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Research paper

A novel spring gravity-balance method for spatial articulated manipulators without auxiliary links

Chia-Wei Juang, Chi-Shiun Jhuang, Dar-Zen Chen *

Dept. of Mechanical Engineering, National Taiwan University, Taipei, 10617, Taiwan

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ABSTRACT

Previous studies developed spring-gravity balancing methods for articulated manipulators using auxiliary links. However, these links caused extra inertia and motion interference. To address these issues, a spring balancing method based on the quadratic form was proposed. In this form, matrix components represent the energy change due to the manipulator's posture, i.e., stiffnesses. Gravity balancing can be simplified as the summation of matrices that remain unchanged. However, the matrix component contains the polar angles, which can only describe the links' direction on a plane, thus making this form applicable only to planar manipulators. To extend its applicability to spatial manipulators, we have reformulated it using local coordinates to describe the manipulator's posture in space. The improved quadratic form can be applied to both planar and spatial manipulators, unifying energy representations of articulated manipulators. By arranging the springs to maintain a constant summation of matrices, energy balance is achieved. The criteria of spring attachment and the rules of using springs are proposed. Simulation of a spatial four-link manipulator show perfect balance can be achieved without auxiliary links using our approach.

1. Introduction

The technology of gravity compensation for physical mechanisms has been widely used for decades. It offers several advantages that have been discussed in the literature, such as decreasing the loading of actuators and improving machine efficiency.

Many of the methods use counterweights or elastic elements such as springs to compensate for the gravitational force. For example, Woo [1] proposed a gravity balanced surgical manipulator that uses counterweights and springs. Agrawal [2] developed a method that adds auxiliary links to the manipulator to form parallelogram mechanisms that can identify the center-of-mass (COM) of the manipulators and the manipulator's center of gravity can then be perfectly balanced by the attachment of springs. Also using an auxiliary parallelogram mechanism with the manipulator but instead of identifying the COM, Nathan [3] and Rahman [4] implemented it so as to form a "pseudo-base" that is always parallel to the ground-link; and with these pseudo-bases, the springs can individually balance each link. Nguyen [5] developed a method that uses gear-spring modules to compensate for the gravitational force of a planar manipulator. Jamshidifar [6] used a device composed of cables and pulleys and attached them to links, compensating for gravity using balancing elements such as a counterweight or spring. Li [7] developed an active and passive combined gravity compensation approach for a 6-DOF hybrid force feedback device. Kuo [8] proposed a balancing method using springs and cardan-gear mechanisms. Yet these methods require the use of counterweights, auxiliary links or additional devices, which brings other defects such as extra

* Corresponding author: Dar-Zen Chen, Dept. of Mechanical Engineering, National Taiwan University, Taipei 10617, Taiwan.
E-mail addresses: f06522638@ntu.edu.tw (C.-W. Juang), d06522028@ntu.edu.tw (C.-S. Jhuang), dzchen@ntu.edu.tw (D.-Z. Chen).

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inertia, motion interference and a reduced workspace. To fill in some of the gaps in the literature, Lee and Juang [9,10] developed a spring gravity balance method based on a quadratic form of energy representation, in which multiple springs can be directly attached to the manipulator, thereby systematically balancing the gravitational forces.

The above methods can in fact perfectly balance the gravitational forces. However, they can only be applied on planar manipulators. And for those balance methods on spatial manipulators, extra auxiliary links or devices are needed. For example, applying a similar concept as [2], Agrawal [11] further used an auxiliary parallelogram mechanism to identify a system's COM and then used springs to achieve gravity balancing (as shown in Fig. 1. (a)), thereby expanding the usage of auxiliary parallelogram mechanisms to the balance of spatial manipulators. Lin [12] introduced an approach that extended the concept of the pseudo-base, as proposed by Nathan [3] and Rahman [4], to spatial manipulators. This method involves utilizing a spatial parallelogram revolute-spherical-spherical-revolute (RSSR) mechanism to create a pseudo-base (as depicted in Fig. 1 (b)). The pseudo-base is designed to follow the movement of each link and maintain perpendicularity to the ground. By attaching springs between the pseudo-bases and the links, it becomes possible to achieve individual balance for each link. This means that the manipulator can be balanced by attaching pseudo-bases and springs to all of its links.

Several studies have applied the gravity balance concept to a wide variety of fields. For example, Wang [13] proposed the gravity compensation of a spatial parallel mechanism by using spring and counterweight; and Nguyen [14,15] balanced it by using spring-gear module. Banala [16] and Zhou [17] proposed a gravity balanced exoskeleton for a lower-limb, thereby decreasing user muscle work during walking. The similar concept was used on an upper-limb exoskeleton developed by Peng [18]. Several papers [19–22] studied the application of gravity compensation on human limb rehabilitation. For industrial applications, Alabdulkarim [23] developed an exoskeleton that balanced the weight of heavy tools using springs and counterweights. By using the gravity-balanced exoskeleton, workers are able to reduce the risk of being injured. Aldanmaz [24] developed a gravity balanced 2R1T (two rotations and one translation) mechanism composed of springs and counterweights and used it for surgical applications. Jin [25] proposed a gravity compensated haptic device for surgical robotics. In short, the technology of gravity compensation can be widely used and Fig. 2 shows several of its applications.

Although Agrawal [11] and Lin [12] have proposed gravity balancing methods for spatial articulated manipulators, it should be noted that the inclusion of additional devices or auxiliary links can introduce additional drawbacks such as increased inertia in the mechanism. In contrast, our research focuses on developing a novel spring-gravity balance method that explicitly avoids the use of auxiliary devices and instead directly attaches springs to the spatial articulated manipulator. Furthermore, our method differs from Lin's work [12], in that we do not balance each link individually. Instead, we employ a systematic approach to balance the entire spatial articulated manipulator using multiple springs. Our method is based on energy representation in quadratic form. The quadratic form proposed by [9,10] was originally applicable only to planar manipulators, as it was represented in terms of cosine with a polar angle. However, we have successfully extended the quadratic form to spatial articulated manipulators. This extension involved replacing the column vectors in the original form with unit vectors representing the local coordinates of each link in space. By doing so, we can accurately describe the positions of the links in three-dimensional space, making the quadratic form applicable to spatial manipulators for the first time. The extended quadratic form can also be applied to planar manipulators, as planar articulated manipulators can be seen as a special case of spatial articulated manipulators. This unifies the quadratic forms of both planar and spatial manipulators. The manipulators generally connected in series with spherical joints are considered in this paper. The extended quadratic form shown that the ground-connected springs cancel out the gravity. At the same time, they contribute part of the redundant energy. Therefore, non-ground-connected springs are used to balance this. The energy of the springs is a function of their parameters (including spring stiffness and the location at which they are attached). They determine whether the system is balanced or not. The criteria of spring attachment and rules for using springs are also proposed. By following them, the gravitational force of an arbitrary n-link spatial articulated manipulator can be perfectly balanced.

