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Spring Configurations and Attachment Angles Determination for Statically Balanced Planar Articulated Manipulators

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Admissible spring configurations for statically balanced planar articulated manipulators have been investigated in previous studies. However, in these spring configurations, springs are only identified by the connection between links. The attachment angles and distance for springs to be properly installed remain unaddressed. In this study, a method to determine attachment angles and distance for springs is developed to ensure all the springs are acting for the benefit of static balancing. Here, the gravitational and elastic potential energies are represented in stiffness matrix form, it is shown that term by term compatibility exists between the first row of gravitational stiffness matrix and the first row of the elastic stiffness matrix. In accordance with these compatibility conditions, the admissible spring attachment angles are found to ensure all the ground-connected springs are acting for the benefit of gravity balancing. And the remained components below the first row of the elastic stiffness matrix are offset by the non-ground-connected springs. In accordance with the compatibility between the remained components and the elastic stiffness matrix of non-ground-connected springs, the spring attachment angles to ensure all the non-ground-connected springs acting for the benefit of elastic balancing are found. The determination of the admissible spring configurations is revisited in addition to the connection between links, and the attachment angles of springs are also specified. The admissible spring configurations of statically balanced planar articulated three- and four-link manipulators are derived. A four-link planar manipulator is used as an example for illustration. [DOI: 10.1115/1.4053733]

Keywords: kinematic synthesis, articulated manipulator, static balance, spring configuration, mechanism design, mechanism synthesis

1 Introduction

A static balanced mechanism has several advantages such as the improvement of control and efficiency since working against gravity is not required. Consequently, it is widely applied to various fields, such as robots and manipulators [1,2], rehabilitation devices [3,4], and wearable exoskeleton [5].

Over the years, several methods to achieve static balancing in manipulators have been proposed. The counterweight method [6,7], with direct principle, is one of the most commonly used static balancing methods as it utilized additional counterweight to balance the gravity. Balancing methods by adding springs on the manipulator can be subdivided into several types: one spring-balancing method [8,9] is through adding auxiliary parallelogram mechanism on each link of the manipulator, and the auxiliary parallelogram mechanism is always vertical to the ground link, forming a pseudobase; that is, each link can be balanced independently by one spring; another spring-balancing method [10–12] is to locate the center of mass by adding an auxiliary device to the manipulator and then attaching a spring to the centers of mass to keep the potential energy constant. Both methods [8–12] mentioned previously need an auxiliary mechanism to be attached to the manipulator, and then motion interference caused by additional linkages may occur; also, the additional mass may increase the burden of springs. In terms of the methods regarding gravity compensator for three-RPS (R, P and S denote revolute, prismatic and spherical joint) parallel robot proposed [13,14], adding springs or gear-spring modules as gravity compensator on the robot can balance the gravity partially.

The other commonly used spring-balancing methods without auxiliary parallelogram mechanisms or additional devices [15–19] are developed. Since the springs are directly attached to the manipulator, it mitigates the disadvantages caused by additional linkages of the auxiliary mechanism. To achieve perfect static balancing without adding auxiliary mechanisms, the springs are directly attached to the planar manipulator in the method used in this study.

To achieve static balancing, the summation of the potential energy should be constant

$$\sum U = \text{constant} \tag{1}$$

For simplicity, Lin [17] re-expressed the potential energy in a compatible form, in which the unit vectors of the system are separated and the potential energy is represented in a quadratic matrix form; Eq. (1) can thus be rewritten as

$$\mathbf{G} + \sum \boldsymbol{K}_{S(i,j)} = \text{constant} \tag{2}$$

where $\mathbf{G}$ denotes the matrix form of the gravitational potential energy and $\boldsymbol{K}_{S(i,j)}$ denotes the matrix form of the elastic potential energy. Assuming that only one spring is installed between each pair of links, Lin [18] proposed that by arranging the equations in potential energy matrices to satisfy Eq. (2), the admissible spring configurations (ASCs) are determined.

Unlike Lin [18], Lee [19] allowed the installation of multiple springs between a pair of links. Therefore, the number of ASCs proposed by Lee [19] was much more than that proposed by Lin [18]. ASCs with a minimum number of springs and ASCs with a minimum number of joints which springs span over on are further revealed by Lee [19].

Both Lin [18] and Lee [19] proposed ASCs. However, neither Lin [18] nor Lee [19] discussed the effect of the spring attachment angles; that is, some of the attachment angles solutions bring about a negative effect on balancing. Following the criteria in previous studies [18,19], one can only know which links are attached with springs. Yet, the springs with negative effects on balancing still may be used, and as a result, the burden of the other springs increases.

In this study, we first discuss the effect of the spring installation parameters in the matrix elements. The spring installation parameters (including spring attachment distance and spring attachment angle) are expressed in a polar coordinate system. The spring attachment angles determine the sign of the elastic stiffness matrix components contributed by the spring, and the attachment distance determines the magnitude of terms. The admissible spring attachment angles are determined by the compatibility between the corresponding stiffness matrix components and by the assumed requirement that each spring contributes the most of offset elastic stiffness matrix components. Spring installation

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rules are then proposed on the basis of the arrangement of the springs with admissible spring attachment angles. Finally, ASCs for statically balanced planar articulated manipulators are proposed, and an example is provided.

