k} \tag{21a}$$

$$\mathbf{M}_{j/3})_{2,k} = \mathbf{M}_{j/3})_{3,k} = \pm I_j b_{j,k} \tag{21b}$$

Hence, from Eq. (3), we have

Axiom 6: *The contribution to the system's dynamic equations of a secondary link j carried by primary link 3 is configuration-invariant if its orientation is parallel to those of joints 1 and 2, i.e., $\mathbf{e}_j // \mathbf{z}_2 // \mathbf{z}_1$.*

3. Twist angle $\alpha_{1,0} = 0$ and ${}^3\mathbf{e}_j = \pm[0, S\alpha_{3,2}, C\alpha_{3,2}]^T$: With the condition, elements of

Table II. Design conditions

Primary link i	Design conditions
0	arbitrary $\mathbf{e}_j$
1	arbitrary $\mathbf{e}_j$
2	(i) arbitrary $\mathbf{e}_j$ if $\mathbf{z}_0//\mathbf{z}_1$
	(ii) $\mathbf{e}_j//\mathbf{z}_1$
3	(i) arbitrary $\mathbf{e}_j$ if $\mathbf{z}_0//\mathbf{z}_1//\mathbf{z}_2$
	(ii) $\mathbf{e}_j//\mathbf{z}_1//\mathbf{z}_2$
	(iii) $\mathbf{e}_j//\mathbf{z}_2$ and $\mathbf{z}_0//\mathbf{z}_1$

region 3 in $\mathbf{M}_{j/3}$ from Eqs. (19a–c), can be rewritten as

$$\mathbf{M}_{j/3})_{1,k} = \mathbf{M}_{j/3})_{2,k} = \pm I_j C\alpha_{2,1} b_{j,k} \quad (22a)$$

$$\mathbf{M}_{j/3})_{3,k} = \pm I_j b_{j,k} \quad (22b)$$

Hence, from Eq. (3), we have

Axiom 7: *The contribution to the system's dynamic equations of a secondary link j carried by primary link 3 is configuration-invariant if its orientation is parallel to that of joint 3 and joints 1 and 2 are parallel to each other, i.e.,* $\mathbf{e}_j//\mathbf{z}_2$ *and* $\mathbf{z}_0//\mathbf{z}_1$.

Table II shows the design conditions for a secondary link j carried by primary link i to have configuration-invariant contribution to the system's dynamic equations. Note that Axiom 5 is an extension of Axiom 3, and Axioms 6 and 7 are extensions of Axiom 4.

6. EXAMPLES

6.1. Redesign an Existing GCM

For the 3-DOF GCM shown in Figure 1, links 0, 1, 2, and 3 are primary links, and links 4, 5, 6, and 7 are secondary links. From the canonical graph shown in Figure 1(b), it can be seen that secondary links 4 and 5 are carried by primary link 0, secondary link 6 is carried by primary link 1 and secondary link 7 is carried by primary link 2. From Figure 1(a), it can be seen that $\mathbf{z}_0$ is perpendicular to $\mathbf{z}_1$, $\mathbf{z}_1$ is parallel to $\mathbf{z}_2$, $\mathbf{e}_4$ and $\mathbf{e}_5$ are parallel to $\mathbf{z}_0$, $\mathbf{e}_6$ is parallel to $\mathbf{z}_1$, and $\mathbf{e}_7$ is perpendicular to $\mathbf{z}_2$ in the existing design.

From Axioms 1 and 2, it can be concluded that the contribution to the system's dynamic equations due to relative motion of secondary links 4 and 5 with respect to primary link 0 and that of secondary link 6 with respect to primary link 1 is configuration-invariant. However, the contribution to the system's dynamic equations due to motion of secondary link 7 relative to primary link 2 is configuration-variant.

By making $\mathbf{z}_0$ parallel to $\mathbf{z}_1$, from Axiom 3, or $\mathbf{e}_7$ parallel to $\mathbf{z}_1$, from Axiom 4, the contribution to the system's dynamic equations due to motion of secondary link 7 relative to primary link 2 will be configuration-invariant. Figure 3 shows a possible redesign of the 3-DOF GCM shown in Figure 1 with $\mathbf{e}_7$ parallel to $\mathbf{z}_1$.

6.2. Design a New GCM

Figure 4a shows the canonical graph representation of a 4-DOF GCM. From Figure 4, it can be seen that links 0, 1, 2, 3, and 4 are primary links, and links 5, 6, 7, 8, 9, and 10 are secondary links. Secondary links 5 and 6 are carried by primary link 0, secondary link 7 is carried by primary link 1, secondary links 8 and 9 are carried by primary link 2, and secondary link 10 is carried by primary link 3.