The structure of this paper is as follows: first, the model of an arbitrary spatial articulated manipulator is built. Based on it, the gravitational energy of the manipulator and the elastic energy of the springs are derived. The energies are represented in quadratic form by a matrix representation, which shows the relationship between the energy and the posture of the manipulator clearly. Then, by comparing the gravitational energy matrix with the elastic energy matrix, the balancing conditions are determined. Following these conditions, the criteria of spring attachment and the rules for spring application are proposed, and the acceptable spring configurations to achieve gravity balance for three- and four-link spatial manipulators are listed. For demonstration purposes, a simulation of a spring-

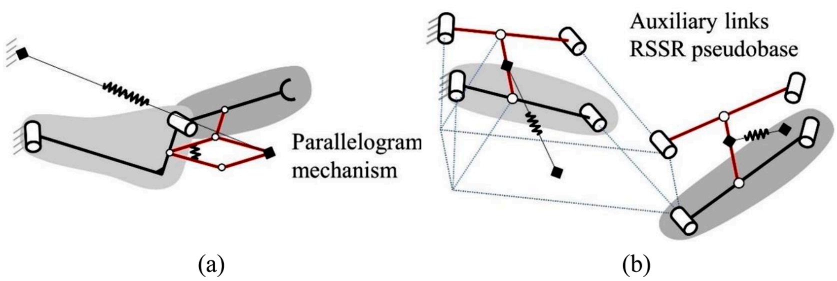

Fig. 1. Gravity balance of spatial articulated manipulators: (a) auxiliary parallelogram mechanism [11] and (b) pseudo-base formed by the RSSR auxiliary mechanism [12].

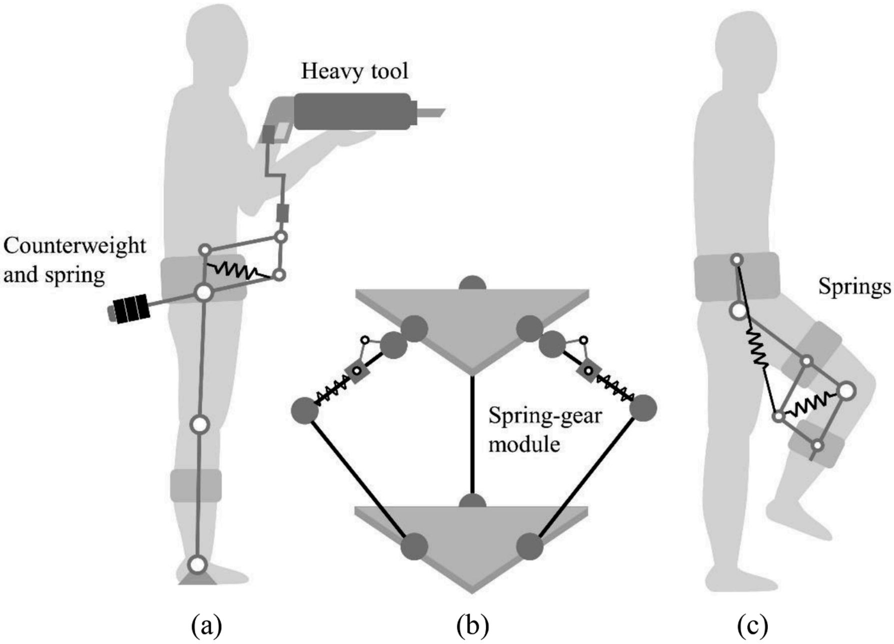

Fig. 2. Applications of gravity compensation technology: (a) industrial exoskeleton [23]; (b) spatial parallel mechanisms with spring-gear modules [14]; (c) spring balanced lower-limb exoskeleton [16].

balanced four-link spatial manipulator with springs is given as an example at the end of the paper, which verifies that the approach described in this study can balance the gravitational forces perfectly by attaching springs directly to the manipulator without additional links or auxiliary devices.

2. Quadratic form of articulated manipulators

2.1. Quadratic form of the planar articulated manipulators

The past studies [9,10] have developed the spring balancing method that directly attached the springs to the manipulator without auxiliary devices. The method is based on the quadratic form of energy representation. The quadratic form is composed of column matrix and stiffness matrix. The column matrix describes the links' distances, and the represents the energy change due to the manipulator's position, i.e., stiffnesses. The stiffness matrix was used to be applied on spring system analysis. [9,10] have applied it to descript the gravitational energy, thereby expressed the gravitational energy and the springs' energy in a compatible form. Gravity balancing can be simplified as the summation of matrices that remain unchanged.

An example is given here to illustrate the gravitational energy representation in quadratic form. A planar articulated 4-link

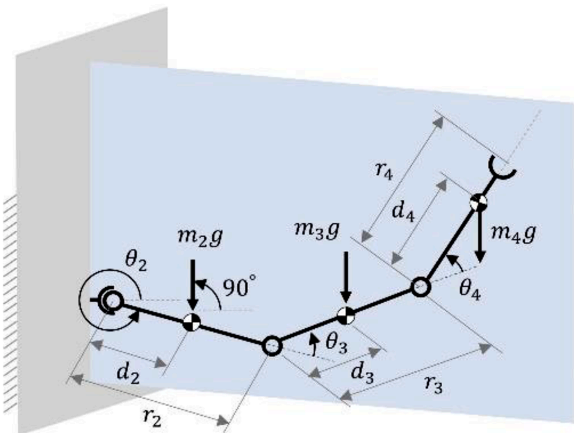

Fig. 3. An example planar articulated 4-link manipulator.

manipulator is shown in Fig. 3. Where g is the gravitational acceleration. r_j is the length of link j and m_j is the mass of link j. The center of mass (COM) of a link was assumed to be on the connecting line of the link's proximal joint and the distal joint. d_j is the distance between the proximal joint and the COM of link j. θ_j is the rotation angle on the plane of link j.

The gravitational energy of the example manipulator is

$$U_g = m_2 g h_2 + m_3 g h_3 + m_4 g h_4 \tag{1}$$

Where h_2, h_3 and h_4 are the altitudes of the links 2, 3 and 4 respectively.

$$h_2 = d_2 \sin(\theta_2) \tag{2a}$$

$$h_3 = r_2 \sin(\theta_2) + d_3 \sin(\theta_2 + \theta_3) \tag{2b}$$

$$h_4 = r_2 \sin(\theta_2) + r_3 \sin(\theta_2 + \theta_3) + d_4 \sin(\theta_2 + \theta_3 + \theta_4) \tag{2c}$$

The gravitational energy of the manipulator can be written as

$$\begin{aligned} U_g &= g(d_2 m_2 + r_2(m_3 + m_4))\sin(\theta_2) \\ &+ g(d_3 m_3 + r_3 m_4)\sin(\theta_2 + \theta_3) \\ &+ g d_4 m_4 \sin(\theta_2 + \theta_3 + \theta_4) \end{aligned} \tag{3}$$

which is rewritten as,

$$\begin{aligned} U_g &= r_1 \left[\left(\frac{m_2 g d_2}{r_1 r_2} + \frac{(m_3 + m_4) g}{r_1} \right) \sin(\theta_2) \right] r_2 \\ &+ r_1 \left[\left(\frac{m_3 g d_3}{r_1 r_3} + \frac{m_4 g}{r_1} \right) \sin(\theta_2 + \theta_3) \right] r_2 \\ &+ r_1 \left[\frac{m_4 g d_4}{r_1 r_4} \sin(\theta_2 + \theta_3 + \theta_4) \right] r_4 \end{aligned} \tag{4}$$