2 Potential Energy Stiffness Matrix Representation

2.1 Spring Attachment Parameters and Coordinate System of a Planar Articulated Manipulator. Figure 1 illustrates the coordinate system of an n-link planar articulated manipulator, where the Denavit–Hartenberg representation [20] is employed, and the point spring attached to is described in the polar coordinate system. The spring attachment parameters are: For spring with a stiffness $k_{S(i,j)}$ attached between links i and j, the attachment parameters include the attachment distance ($a_{S(i,j)}$ for proximal attached link i and $b_{S(i,j)}$ for distal attached link j) and attachment angles ($\alpha_{S(i,j)}$ for proximal attached link i and $\beta_{S(i,j)}$ for distal attached link j) in which the values in the counterclockwise rotation are positive.

2.2 Representation of Gravitational Potential Energy Stiffness Matrix. The gravitational potential energy U_g of an n-link planar articulated manipulator is expressed as follows:

$$U_g = \sum_{d=2}^{n} m_d g h_d \tag{3}$$

where g is the acceleration of gravity, m_d denotes the mass of link d, and $h_d h_w h_j$ denotes the height of link d from the ground

$$h_d = s_d \cos\left(\sum_{t=2}^{d} \phi - \theta_t\right) + \sum_{w=2}^{d-1} r_w \cos\left(\sum_{t=2}^{w} \phi - \theta_t\right) \quad \text{for } n \geq d > w \geq 2 \tag{4}$$

where s_d denotes the length from the joint of link d to the mass center of link d; r_w denotes the length of link w for $d > w \geq 2$; and ϕ denotes the direction of g.

Referring to Lin [17], the gravitational potential energy is then arranged in matrix form as follows:

$$U_g = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_d \\ \vdots \\ r_n \end{bmatrix}^T \mathbf{G} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_d \\ \vdots \\ r_n \end{bmatrix} \tag{5}$$

The matrix $\mathbf{G}$ is called the gravitational stiffness matrix. As those elements that lie on the diagonal in the matrix and the elements below the main diagonal are all zero, only the upper triangular matrix is considered

$$\mathbf{G} = \begin{bmatrix} * & G_{1,2} & G_{1,3} & \cdots & G_{1,n} \\ & * & 0 & 0 & 0 \\ & & * & 0 & \vdots \\ & & & * & 0 \\ & & & & * \end{bmatrix} \tag{6}$$

According to Eqs. (3)–(5), the components of the gravitational stiffness matrix are given as follows:

$$\begin{cases} G_{1,2} = -\left(m_2 \dfrac{g}{r_1}\dfrac{s_2}{r_2} + \dfrac{g}{r_1}\displaystyle\sum_{u=3}^{n} m_u\right)\cos(\phi - \theta_2) \\ G_{1,3} = -\left(m_3 \dfrac{g}{r_1}\dfrac{s_3}{r_3} + \dfrac{g}{r_1}\displaystyle\sum_{u=4}^{n} m_u\right)\cos(\phi - \theta_2 - \theta_3) \\ \vdots \\ G_{1,n} = -m_n \dfrac{g}{r_1}\dfrac{s_n}{r_n}\cos\left(\phi - \displaystyle\sum_{t=2}^{n}\theta_t\right) \end{cases} \tag{7}$$

The general formula of the gravitational stiffness matrix components is as follows:

$$G_{1,q} = -\left(m_q \frac{g}{r_1}\frac{s_q}{r_q} + \frac{g}{r_1}\sum_{w=q+1}^{n} m_w\right)\cos\left(\phi - \sum_{t=2}^{q}\theta_t\right) \quad \text{for } n \geq q \geq 2 \tag{8}$$

2.3 Representation of Elastic Potential Energy Stiffness Matrix. For a zero-free-length spring attached between links i and j (expressed as $S_{(i,j)}$), the elastic potential energy $U_{S(i,j)}$ is as

$$U_{S(i,j)} = \frac{1}{2} k_{S(i,j)} l_{(i,j)}^2 \tag{9}$$

where $k_{S(i,j)}$ is the spring stiffness of $S_{(i,j)}$; $l_{(i,j)}$ is the elongation of the spring is determined by the positions of links articulated between links i and j

$$l_{(i,j)} = \left| \vec{b}_{(i,j)} - \vec{a}_{(i,j)} + \sum_{t=i+1}^{j-1} r_t \hat{x}_t \right| \tag{10}$$

The square of the spring elongation is expressed as follows:

$$l_{(i,j)}^2 = \left(\vec{b}_{S(i,j)} - \vec{a}_{S(i,j)} + \sum_{t=i+1}^{j-1} r_t \hat{x}_t\right) \cdot \left(\vec{b}_{S(i,j)} - \vec{a}_{S(i,j)} + \sum_{t=i+1}^{j-1} r_t \hat{x}_t\right) \tag{11}$$

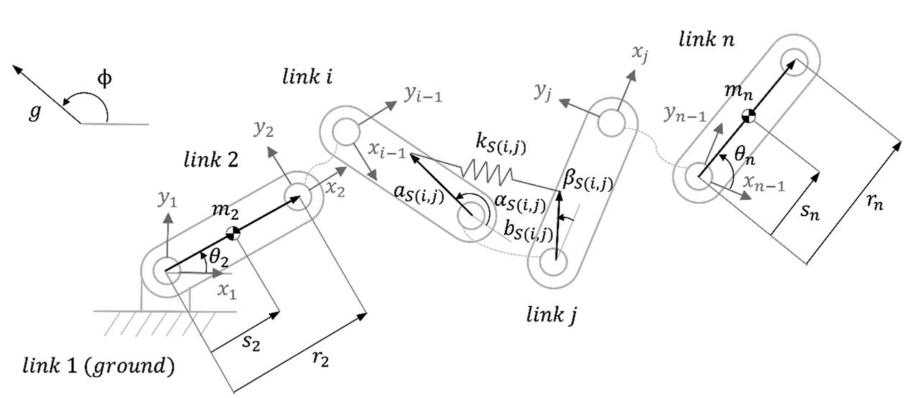

Fig. 1 Coordinate system of an n-link manipulator and spring attachment parameters