With the given canonical graph, assume the twist angle $\alpha_{1,0}$ is equal to 90°, $\alpha_{2,1}$ and $\alpha_{3,1}$ are equal to 0°, i.e., $\mathbf{z}_0$ is perpendicular to $\mathbf{z}_1$, and $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ are parallel to each other. From Axioms 1 and 2, it is clear that the contribution to the system's dynamic equations due to motion of secondary links 5 and 6 relative to primary link 0 and that of secondary link 7 relative to primary link 1 is configuration-invariant with arbitrarily assigned orienta-

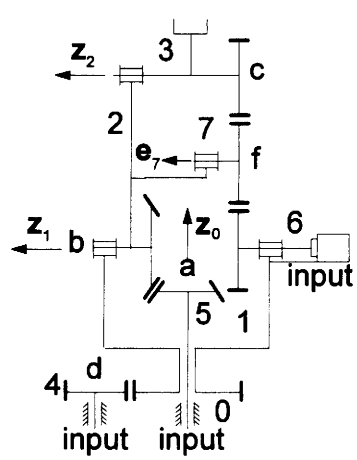

Figure 3. A modified three-dof GCM.

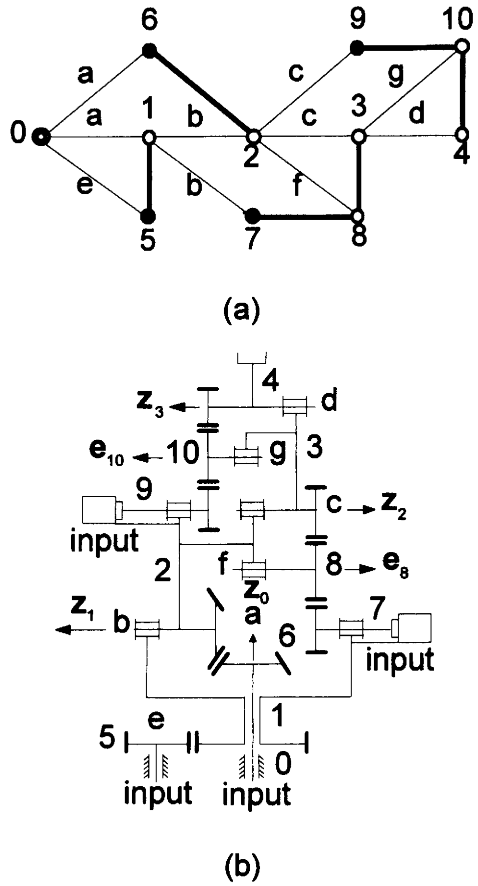

Figure 4. A 4-DOF GCM; (a) Canonical graph representation, (b) A possible functional representation.

tions of secondary links 5, 6, and 7. However, the contribution to the system's dynamic equations due to motion of secondary links 8 and 9 relative to primary link 2 and that of secondary link 10 relative to primary link 3 may be configuration-variant.

In order to make the contribution to the system's dynamic equations due to the motion of secondary links 8 and 9 relative to primary link 2 be configuration-invariant, we have the following conditions: (1) Let $\mathbf{z}_0$ be parallel to $\mathbf{z}_1$, or (2). Let $\mathbf{e}_8$ and $\mathbf{e}_9$ be parallel to $\mathbf{z}_1$. Since we have assumed to have $\mathbf{z}_0$ perpendicular to $\mathbf{z}_1$, the latter condition will be accepted as the design condition. The conditions of configuration-invariant contribution to the system's dynamic equations due to motion of secondary link 10 relative to primary link 3 are: (1) Let $\mathbf{z}_0$, $\mathbf{z}_1$, and $\mathbf{z}_2$ be parallel to each other, (2). Let $\mathbf{z}_0$ be parallel to $\mathbf{z}_1$ and $\mathbf{e}_{10}$ be parallel to $\mathbf{z}_2$, or (3). Let $\mathbf{e}_{10}$, $\mathbf{z}_1$, and $\mathbf{z}_2$ be parallel to each other. Since we have assumed to have $\mathbf{z}_0$ perpendicular to $\mathbf{z}_1$, and $\mathbf{z}_1$, $\mathbf{z}_2$ and $\mathbf{z}_3$ are parallel to each other, the latter condition is accepted as design condition. A possible functional representation of the 4-DOF GCM is shown in Figure 4(b).

7. SUMMARY

A systematic methodology for the drive train design of gear coupled manipulators is developed. It is shown that the design of a GCM can be treated as the design of a VOLC and the design of the drive train which drives the joint of the VOLC. For GCMs, the drive train is formed by the collection of secondary links including input actuators and intermediate secondary links. It is shown that the contribution of the drive train to the system's dynamic equations due to the relative motion of a secondary link with respect to a primary link is a function of the orientations of its rotational axes and those of joints. Based on configuration-invariant dynamic contribution, design conditions such as: (1) the locations of secondary links, (2) the orientations of secondary links, and (3) the compatibility between orientations of the secondary links and those of joints are derived. The approach, together with the developed methodologies for the open-loop chain manipulators, provides a complete design methodology in designing new robots or improving the dynamic performance of existing ones.

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