And according to [9,10] it can be rearranged to quadratic form in matrix representation as follows

$$U_g = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix}^T \mathbf{G}^p \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} \tag{5}$$

where

$$\mathbf{G}^p = \begin{bmatrix} 0 & G^p_{1,2} & G^p_{1,3} & G^p_{1,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

and

$$G^p_{1,2} = \left(\frac{m_2 g d_2}{r_1 r_2} + \frac{(m_3 + m_4) g}{r_1} \right) \sin(\theta_2) \tag{7a}$$

$$G^p_{1,3} = \left(\frac{m_3 g d_3}{r_1 r_3} + \frac{m_4 g}{r_1} \right) \sin(\theta_2 + \theta_3) \tag{7b}$$

$$G^p_{1,4} = \frac{m_4 g d_4}{r_1 r_4} \sin(\theta_2 + \theta_3 + \theta_4) \tag{7c}$$

The energy is length squared times stiffness, in Eq. (5), the column matrix is the link length, and $\mathbf{G}^p$ is the gravitational stiffness matrix. The components (Eq. (7a~c)) represent the energy change due to the manipulator's posture, i.e., stiffnesses. The springs' energy can also be represented in the quadratic form with stiffness matrix. Therefore, the gravitational energy and the springs' energy are compatible. Gravity balancing can be simplified as the summation of stiffness matrices that remain unchanged. By arranging the springs' installation, as all the components $G^p_{1,2}$ and $K^{S(i,j)}_{i,j}$ are offset, the gravity is balanced.

In this form, the polar angles θ_2, θ_3 and θ_4 in cosine are used to describe the posture of links. However, they can only be applied to planar articulated manipulators. The quadratic form is for the first time extended to spatial articulated manipulator in the following section.

2.2. Quadratic form of the spatial articulated manipulator

To extend the quadratic form to spatial articulated manipulator, a model of the spatial articulated manipulator is built. Fig. 4 shows a general spatial articulated manipulator. We assumed that the links of the manipulator were connected in series with spherical joints, which completely contained all three rotational degrees of freedom (DOF) of the space. The friction in the joints was neglected. As shown in Fig. 4, the Denavit-Hartenberg representation was applied. The local coordinate system of link j sets the unit vector $\hat{x}_j$ as the direction in which proximal joints point to the distal joints. g is the gravitational acceleration, which direction is negative in relation to the unit vector $\hat{x}_1$. r_j is the length of link j and m_j is the mass of link j. To simplify the model, the center of mass (COM) of a link was assumed to be on the connecting line of the link's proximal joint and the distal joint. d_j is the distance between the proximal joint and the COM of a link j and h_j is the altitude of the COM of link j from the zero potential plane (here we set the $\hat{y}_1$-$\hat{z}_1$ plane as the zero potential plane).

Based on the model, the energy representation was developed as shown in the following sections.

2.2.1. Gravitational energy in quadratic form

For a link j, its gravitational energy is $U_g^j = m_j g h_j$, where h_j can be expressed as the projection of links' position to the vector $\hat{x}_1$. According to Fig. 4, the altitude of link j can be expressed as:

$$h_j = \left(d_j \hat{x}_j + \sum_{w=2}^{j-1} r_w \hat{x}_w \right) \cdot \hat{x}_1 \tag{8}$$

The total gravitational energy of an n-link spatial manipulator is $U_g = \sum m_j g h_j$, from Eq. (8), it can be generally expressed as:

$$U_g = \sum_{t=2}^{n} m_t g \left(d_t \hat{x}_t + \sum_{w=2}^{t-1} r_w \hat{x}_w \right) \cdot \hat{x}_1 \tag{9}$$

Equation (9) can be expanded as:

$$U_g = \left(m_2 g d_2 + r_2 g \sum_{t=3}^{n} m_t \right) \hat{x}_1 \cdot \hat{x}_2 + \left(m_3 g d_3 + r_3 g \sum_{t=4}^{n} m_t \right) \hat{x}_1 \cdot \hat{x}_3 + \ldots$$
$$+ \left(m_j g d_j + r_j g \sum_{t=j+1}^{n} m_t \right) \hat{x}_1 \cdot \hat{x}_j + \ldots + (m_n g d_n) \hat{x}_1 \cdot \hat{x}_n \tag{10}$$

Fig. 5 shows an illustrative example of a four-link spatial articulated manipulator.

From Eq. (10), its gravitational energy is:

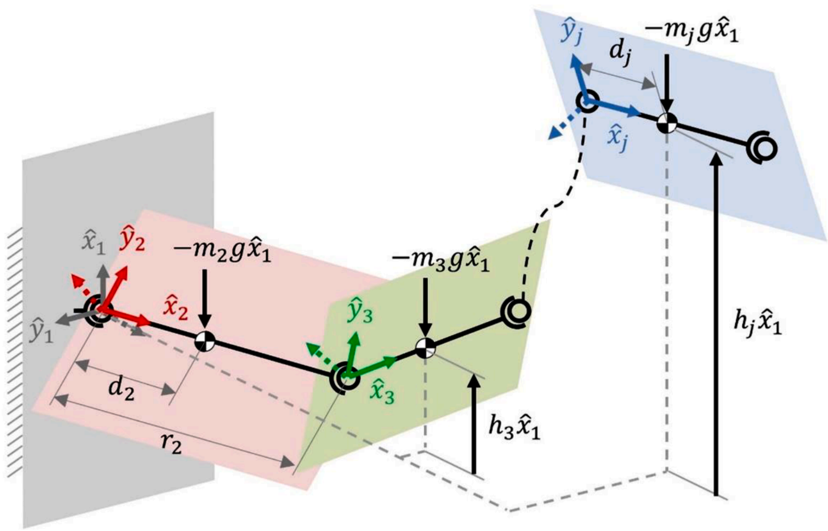

Fig. 4. Modeling of a general spatial articulated manipulator with spherical joints.

$$U_g = (m_2 g d_2 + (m_3 + m_4) r_2 g)\widehat{x}_1 \cdot \widehat{x}_2 + (m_3 g d_3 + m_4 r_3 g)\widehat{x}_1 \cdot \widehat{x}_3 + (m_4 g d_4)\widehat{x}_1 \cdot \widehat{x}_4 \tag{11}$$

In it, each term denotes a part of the gravitational energy, the inner production $\widehat{x}_1 \cdot \widehat{x}_2$ denotes the relative posture between Link 1 and Link 2 an $(m_2 g d_2 + (m_3 + m_4) r_2 g)$ is its coefficient; and so on for the rest of the terms. We can express Eq. (11) by the matrix representation:

$$U_g = \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \\ \widehat{x}_3 \\ \widehat{x}_4 \end{bmatrix} \begin{bmatrix} 0 & G_{1,2} & G_{1,3} & G_{1,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \\ \widehat{x}_3 \\ \widehat{x}_4 \end{bmatrix} \tag{12}$$

where the components in the matrix are:

$$\begin{cases} G_{1,2} = m_2 g d_2 + (m_3 + m_4) r_2 g \\ G_{1,3} = m_3 g d_3 + m_4 r_3 g \\ G_{1,4} = m_4 g d_4 \end{cases} \tag{13}$$

Thus, the matrix representation clearly shows the relationship between the gravitational energy and the posture of the manipulator. Each component in the matrix denotes the coefficient of a part of the energy that relates to the links' relative positions in the space. For example, $G_{1,2}$ denotes the coefficient of partial gravitational energy that depends on the relative posture between Link 1 (the ground link) and Link 2.