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Referring to Lin [17], the elastic potential energy Eq. (9) can also be expressed in matrix form as

$$U_{S(i,j)} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_i \\ \vdots \\ r_j \\ \vdots \\ r_n \end{bmatrix}^T \boldsymbol{K}_{S(i,j)} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_i \\ \vdots \\ r_j \\ \vdots \\ r_n \end{bmatrix} \tag{12}$$

The matrix $\boldsymbol{K}_{S(i,j)}$ is called the elastic stiffness matrix. Because the elements that lie on the diagonal in the matrix are constant. According to Eq. (2), static balancing is achieved when the summation of the matrices is a constant. Therefore, the constant elements on the diagonal can be neglected and the elements below the main diagonal are all zero, only the upper triangular matrix is considered

$$\boldsymbol{K}_{S(i,j)} = \begin{bmatrix} * & K_{i,i+1}^{S(i,j)} & K_{i,i+2}^{S(i,j)} & \cdots & K_{i,j}^{S(i,j)} \\ & * & K_{i+1,i+2}^{S(i,j)} & \cdots & K_{i+1,j}^{S(i,j)} \\ & & * & K_{u,v}^{S(i,j)} & \vdots \\ & & & * & K_{j-1,j}^{S(i,j)} \\ & & & & * \end{bmatrix} \tag{13}$$

According to Eqs. (9), (11), and (12), the components of the elastic stiffness matrix of spring $S_{(i,j)}$ are given as follows:

$$K_{i,j}^{S(i,j)} = -k_{S(i,j)} \frac{a_{S(i,j)}}{r_i} \frac{b_{S(i,j)}}{r_j} \cos\left(\alpha_{S(i,j)} - \beta_{S(i,j)} - \sum_{t=i+1}^{j} \theta_t\right) \tag{14a}$$

$$K_{i,v}^{S(i,j)} = -k_{S(i,j)} \frac{a_{S(i,j)}}{r_i} \cos\left(\alpha_{S(i,j)} - \sum_{t=i+1}^{v} \theta_t\right) \quad \text{for} \quad v < j \tag{14b}$$

$$K_{u,j}^{S(i,j)} = k_{S(i,j)} \frac{b_{S(i,j)}}{r_j} \cos\left(-\beta_{S(i,j)} - \sum_{t=u+1}^{j} \theta_t\right) \quad \text{for} \quad u > i \tag{14c}$$

$$K_{u,v}^{S(i,j)} = k_{S(i,j)} \cos\left(-\sum_{t=u+1}^{v} \theta_t\right) \quad \text{for} \quad u > i;\ v < j \tag{14d}$$

Four types of general formulas are depended on the location of elastic stiffness matrix components in the matrix. The distribution of the elastic stiffness matrix components in the matrix is shown in Fig. 2.

To achieve static balancing, Eq. (2) must be satisfied, where the elastic stiffness matrix components are arranged to offset the corresponding gravitational stiffness matrix components.

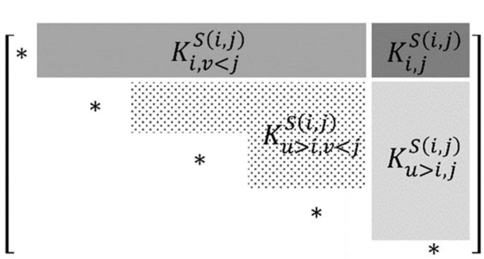

Fig. 2 Distribution of elastic stiffness matrix components

3 Springs for Static Balancing

3.1 Admissible Ground-Connected Spring Attachment Angles for Balancing Gravitational Energy. To offset the gravitational stiffness matrix components, the sign of the elastic stiffness matrix components must be opposite to the sign of the gravitational stiffness matrix components. Two assumptions are made in this study, the first assumption is that the spring attachment distances $a_{S(1,j)}$, $b_{S(1,j)}$ and spring stiffness $k_{S(1,j)}$ are positive. The second assumption is that the direction of the gravitational acceleration g is toward the ground; for the second assumption, the direction is given as

$$\phi = \frac{3\pi}{2} \tag{15}$$

According to Eqs. (14a)–(14d), it is found that the spring attachment angles $\alpha_{S(i,j)}$ and $\beta_{S(i,j)}$ determine the signs of the elastic stiffness matrix components, and the spring attachment distances $a_{S(1,j)}$, $b_{S(1,j)}$ and spring stiffness $k_{S(1,j)}$ determine the magnitude of the elastic stiffness matrix components.

According to Eq. (6), the gravitational stiffness matrix components are distributed in the first row of the matrix, and according to Eq. (13), the corresponding elastic stiffness matrix components distributed in the first row must be $K_{1,j}^{S(1,j)}$ and $K_{i,v}^{S(i,j)}$ for $v < j$ (i.e., Eqs. (14a) and (14a) with $i = 1$), which must be contributed by a ground-connected spring $S_{(1,j)}$.