According to the example mentioned above, the gravitational energy of a general n-link spatial manipulator (Eq. (10)) can be presented in quadratic form by the matrix representation:

$$U_g = \mathbf{X}^T \mathbf{G} \mathbf{X} \tag{14}$$

where $\mathbf{X}$ is an $n \times 1$ column vector:

$$\mathbf{X} = \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \\ \vdots \\ \widehat{x}_j \\ \vdots \\ \widehat{x}_n \end{bmatrix} \tag{15}$$

and $\mathbf{G}$ is an $n \times n$ square matrix with non-zero components in the first row only:

$$\mathbf{G} = \begin{bmatrix} 0 & G_{1,2} & G_{1,3} & \dots & G_{1,n} \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{n \times n} \tag{16}$$

where the non-zero components in $\mathbf{G}$ are as follows, being functions of the manipulator parameters (i.e., link length and mass):

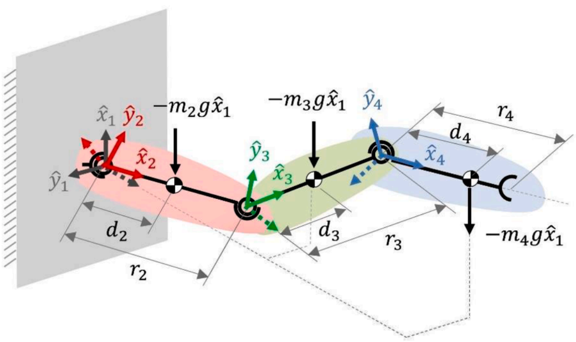

Fig. 5. Example of a four-link spatial articulated manipulator with spherical joints.

$$\begin{cases} G_{1,2} = m_2 g d_2 + r_2 g \sum_{t=3}^{n} m_t \\ G_{1,3} = m_3 g d_3 + r_3 g \sum_{t=4}^{n} m_t \\ \vdots \\ G_{1,n} = g m_n d_n \end{cases} \tag{17}$$

which can be generally expressed as:

$$G_{1,j} = g\left(m_j d_j + r_j \sum_{t=j+1}^{n} m_t \right) \tag{18}$$

Here, the gravitational energy is reformulated, where the relative postures of the links are expressed in terms of the inner product of the links' local coordinates. The representation of gravitational energy in quadratic form is thus simplified and extended to spatial articulated manipulators.

The springs' elastic energy can also be expressed in quadratic form as discussed in the following section.

2.2.2. Elastic energy of a spring in quadratic form

Fig. 6 shows an arbitrary tension spring attached between Links i and j ($i < j$), represented as $S_{i,j}$. Where Link i is the proximally attached link and Link j is the distally attached link. $a_{S(i,j)}$ is the distance between the proximal point at which the spring is attached and the distal joint of Link i and $\widehat{a}_{S(i,j)}$ is its direction. $b_{S(i,j)}$ is the distance between the distal point at which the spring is attached and the proximal joint of Link j and $\widehat{b}_{S(i,j)}$ is its direction.

The spring is assumed as an ideal zero-free-length spring (ZFL), which means that its elongation is assumed to be equal to its length, and it has no initial length when it is in a relaxed state. This idealized model allows for simplified analysis and calculations in this paper of spring balance methods for manipulators.

The elongation of $S_{i,j}$ is expressed as:

$$l_{S(i,j)} = \left| - a_{S(i,j)} \widehat{a}_{S(i,j)} + b_{S(i,j)} \widehat{b}_{S(i,j)} + \sum_{t=i+1}^{j-1} r_t \widehat{x}_t \right| \tag{19}$$

The elastic energy of spring $S_{i,j}$ is $U_{S(i,j)} = \frac{1}{2} k_{S(i,j)} \, l^2_{S(i,j)}$, and by substituting in Eq. (19), the elastic energy can be expressed as:

$$U_{S(i,j)} = \frac{1}{2} k_{S(i,j)} \left(- a_{S(i,j)} \widehat{a}_{S(i,j)} + b_{S(i,j)} \widehat{b}_{S(i,j)} + \sum_{t=i+1}^{j-1} r_t \widehat{x}_t \right) \cdot \left(- a_{S(i,j)} \widehat{a}_{S(i,j)} + b_{S(i,j)} \widehat{b}_{S(i,j)} + \sum_{t=i+1}^{j-1} r_t \widehat{x}_t \right) \tag{20}$$

Expanding Eq. (20) as:

$$U_{S(i,j)} = C^{S(i,j)} + K^{S(i,j)}_{i,j} \widehat{a}_{S(i,j)} \cdot \widehat{b}_{S(i,j)} + \sum_{v=i+1}^{j-1} K^{S(i,j)}_{i,v} \widehat{a}_{S(i,j)} \cdot \widehat{x}_v$$

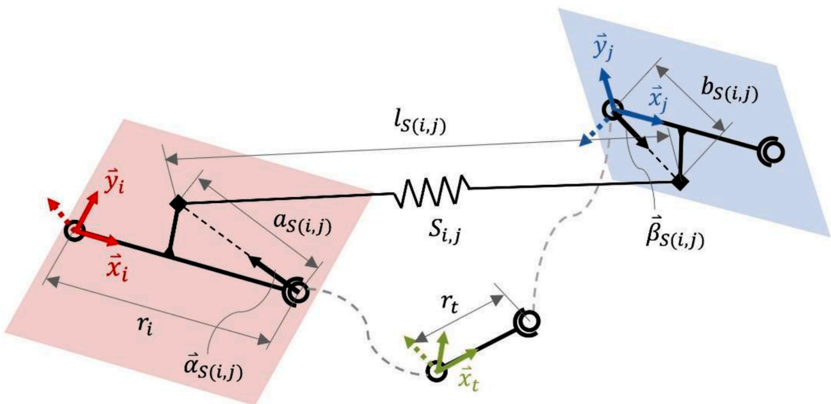

Fig. 6. Tension spring attached to a spatial articulated manipulator.

$$+\sum_{u=i+1}^{j-1} K_{u,j}^{S(i,j)} \widehat{x}_u \cdot \widehat{b}_{S(i,j)} + \sum_{u=i+1}^{j-2} \sum_{v=u+1}^{j-1} K_{u,v}^{S(i,j)} \widehat{x}_u \cdot \widehat{x}_v \tag{21}$$

where

$$C^{S(i,j)} = \frac{1}{2} k_{S(i,j)} \left(a_{S(i,j)}^2 + b_{S(i,j)}^2 + \sum_{t=i+1}^{j-1} r_t^2 \right) \tag{22}$$

$$K_{i,j}^{S(i,j)} = -k_{S(i,j)} a_{S(i,j)} b_{S(i,j)} \tag{23a}$$

$$K_{i,v}^{S(i,j)} = -r_v k_{S(i,j)} a_{S(i,j)} \quad \text{for } v < j \tag{23b}$$

$$K_{u,v}^{S(i,j)} = r_u r_v k_{S(i,j)} \quad \text{for } u > i; v < j \tag{23c}$$

$$K_{u,j}^{S(i,j)} = r_u k_{S(i,j)} b_{S(i,j)} \quad \text{for } u > i \tag{23d}$$

Comparing Eq. (21) with Eq. (10), the spring elastic energy is observed to be in similar form to the gravitational energy. Therefore, it can also be represented in the quadratic form:

$$U_{S(i,j)} = \mathbf{Y}^T \mathbf{K}_{S(i,j)} \mathbf{Y} \tag{24}$$

where $\mathbf{Y}$ is an $n \times 1$ column vector:

$$\mathbf{Y} = \begin{bmatrix} \widehat{x}_1 \\ \vdots \\ \widehat{a}_{S(i,j)} \\ \widehat{x}_{i+1} \\ \vdots \\ \widehat{x}_{j-1} \\ \widehat{b}_{S(i,j)} \\ \vdots \\ \widehat{x}_n \end{bmatrix} \tag{25}$$

and $\mathbf{K}_{S(i,j)}$ is an $n \times n$ square matrix with non-zero components located at the area bounded by row i, column j, and the diagonal. $\mathbf{K}_{S(i,j)}$ is as:

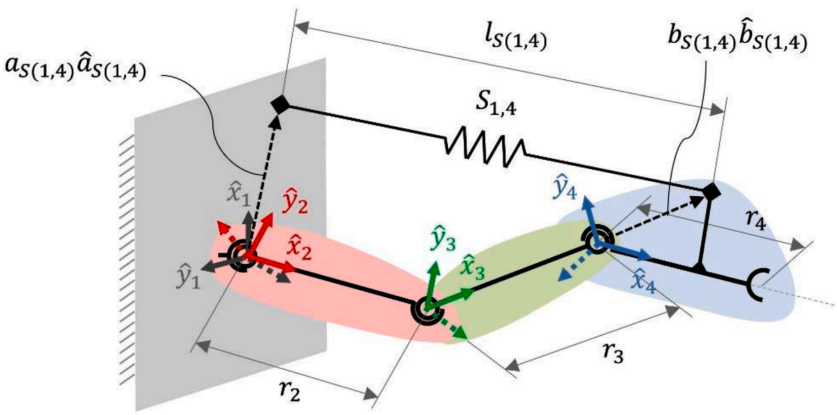

Fig. 7. Example spring attached between the ground link and the fourth link.

$$\mathbf{K}_{S(i,j)} = \begin{bmatrix} 0 & 0 & 0 & & \cdots & & 0 & 0 \\ 0 & 0 & \ddots & & \cdots & & & 0 \\ 0 & \vdots & * & K_{i,i+1}^{S(i,j)} & \cdots & K_{i,j}^{S(i,j)} & & \\ & & & * & K_{u,v}^{S(i,j)} & \vdots & \vdots & \vdots \\ \vdots & \vdots & & & * & K_{j-1,j}^{S(i,j)} & & \\ & & & & & * & \ddots & 0 \\ & & & & & & 0 & 0 \\ 0 & 0 & & & \cdots & & 0 & 0 \end{bmatrix} \tag{26}$$

The non-zero components of the matrix $\mathbf{K}_{S(i,j)}$ are presented in Eqs. (22) and (23a~d), which are functions of the spring parameters (i.e., the spring stiffness $k_{S(i,j)}$ and the spring attachment distances $a_{S(i,j)}$ and $b_{S(i,j)}$). The position of these components shows that $S_{i,j}$ has an effect on all the links between links i and j. The "$*$" components in the diagonal are the constant terms in Eq. (22).

A spring attached between the ground link and the fourth link ($S_{1,4}$) as shown in Fig. 7 is given as an example below.

According to Eq. (21), the elastic energy of $S_{1,4}$ is:

$$U_{S(1,4)} = C^{S(1,4)} + K_{1,4}^{S(1,4)}\widehat{a}_{S(1,4)} \cdot \widehat{b}_{S(1,4)} + K_{1,3}^{S(1,4)}\widehat{a}_{S(1,4)} \cdot \widehat{x}_3 + K_{1,2}^{S(1,4)}\widehat{a}_{S(1,4)} \cdot \widehat{x}_2$$

$$+K_{2,4}^{S(1,4)}\widehat{x}_2 \cdot \widehat{b}_{S(1,4)} + K_{3,4}^{S(1,4)}\widehat{x}_3 \cdot \widehat{b}_{S(1,4)} + K_{2,3}^{S(1,4)}\widehat{x}_2 \cdot \widehat{x}_3 \tag{27}$$

where

$$C^{S(1,4)} = \frac{1}{2}k_{S(1,4)}\left(a_{S(1,4)}^2 + b_{S(1,4)}^2 + r_2^2 + r_3^2\right) \tag{28a}$$

$$K_{1,4}^{S(1,4)} = -k_{S(1,4)}a_{S(1,4)}b_{S(1,4)} \tag{28b}$$

$$K_{1,3}^{S(1,4)} = -k_{S(1,4)}a_{S(1,4)}r_3 \tag{28c}$$

$$K_{1,2}^{S(1,4)} = -k_{S(1,4)}a_{S(1,4)}r_2 \tag{28d}$$

$$K_{2,4}^{S(1,4)} = k_{S(1,4)}r_2 b_{S(1,4)} \tag{28e}$$

$$K_{3,4}^{S(1,4)} = k_{S(1,4)}r_3 b_{S(1,4)} \tag{28f}$$

$$K_{2,3}^{S(1,4)} = k_{S(1,4)}r_2 r_3 \tag{28g}$$

This can be expressed in quadratic form.

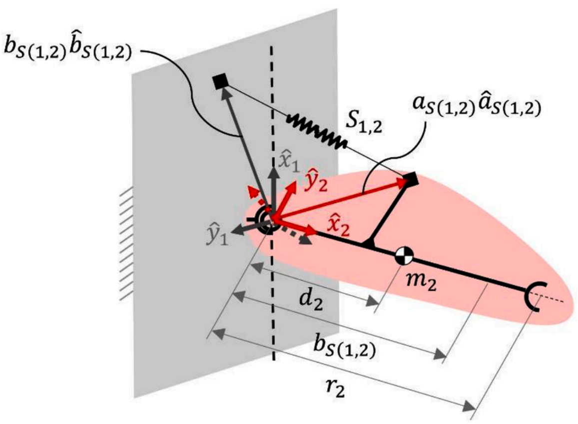

Fig. 8. Two-link spatial manipulator with spring $S_{1,2}$.

$$U_{S(1,4)} = \begin{bmatrix} \widehat{a}_{S(1,4)} \\ \widehat{x}_2 \\ \widehat{x}_3 \\ \widehat{b}_{S(1,4)} \end{bmatrix} \begin{bmatrix} * & K_{1,2}^{S(1,4)} & K_{1,3}^{S(1,4)} & K_{1,4}^{S(1,4)} \\ 0 & * & K_{2,3}^{S(1,4)} & K_{2,4}^{S(1,4)} \\ 0 & 0 & * & K_{3,4}^{S(1,4)} \\ 0 & 0 & 0 & * \end{bmatrix} \begin{bmatrix} \widehat{a}_{S(1,4)} \\ \widehat{x}_2 \\ \widehat{x}_3 \\ \widehat{b}_{S(1,4)} \end{bmatrix} \tag{29}$$

The quadratic form of the energies has thus been presented. How to use springs to achieve the gravity balancing of a spatial articulated manipulator is discussed in the next section.