Therefore, according to Eqs. (8), (14a), and (15), if the sign of components are opposite, in other words, the elastic stiffness matrix component $K_{1,j}^{S(1,j)}$ is proposed to offset the gravitational stiffness matrix components when

$$\alpha_{S(1,j)} - \beta_{S(1,j)} = \frac{\pi}{2} \tag{16}$$

and according to Eqs. (8), (14b) and (15) the elastic stiffness matrix component $K_{i,v}^{S(i,j)}$ is proposed to offset the gravitational stiffness matrix components when

$$\alpha_{S(1,j)} = \frac{\pi}{2} \tag{17}$$

For a ground-connected spring $S_{(1,j)}$, if both the elastic stiffness matrix components $K_{1,j}^{S(1,j)}$ and $K_{i,v}^{S(i,j)}$ are proposed to simultaneously offset the gravitational stiffness matrix components, substitute Eqs. (17) into (16), and $\beta_{S(1,j)}$ is solved; the spring attachment angles of the ground-connected spring are then determined as follows:

$$(\alpha_{S(1,j)}, \beta_{S(1,j)}) = \left(\frac{\pi}{2}, 0\right) \tag{18}$$

A spring installed between a pair of adjacent links is called a mono-articulated spring, and a spring that spans over multiple articulated joints is called a multi-articulated spring. A mono-articulated spring contributes only one elastic stiffness matrix component $K_{1,2}^{S(1,2)}$. Therefore, for the mono-articulated ground-connected spring $S_{(1,2)}$, Eq. (16) is the only constraint for the spring attachment angle. The spring attachment angles of the mono-articulated ground-connected spring $S_{(1,2)}$ satisfy

$$\alpha_{S(1,2)} - \beta_{S(1,2)} = \frac{\pi}{2} \tag{19}$$

The spring attachment angles of the ground-connected springs are then determined. The system of ground-connected springs attachment angles is as follows.

$S1$: To ensure that most components offset the gravitational stiffness matrix components, the attachment angles of a multi-articulated ground-connected spring $S_{(1,j)}$ for $j > 2$ are $(\pi/2, 0)$. And the attachment angles of the mono-articulated ground-connected spring $S_{(1,2)}$ satisfy $\alpha_{S(1,2)} - \beta_{S(1,2)} = \pi/2$.

According to Fig. 2, the elastic stiffness matrix components of a ground-connected spring are not only distributed in the first row of the matrix. The excess elastic stiffness matrix components below

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the first row are also remained by the ground-connected springs. Therefore, non-ground-connected springs must be installed to offset the elastic stiffness matrix components that remained by ground-connected springs.

3.2 Excess Elastic Stiffness Matrix Components Remained by Ground-Connected Springs. Two types of excess elastic stiffness matrix components are remained by ground-connected springs: $K_{u,j}^{S(i,j)}$ for $u>2$ and $K_{u,v}^{S(i,j)}$ for $u>2$; $v<j$. Substituting the spring attachment angles in Eqs. (18) into (14*c*) and (14*d*), the excess elastic stiffness matrix components are given as

$$K_{u,j}^{S(1,j)} = k_{S(1,j)} \frac{b_{S(1,j)}}{r_j} \cos\left(-\sum_{t=u+1}^{j} \theta_t\right) \quad \text{for } u>2 \tag{20a}$$

and

$$K_{u,v}^{S(1,j)} = k_{S(1,j)} \cos\left(-\sum_{t=u+1}^{v} \theta_t\right) \quad \text{for } u>2;\ v<j \tag{20b}$$

To satisfy Eq. (2), non-ground-connected springs should be installed to offset the excess elastic stiffness matrix components.

3.3 Admissible Non-ground-Connected Spring Attachment Angles for Balancing Excess Elastic Energy. According to Eqs. (14*a*)–(14*d*) and Fig. 2, for non-ground-connected springs, up to four types of elastic stiffness matrix components exist: $K_{i,j}^{S(i,j)} K_{i,v<j,}^{S(i,j)}\ K_{u>i,j}^{S(i,j)}$ and $K_{u>i,v<j}^{S(i,j)}$; they correspond to the excess elastic stiffness matrix components.

To offset the excess elastic stiffness matrix components that the ground-connected spring remained, the sign of the elastic stiffness matrix components of the non-ground-connected spring must be opposite to the sign of the excess elastic stiffness matrix components.

Comparing Eq. (14*a*) with Eqs. (20*a*) and (20*b*), the elastic stiffness matrix component $K_{i,j}^{S(i,j)}$ of a non-ground-connected spring is proposed to offset the excess elastic stiffness matrix components when

$$\alpha_{S(i,j)} - \beta_{S(i,j)} = 0 \tag{21}$$

Similarly, comparing Eqs. (14*b*) and (14*c*) with Eqs. (20*a*) and (20*b*), the elastic stiffness matrix components $K_{i,v}^{S(i,j)}$ and $K_{u,j}^{S(i,j)}$ are individually proposed to offset the excess elastic stiffness matrix components when

$$\alpha_{S(i,j)} = 0 \tag{22}$$

and

$$\beta_{S(i,j)} = \pi \tag{23}$$

The sign of $K_{u,v}^{S(i,j)}$ is always identical to the sign of the excess elastic stiffness matrix components. In other words, the elastic stiffness matrix component $K_{u,v}^{S(i,j)}$ of a non-ground-connected spring cannot offset the excess elastic stiffness matrix components.