3. Gravity balancing of spatial articulated manipulators by use of springs

3.1. Gravity balancing conditions

To achieve the gravity balancing of the spatial manipulator by use of springs, the summation of the manipulator's gravitational energy and the springs' elastic energy should be constant, which can be expressed as:

$$U_g + \sum U_{S(i,j)} = \text{const.} \tag{30}$$

An example of a two-link spatial manipulator with a spring $S_{1,2}$ is shown in Fig. 8, given to illustrate the balancing conditions: According to Eq. (14) and Eq. (24) the gravitational energy and the elastic energy can be expressed as:

$$U_g = \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{bmatrix} \begin{bmatrix} 0 & G_{1,2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{bmatrix} \tag{31}$$

$$U_{S(1,2)} = \begin{bmatrix} \widehat{a}_{S(1,2)} \\ \widehat{b}_{S(1,2)} \end{bmatrix} \begin{bmatrix} * & K_{1,2}^{S(1,2)} \\ 0 & * \end{bmatrix} \begin{bmatrix} \widehat{a}_{S(1,2)} \\ \widehat{b}_{S(1,2)} \end{bmatrix} \tag{32}$$

where

$$G_{1,2} = m_2 g d_2 \tag{33a}$$

$$K_{1,2}^{S(1,2)} = -k_{S(1,2)} a_{S(1,2)} b_{S(1,2)} \tag{33b}$$

Then, the balancing condition can be expressed as:

$$\begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{bmatrix} \begin{bmatrix} 0 & G_{1,2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{bmatrix} + \begin{bmatrix} \widehat{a}_{S(1,2)} \\ \widehat{b}_{S(1,2)} \end{bmatrix} \begin{bmatrix} * & K_{1,2}^{S(1,2)} \\ 0 & * \end{bmatrix} \begin{bmatrix} \widehat{a}_{S(1,2)} \\ \widehat{b}_{S(1,2)} \end{bmatrix} = \text{const.} \tag{34}$$

Since the terms "$*$" on the diagonal of Eq. (34) are constants, they are negligible. The coefficients $G_{1,2}$ and $K_{1,2}^{S(1,2)}$ are functions of the manipulator and the spring parameters. And the bases of them are $\widehat{x}_1 \cdot \widehat{x}_2$ and $\widehat{a}_{S(1,2)} \cdot \widehat{b}_{S(1,2)}$ respectively, which are change due to the posture of manipulator. Therefore, to keep Eq. (34) a constant, the bases should be the same and the coefficients $G_{1,2}$ and $K_{1,2}^{S(1,2)}$ should cancel each other out. The balancing condition can then be simplified as:

$$\begin{bmatrix} \widehat{a}_{S(1,2)} \\ \widehat{b}_{S(1,2)} \end{bmatrix} = \begin{bmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{bmatrix} \tag{35a}$$

$$G_{1,2} + K_{1,2}^{S(1,2)} = 0 \tag{35b}$$

To attain Eq. (35a), the spring must be installed along $\widehat{x}_1$ and $\widehat{x}_2$ (i.e., the spring must be installed at the connecting line of the link's joints), which is at $\widehat{a}_{S(1,2)} = \widehat{x}_1$ and $\widehat{b}_{S(1,2)} = \widehat{x}_2$.

For Eq. (35b), according to Eqs. (33a, b), the required spring parameters are found:

$$k_{S(1,2)} a_{S(1,2)} b_{S(1,2)} = m_2 g d_2 \tag{36}$$

Compared with previous works [9], the spring installation requirement (Eq. (36)) to balance a two-link spatial manipulator as solved above has the same requirements, and here we offer further demonstration that the spring must be installed along the connecting line of the link's joints.

The spring installation requirements for balancing can be further extended to an arbitrary spatial articulated manipulator where $\widehat{a}_{S(i,j)} = \widehat{x}_i$ and $\widehat{b}_{S(i,j)} = \widehat{x}_j$, making the column vectors **X** and **Y** compatible. According to Eq. (14) and Eq. (24), the balancing condition of a spatial manipulator can be presented in quadratic form as:

$$\mathbf{X}^T\mathbf{G}\mathbf{X} + \sum \mathbf{Y}^T\mathbf{K}_{S(i,j)}\mathbf{Y} = \text{const.} \tag{37}$$

To satisfy this, the column vectors **X** and **Y** should be compatible, that is:

$$\mathbf{X} = \mathbf{Y} \tag{38}$$

and the summation of components of the square matrices **G** and $\mathbf{K}_{S(i,j)}$ should cancel each other out. As mentioned in Section 2, it is known that the non-zero components of **G** are located in the first row only (Eq. (16)); and the non-zero components of $\mathbf{K}_{S(i,j)}$ are located in the area bounded by row i, column j, and the diagonal (Eq. 26). Therefore, the condition can be further rewritten as:

$$G_{1,j} + \sum K_{1,j}^{S(1,v)} = 0 \text{ for } v \geq j > 1 \tag{39a}$$

$$\sum K_{i,j}^{S(u,v)} = 0 \text{ for } i \geq u \geq 1; v \geq j > 1 \tag{39b}$$

Equations (39a, b) are the balancing equations which determine the spring parameters.
In summary, the basic principles to balance a spatial manipulator using springs are:

P1. The springs must be installed on the connecting line of the joints (Eq. (38)).
P2. The spring parameters must satisfy the balancing equations to completely cancel out the gravitational energy (Eqs. (39a, b)).

A detailed description of applying these principles to an arbitrary spatial manipulator are discussed in the following sections.

3.2. Criteria of spring attachment

From Eq. (26), it is known that only a spring connected to the ground (referred to as a ground-connected spring) can contribute the components ($K_{1,j}^{S(1,v)}$) that locate at the first row and correspond to the components of gravity ($G_{1,j}$). Therefore, only the ground-connected springs are related to the balancing equation Eq. (39a). To ensure the ground-connected springs contribute to balancing rather than burdening the system, all the components contributed by the ground-connected springs ($K_{1,j}^{S(1,v)}$) should be negative in relation to the components of gravity ($G_{1,j}$), which are always positive. Therefore, the requirements of the ground-connected spring components are known:

$$\begin{cases} K_{1,j}^{S(1,j)} < 0 \\ K_{1,v}^{S(1,j)} < 0 \end{cases} \tag{40}$$

From Eqs. (23a, b) and (40), to guarantee that a ground-connect spring contributes to balancing, the attachment distance must satisfy:

$$\begin{cases} a_{S(1,j)} > 0 \\ b_{S(1,j)} > 0 \end{cases} \tag{41}$$

Here we present Eq. (40) as $(a_{S(1,j)}, b_{S(1,j)}) = (+,+)$, taking into account *P1,* where the springs must be attached on the connecting line of the link joints, and an arbitrary ground-connected springs must be attached as shown in Fig. 9.

Note that a special case can be considered for a spring $S_{1,2}$ that contributes only one component $K_{1,2}^{S(1,2)}$, and therefore a spring attachment with $(a_{S(1,2)}, b_{S(1,2)}) = (+,+)$ or $(-,-)$ is also allowed.

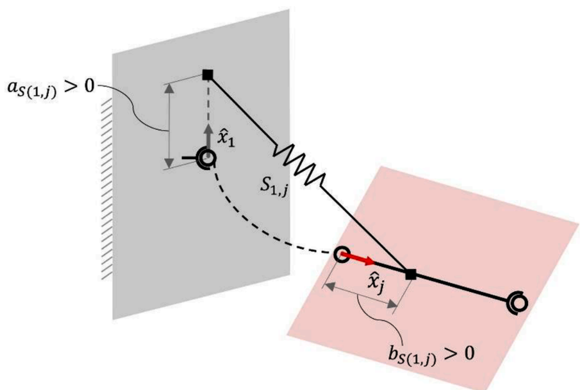

Fig. 9. Attachment of a ground-connected spring with $(a_{S(1,j)}, b_{S(1,j)}) = (+,+)$.

Other than the components in the first row of the matrix, which correspond to $G_{1,j}$, a ground-connected spring contains non-first row components that need to be balanced by other springs. Therefore, springs must be installed between two non-ground links (referred to as non-ground-connected springs).

Similar to the ground-connected springs, the non-ground-connected springs need to contribute to balancing. Therefore, the components of a non-ground-connected spring need to be negative in relation to the remaining components of the ground-connected springs. According to Eqs. (23a~d), the components of a non-ground-connected spring cannot all be negative at the same time but there can be at most two types of components with a negative value at the same time. The acceptable attachments for an arbitrary non-ground-connected spring are shown in Fig. 10.

The criteria of spring attachment for a spatial manipulator are thus provided. The past study [10] has proposed the spring attachment criteria for planar manipulators that use springs to achieve gravity balance without auxiliary links. A comparison of spring attachment criteria between the planar manipulator and the spatial manipulator is discussed below.

For a planar manipulator, the manipulator's links move on a plane only (here we take the $\hat{x}_i$-$\hat{y}_i$ plane). The column vectors $\mathbf{X}$ and $\mathbf{Y}$ are still need to be compatible, that is, the springs are attached on the connecting line of the joints generally, which is the same as for a spatial manipulator. However, there is a special case: when a spring is attached between the two adjacent links i and $i+1$ as shown in Fig. 11 (referred to as "mono-articulated springs"; springs attached between two non-adjacent links are referred to as "multi-articulated springs"), where $\Theta_{\hat{a},\hat{b}}$ denotes the angles between the unit vectors $\hat{a}_{S(i,i+1)}$ and $\hat{b}_{S(i,i+1)}$ and $\Theta_{\hat{x}i,\hat{x}i+1}$ denotes the angles between the unit vectors $\hat{x}_i$ and $\hat{x}_{i+1}$.

The bases should be the same, that is $\hat{a}_{S(i,i+1)} \cdot \hat{b}_{S(i,i+1)} = \hat{x}_i \cdot \hat{x}_{i+1}$, it can be rewritten as:

$$\left|\hat{a}_{S(i,i+1)}\right|\left|\hat{b}_{S(i,i+1)}\right|\cos\left(\Theta_{\hat{a},\hat{b}}\right) = |\hat{x}_i||\hat{x}_{i+1}|\cos\left(\Theta_{\hat{x}i,\hat{x}i+1}\right) \tag{42}$$

$\hat{a}_{S(i,i+1)}$, $\hat{b}_{S(i,i+1)}$, $\hat{x}_i$, and $\hat{x}_{i+1}$ are unit vectors, therefore, Eq. (42) can be simplified as:

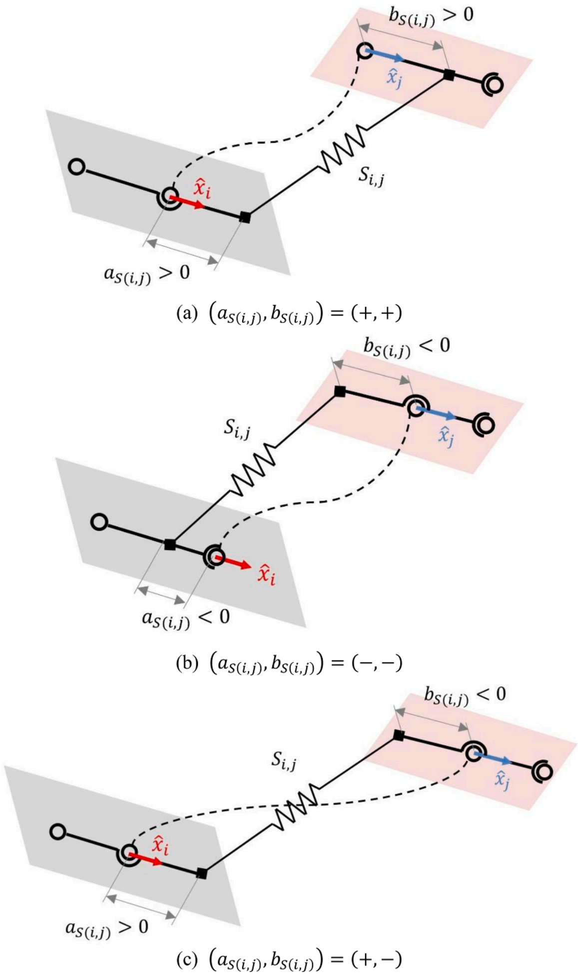

Fig. 10. Acceptable attachments for an arbitrary non-ground-connected spring.

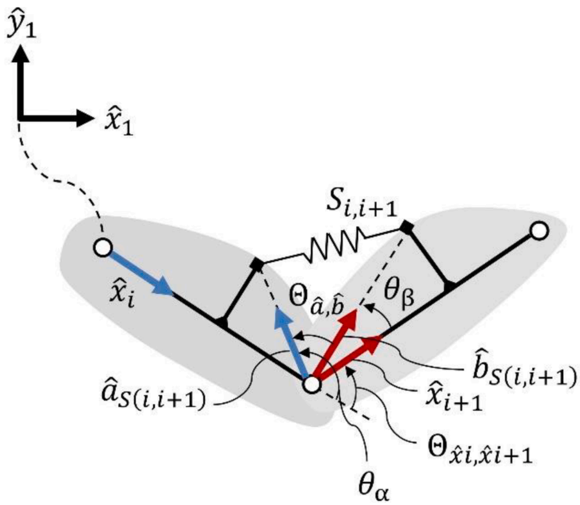

Fig. 11. Mono-articulated spring attached to a planar manipulator.

$$\Theta_{\hat{a},\hat{b}} = \pm\Theta_{\hat{x}i,\hat{x}i+1} \tag{43}$$

and from Fig. 11 it is known that:

$$\Theta_{\hat{a},\hat{b}} = \theta_\alpha - \left(\Theta_{\hat{x}i,\hat{x}i+1} + \theta_\beta\right) \tag{44}$$

Solving Eq. (43) and Eq. (44), the required mono-articulated spring attachment angles are thus determined:

$$\theta_\alpha - \theta_\beta = 0 \tag{45}$$

It is thus shown that if a mono-articulated spring is used on a planar manipulator, its attachment should satisfy the condition $\theta_\alpha - \theta_\beta = 0$. This means that a mono-articulated spring does not need to be attached on the connecting line of the joints. Fig. 12. shows an example of a feasible mono-articulated spring attachment to a planar manipulator with $\theta_\alpha = \theta_\beta = 45^\circ$.

The attachment condition of a mono-articulated spring to planar manipulator is thus shown here as the same as that determined by [10].

In summary, the comparison of spring attachments between spatial and planar manipulators revealed that they differ in their required attachments of mono-articulated springs. For planar manipulators, multi-articulated springs must be attached on the connecting line of the joints, whereas mono-articulated springs do not need to be, however, the attachment angles must meet the condition $\theta_\alpha - \theta_\beta = 0$. For spatial manipulators, both mono-articulated springs and multi-articulated springs are required to be attached on the connecting line of the joints.

The criteria of spring attachment are thus provided. For a spatial-articulated manipulator, the ground-connected springs are attached with $(a_{S(1,j)}, b_{S(1,j)}) = (+,+)$; and the non-ground-connected springs are attached with $(a_{S(i,j)}, b_{S(i,j)}) = (+,+),\ (-,-)$ or $(+,-)$. It is observed that there are multiple options for attaching a spring. The next section will discuss how to select the spring attachments and determine which links should have springs installed in order to form a gravity-balancing spring configuration.