If the elastic stiffness matrix components of a non-ground-connected spring $K_{i,j}^{S(i,j)}$, $K_{i,v}^{S(i,j)}$, and $K_{u,j}^{S(i,j)}$ are all proposed to offset the excess elastic stiffness matrix components that the ground-connected spring remained, according to Eqs. (21)–(23), the solution is contradictory. A non-ground-connected spring with up to two types of elastic stiffness matrix components to balance the excess elastic stiffness matrix components is ideal. If $K_{i,j}^{S(i,j)}$ and $K_{i,v}^{S(i,j)}$ are offset elastic stiffness matrix components, according to Eqs. (21) and (22), the spring attachment angles satisfy

$$(\alpha_{S(i,j)}, \beta_{S(i,j)}) = (0, 0) \tag{24}$$

If $K_{i,j}^{S(i,j)}$ and $K_{u,j}^{S(i,j)}$ are offset elastic stiffness matrix components, according to Eqs. (21) and (23), the spring attachment angles satisfy

$$(\alpha_{S(i,j)}, \beta_{S(i,j)}) = (\pi, \pi) \tag{25}$$

If $K_{i,v}^{S(i,j)}$ and $K_{u,j}^{S(i,j)}$ are offset elastic stiffness matrix components, according to Eqs. (22) and (23), the spring attachment angles satisfy

$$(\alpha_{S(i,j)}, \beta_{S(i,j)}) = (0, \pi) \tag{26}$$

For a mono-articulated non-ground-connected spring $S_{(i,i+1)}$, which contributes only one elastic stiffness matrix component $K_{i,i+1}^{S(i,i+1)}$, Eq. (21) is the only constraint when the $K_{i,i+1}^{S(i,i+1)}$ is an offset elastic stiffness matrix component. The spring attachment angles of the mono-articulated non-ground-connected spring satisfy

$$\alpha_{S(i,i+1)} - \beta_{S(i,i+1)} = 0 \tag{27}$$

The systems of non-ground-connected springs attachment angles are as follows:

*S*2: To ensure that the most of the components offset the excess elastic stiffness matrix components that remained by ground-connected springs, the attachment angles of a multi-articulated non-ground-connected spring $S_{(i,j)}$ for $i>2$; $j-i>1$ must be (0, 0), (π, π), or (0, π). And the attachment angles of a mono-articulated non-ground-connected spring $S_{(i,i+1)}$ need to satisfy $\alpha_{S(i,i+1)} - \beta_{S(i,i+1)} = 0$.

4 Admissible Spring Configurations With Spring Attachment Angles

4.1 Installation Rules for Ground-Connected Spring. To achieve static balancing, each nonzero gravitational stiffness matrix component must correspond to an elastic stiffness matrix component, where the gravitational stiffness matrix component $G_{1,j}$ can be offset by the elastic stiffness matrix component $K_{1,j}^{S(1,j)}$ contributed by the spring $S_{(1,j)}$ or the elastic stiffness matrix component $K_{1,j}^{S(1,j+1)}$ contributed by the spring $S_{(1,j+1)}$. The general rules for installing ground-connected springs for an n-link articulated manipulator are as follows:

*R*1–1: To offset $G_{1,n}$, at least one ground-connected spring $S_{(1,n)}$ with (π/2, 0) must be installed.

*R*1–2: To offset $G_{1,j}$ for $n>j>2$, at least one ground-connected spring $S_{(1,j)}$ or $S_{(1,j+1)}$ with (π/2, 0) must be installed.

According to the system of a mono-articulated ground-connected spring (*S*1), the rule for balancing $G_{1,2}$, which can be balanced by a mono-articulated ground-connected spring, is as follows:

*R*2: To offset $G_{1,2}$, at least one multi-articulated ground-connected spring $S_{(1,3)}$ with (π/2, 0) or a mono-articulated ground-connected spring $S_{(1,2)}$ with attachment angles that satisfy $\alpha_{S(1,2)} - \beta_{S(1,2)} = \pi/2$ must be installed.

4.2 Installation Rules for Non-ground-Connected Spring. The excess elastic stiffness matrix components that remained by a ground-connected spring $S_{(1,j)}$ are $K_{u,j}^{S(1,j)}$ for $u>2$ and $K_{u,v}^{S(1,j)}$ for $u>2$ and $v<j$. Referring to the distribution of elastic stiffness matrix components in Fig. 2, we note that the elastic stiffness matrix components in column j differ from those in column v. Thus, the excess elastic stiffness matrix components that remained by all ground-connected springs differ from column to column; however, in the same column, the excess elastic stiffness matrix components are the same.

Referring to *S*2, the non-ground-connected spring has spring attachment angles (0, 0), (π, π), and (0, π), the non-ground-connected spring $S_{(i,j)}$ with spring attachment angles (0, 0) can provide elastic stiffness matrix components, $K_{i,j}^{S(i,j)}$ and $K_{i,j-1}^{S(i,j)}$, to fully offset the excess elastic stiffness matrix components at the corresponding location in the matrix.

For the same reason, the non-ground-connected spring $S_{(i,j)}$ with spring attachment angles (π, π) can provide elastic stiffness matrix components, $K_{i,j}^{S(i,j)}$ and $K_{u,j}^{S(i,j)}$ for $u>i$; the non-ground-connected

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Table 1 ASCs of three- and four-link manipulators with minimum number of springs

N	Spring configuration matrices with minimum number of springs
3	$\begin{bmatrix} * & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} \\ & & * \end{bmatrix}^{*\mathrm{Lee}}$
4	$\begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(\pi,\pi)} \\ & & * & 0 \\ & & & * \end{bmatrix}^{*\mathrm{Lee}} \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,0)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix}^{*\mathrm{Lee}} \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,\pi)}+1^{\frac{(0,0)}{(\pi,\pi)}} \\ & & * & 0 \\ & & & * \end{bmatrix}$ $\begin{bmatrix} * & 0 & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(\pi,\pi)} \\ & & * & 0 \\ & & & * \end{bmatrix} \begin{bmatrix} * & 0 & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,0)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix} \begin{bmatrix} * & 0 & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 0 \\ & & & * \end{bmatrix}$

spring $S_{(i,j)}$ with spring attachment angles $(0, \pi)$ can provide elastic stiffness matrix components, $K_{i,j-1}^{S(i,j)}$ and $K_{u,j}^{S(i,j)}$ for $u > i$, which can fully offset the excess elastic stiffness matrix components at the corresponding location in the matrix.

Each excess elastic stiffness matrix component must correspond to an offsetting elastic stiffness matrix component contributed by non-ground-connected springs. The general rules for installing non-ground-connected springs are as follows:

Table 2 ASCs of three- and four-link manipulators with nonminimum number of springs

N	Spring configuration matrices with nonminimum number of springs
3	$\begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} \\ & & * \end{bmatrix}$
4	$\begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 0 \\ & & & * \end{bmatrix} \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix}$ $\begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix} \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix}$ $\begin{bmatrix} * & 0 & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 0 \\ & & & * \end{bmatrix} \begin{bmatrix} * & 0 & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix} \begin{bmatrix} * & 0 & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix}$ $\begin{bmatrix} * & 0 & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix} \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(\pi,\pi)} \\ & & * & 0 \\ & & & * \end{bmatrix} \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,0)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix}$ $\begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix} \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 0 \\ & & & * \end{bmatrix}$ $\begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 0 \\ & & & * \end{bmatrix} \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix}$ $\begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 1^{(\frac{\pi}{2},0)} & 1^{(\frac{\pi}{2},0)} \\ & * & 1^{\alpha-\beta=0} & 1^{(0,\pi)}+1^{(0,0)/(\pi,\pi)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix}$

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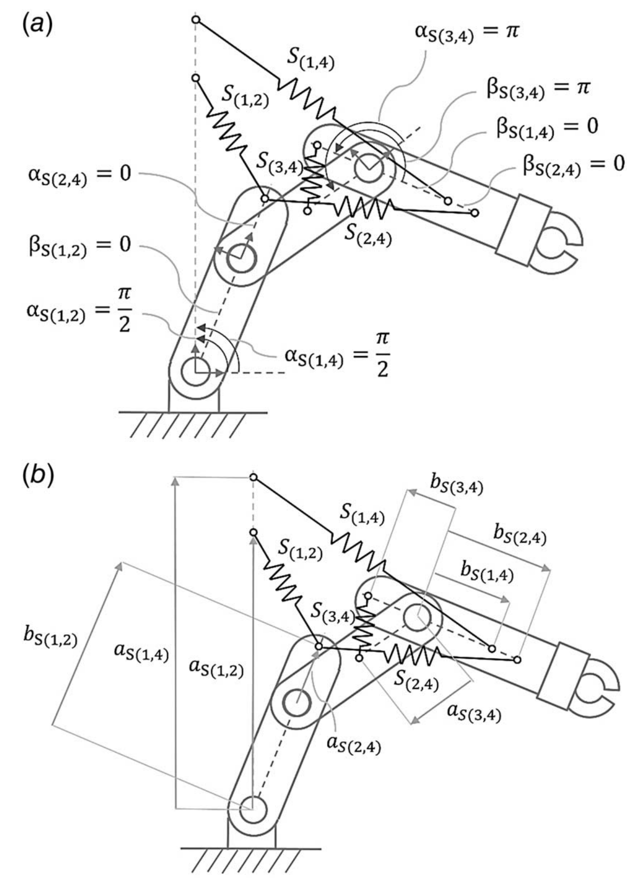

Fig. 3 Spring installation of a statically balanced four-link planar articulated manipulator: (*a*) spring attachment angles and (*b*) spring attachment distances

*R*3-1: To offset the excess components located at (2, n), at least one non-ground-connected spring $S_{(2,n)}$ with attachment angles (0, 0) or (π, π) should be installed.

*R*3-2: To offset the excess components located at (2, j) for $n > j > 2$, at least one non-ground-connected spring $S_{(2,j)}$ with attachment angles (0, 0) or (π, π) spring $S_{(2,j+1)}$ with attachment angles (0, 0) or (0, π) must be installed.

*R*3-3: To offset the excess components located at (i, n) for $i > 2$, at least one non-ground-connected spring $S_{(i,n)}$ with attachment angles (0, 0) or (π, π) or spring $S_{(u,n)}$ for $i > u > 1$ with attachment angles (π, π) or (0, π) must be installed.

*R*3-4: To offset the excess components located at (i, j) for $i > 2$;$j < n$, at least one non-ground-connected spring $S_{(i,j)}$ with attachment angles (0, 0) or (π, π), spring $S_{(i,j+1)}$ with attachment angles (0, 0) or (0, π), or spring $S_{(u,j)}$ for $u < i$ with attachment angles (π, π) or (0, π) must be installed.

According to the system of a mono-articulated non-ground-connected spring (S2), the installation rules for a mono-articulated non-ground-connected spring $S_{(i,i+1)}$ are as follows:

*R*4-1: To offset the excess components located at (2, 3), at least one multi-articulated non-ground-connected spring $S_{(2,4)}$ with attachment angles (0, 0) or (0, π) or one mono-articulated non-ground-connected spring $S_{(2,3)}$ with attachment angles satisfying $\alpha_{S(2,3)} - \beta_{S(2,3)} = 0$ must be installed.

*R*4-2: To offset the excess components located at ($n-1$, n), at least one multi-articulated non-ground-connected spring $S_{(u,n)}$ for $n > u > 1$ with attachment angles (π, π) or (0, π) or one mono-articulated non-ground-connected spring $S_{(n-1,n)}$ with attachment angles satisfying $\alpha_{S(n-1,n)} - \beta_{S(n-1,n)} = 0$ must be installed.

*R*4-3: To offset the excess components located at (i, $i+1$) for $n > i > 2$, at least one multi-articulated non-ground-connected spring $S_{(i,i+2)}$ with attachment angles (0, 0) or (0, π), spring $S_{(u,i+1)}$ for $i > u > 1$ with attachment angles (π, π) or (0, π), or one mono-articulated non-ground-connected spring $S_{(i,i+1)}$ with attachment angles satisfying $\alpha_{S(i,i+1)} - \beta_{S(i,i+1)} = 0$ must be installed.

4.3 Admissible Spring Configurations. Referring to the rules mentioned previously, we arrange the springs with admissible spring attachment angles determined in Sec. 3; consequently, the statically balanced spring configurations are determined.

The ASCs with a minimum number of springs of three- and four-link manipulators are listed in Table 1, and ASCs with a nonminimum number of springs are listed in Table 2. The spring configurations proposed by Lee [19] are indicated using superscripts. Compared to the ASCs in this paper with the spring configurations proposed by Lin and Lee [18,19], besides knowing which links are attached with spring; here, the admissible attachment angles of each spring are marked.

4.4 A Four-Link Planar Articulated Manipulator Illustrative Example. Take a four-link planar articulated manipulator as an example. Figure 3 illustrates the spring installation for a statically balanced four-link planar articulated manipulator. The mass and dimensions are tabulated in Table 3, and a spring configuration matrix in Table 1 is taken for further illustration, where

$$\Lambda = \begin{bmatrix} * & 1^{\alpha-\beta=\frac{\pi}{2}} & 0 & 1^{(\frac{\pi}{2},0)} \\ & * & 0 & 1^{(0,0)} \\ & & * & 1^{\alpha-\beta=0} \\ & & & * \end{bmatrix} \tag{28}$$

According to *R*2 and *R*4-1, the attachment angles of the mono-articulated springs $S_{(1,2)}$ and $S_{(3,4)}$ are set as follows:

$$(\alpha_{S(1,2)}, \beta_{S(1,2)}) = \left(\frac{\pi}{2}, 0\right) \tag{29}$$

$$(\alpha_{S(3,4)}, \beta_{S(3,4)}) = (\pi, \pi) \tag{30}$$

According to Eq. (2), the summations of the gravitational and elastic stiffness matrices components are zero. According to Eqs. (8) and (14*a*), the equation for components locating at (1,4) is as

Table 3 Mass and dimensions of the example manipulator

j	m_j(kg)	r_j(m)	s_4(m)
1	–	1.000	–
2	0.400	0.300	0.150
3	0.320	0.240	0.120
4	0.350	0.360	0.180

Table 4 Spring stiffness and attachment distances of the example manipulator

$S_{(i,j)}$	$k_{S(i,j)}$(N/m)	$a_{S(i,j)}$(m)	$b_{S(i,j)}$(m)
$S_{(1,4)}$	50	0.100	0.124
$S_{(1,2)}$	50	0.053	0.400
$S_{(2,4)}$	500	0.030	0.124
$S_{(3,4)}$	800	0.200	0.102

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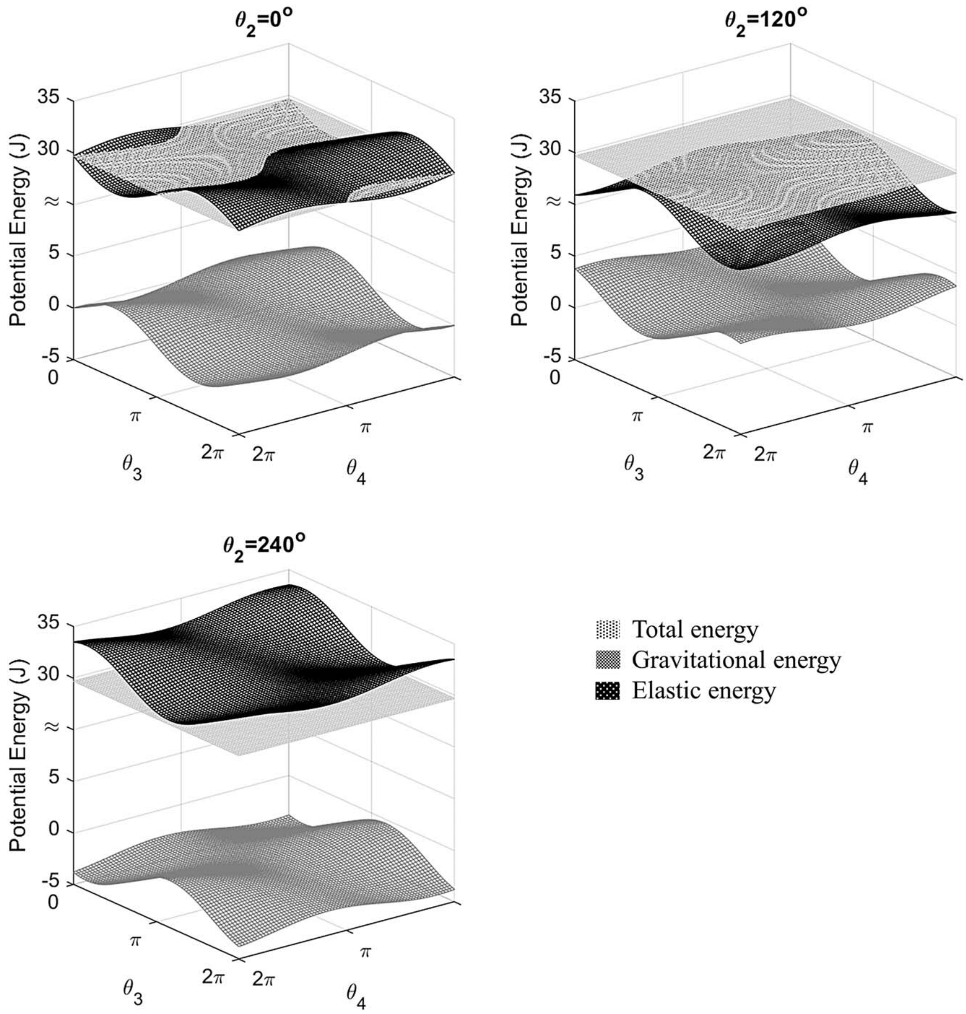

Fig. 4 The gravitational energy, elastic energy, and static balancing ability of a statically balanced four-link planar articulated manipulator in the whole workspace

follows:

$$-\left(m_4 \frac{g}{r_1}\frac{s_4}{r_4}\right)\cos\left(\frac{3\pi}{2}-\sum_{t=2}^{q}\theta_t\right) - k_{S(1,4)}\frac{a_{S(1,4)}}{r_1}\frac{b_{S(1,4)}}{r_4}\cos\left(\alpha_{S(1,4)}-\beta_{S(1,4)}-\sum_{t=2}^{q}\theta_t\right)=0 \quad (31)$$

Bring the predetermined spring attachment angles ($\alpha_{S(1,4)}, \beta_{S(1,4)}$) $=(\pi/2, 0)$ into Eqs. (31) and (32) can be simplified as

$$k_{S(1,4)}\frac{a_{S(1,4)}}{r_1}\frac{b_{S(1,4)}}{r_4}=m_4\frac{g}{r_1}\frac{s_4}{r_4} \quad (32a)$$

Likewise, to bring the predetermined spring attachment angles into the components of elastic stiffness matrices, the equations for components which locate in (1,3), (1,2), (2,4). (2,3) and (3,4) in matrices are listed as follows:

$$k_{S(1,4)}\frac{a_{S(1,4)}}{r_1}=m_3\frac{g}{r_1}\frac{s_3}{r_3}+m_4\frac{g}{r_1} \quad (32b)$$

$$k_{S(1,4)}\frac{a_{S(1,4)}}{r_1}+k_{S(1,2)}\frac{a_{S(1,2)}}{r_1}\frac{b_{S(1,2)}}{r_2}=m_2\frac{g}{r_1}\frac{s_2}{r_2}+m_3\frac{g}{r_1}+m_4\frac{g}{r_1} \quad (32c)$$

$$k_{S(2,4)}\frac{a_{S(2,4)}}{r_2}\frac{b_{S(2,4)}}{r_4}=k_{S(1,4)}\frac{b_{S(1,4)}}{r_4} \quad (32d)$$

$$k_{S(2,4)}\frac{a_{S(2,4)}}{r_2}=k_{S(1,4)} \quad (32e)$$

$$k_{S(3,4)}\frac{a_{S(3,4)}}{r_3}\frac{b_{S(3,4)}}{r_4}=k_{S(1,4)}\frac{b_{S(1,4)}}{r_4}+k_{S(2,4)}\frac{b_{S(2,4)}}{r_4} \quad (32f)$$

Equations (32a)–(32f) can be used to solve the spring stiffness and the attachment distances of all springs; the suitable spring stiffness and attachment distances are listed in Table 4.

Figure 4 presents the gravitational energy, elastic energy, and static balancing ability of a statically balanced four-link planar articulated manipulator in the whole workspace (note that, to present the figure clearly, several θ_2 angles are used to represent the potential energies of the manipulator in the whole workspace).

5 Conclusion

In this study, a method for determining the ASCs for statically balanced planar articulated manipulators was proposed. The potential energy was expressed in the form of a stiffness matrix. We focused on the effect of each spring and discussed the relationship between elastic stiffness matrix components and the components being offset. Also, the compatibility between the two and how they had an impact on the spring attachment angles are discussed. Then, the springs with admissible spring attachment angles which are acting for the benefit of static balancing were proposed. By

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arranging the elastic stiffness matrix components that correspond to the gravitational stiffness matrix components, we proposed rules for ASCs for statically balanced planar articulated manipulators; the ASCs for three- and four-link manipulators were proposed as well. Finally, the spring installation for a four-link statically balanced manipulator was presented as an example.

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