3.3. Spring installation rules and acceptable spring configurations

The springs should be installed on specific links to achieve gravity balance. As mentioned in the previous section, the ground-connected springs are used to balance the gravity. According to Eq. (39a), all the non-zero components of the gravitational energy matrix **G** must be offset by the ground-connected spring components to achieve balance. For an *n*-link manipulator, its gravitational

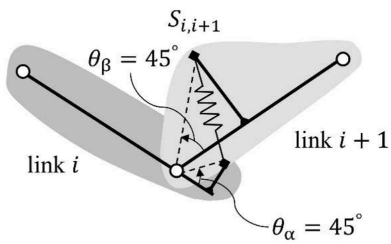

Fig. 12. Example of a mono-articulated spring attachment to a planar manipulator.

energy matrix contains $n-1$ non-zero components: $G_{1,2},\ G_{1,3}\ldots G_{1,n}$. The component $G_{1,n}$ can only be fully offset by a ground-connected spring $S_{1,n}$, which can contribute the corresponding elastic energy matrix component $K_{1,n}^{S(1,n)}$. The first rule of ground-connected spring installation for an n-link spatial articulated manipulator is thus:

R1: A ground-connected spring must be installed between the ground link and the end link (Link n) $S_{1,n}$.

The component $G_{1,n}$ is thereby fully offset. The component $G_{1,n-1}$ can also be fully offset by $S_{1,n}$, since $S_{1,n}$ can contribute the corresponding elastic energy matrix component $K_{1,n-1}^{S(1,n)}$. The next component, $G_{1,n-2}$, can only be partially offset. That is because $G_{1,n-1} \neq G_{1,n-2}$ (according to Eq. (17)) and the balancing condition cannot fulfill both at the same time by arranging the spring attachment. Therefore, another spring $S_{1,n-2}$ is required to fully offset $G_{1,n-2}$ and the next component $G_{1,n-3}$. Generally, for the component $G_{1,j}$ where $j < n$, it must be fully offset by the spring $S_{1,j}$ or $S_{1,j+1}$. The second rule of ground-connected spring installation is thus:

R2: There must be at least one ground-connected spring for every two links.

As the ground-connected springs are installed, the gravitational energy matrix components $G_{1,2},\ G_{1,3}\ldots G_{1,n}$ become fully offset. However, the ground-connected springs still contribute additional components below the first row that need to be offset by the non-ground-connected springs to satisfy the balancing equation Eq. (39b). According to Fig. 10, the non-ground-connected springs can be attached in three different ways, with each type of attachment contributing balancing components at different locations. For the non-ground-connected spring attached with $(a_{S(i,j)}, b_{S(i,j)}) = (+,+)$, it contributes balancing components $K_{i,j}^{S(i,j)}$ and $K_{i,v}^{S(i,j)}$ for $v < j$; for $(a_{S(i,j)}, b_{S(i,j)}) = (-,-)$, it contributes $K_{i,j}^{S(i,j)}$ and $K_{u,j}^{S(i,j)}$ for $u > i$; and for $(a_{S(i,j)}, b_{S(i,j)}) = (+,-)$, it contributes $K_{i,v}^{S(i,j)}$ and $K_{u,j}^{S(i,j)}$. According to Eqs. (23c, d), *R1* and *R2,* the remaining components below the first row are known, and the general rules to follow such that the non-ground-connected springs fully offset the remaining components are as follows:

R3: To offset the unbalanced component $K_{2,n}^{S(1,n)}$, at least one non–ground-connected spring $S_{2,n}$ with $(a_{S(2,n)}, b_{S(2,n)}) = (+,+)$ or $(-,-)$ should be installed.
R4: To offset the unbalanced components located at $(2,j)$ for $n > j > 2$, at least one non–ground-connected spring $S_{2,j}$ with $(a_{S(2,j)}, b_{S(2,j)}) = (+,+)$ or $(-,-)$ or $S_{2,j+1}$ with $(a_{S(2,j+1)}, b_{S(2,j+1)}) = (+,+)$ or $(+,-)$ should be installed.
*R*5: To offset the unbalanced components located at (i,n) for $i > 2$, at least one non–ground-connected spring $S_{i,n}$ with $(a_{S(i,n)}, b_{S(i,n)}) = (+,+)$ or $(-,-)$ or $S_{u,n}$ for $i > u > 1$ with $(a_{S(u,n)}, b_{S(u,n)}) = (-,-)$ or $(+,-)$ should be installed.
R6: To offset the unbalanced components located at (i,j) for $i > 2; j < n$, at least one non–ground-connected spring $S_{i,j}$ with $(a_{S(i,j)}, b_{S(i,j)}) = (+,+)$ or $(-,-)$, or $S_{i,j+1}$ with $(a_{S(i,j+1)}, b_{S(i,j+1)}) = (+,+)$ or $(+,-)$, or $S_{u,j}$ for $u < i$ with $(a_{S(u,j)}, b_{S(u,j)}) = (-,-)$ or $(+,-)$ should be installed.

R3∼*R6* are the installation rules for non-ground-connected springs. The specific rule to apply is determined based on the location of the unbalanced components remaining in the matrix. If all the unbalanced components correspond to the balancing components, it indicates that an acceptable spring configuration has been identified.

A four-link spatial articulated manipulator shown in Fig. 5 is given as an example. According to Eq. (12), the components of **G** are $G_{1,2}$, $G_{1,3}$ and $G_{1,4}$, which should be balanced by the components that contributed by the ground-connected springs. From *R1,* a spring $S_{1,4}$ with $(a_{S(1,4)}, b_{S(1,4)}) = (+,+)$ must be installed, and from *R2,* at least one ground-connected spring must be attached to link 2 or/and link 3, that is, $S_{1,2}$ with $(a_{S(1,2)}, b_{S(1,2)}) = (+,+)$ or $(-,-)$ or/and $S_{1,3}$ with $(a_{S(1,3)}, b_{S(1,3)}) = (+,+)$) must be installed. By installing the ground-connected springs according to the specified rules, the components $G_{1,2}$, $G_{1,3}$ and $G_{1,4}$ can be balanced. However, it should be noted that the ground-connected springs also contribute non-first row components that need to be balanced by the non-ground-connected springs, as mentioned earlier.

According to *R3,* a non-ground-connected spring $S_{2,4}$ with $(a_{S(2,4)}, b_{S(2,4)}) = (+,+)$ or $(-,-)$ must be installed. Two conditions are discussed:

1. If $S_{2,4}$ with $(a_{S(2,4)}, b_{S(2,4)}) = (+,+)$ is installed, it contributes balancing components $K_{2,4}^{S(2,4)}$ and $K_{2,3}^{S(2,4)}$. However, it also contains a component $K_{3,4}^{S(2,4)}$ that needs to be balanced by another non-ground-connected spring. Therefore, *R*5 is applied: $S_{3,4}$ with $(a_{S(3,4)}, b_{S(3,4)}) = (+,+)$ or $(-,-)$ or/and $S'_{2,4}$ with $(a_{S'(2,4)}, b_{S'(2,4)}) = (-,-)$ or $(+,-)$ should be installed.
2. If $S_{2,4}$ with $(a_{S(2,4)}, b_{S(2,4)}) = (-,-)$ is installed, it contributes balancing components $K_{2,4}^{S(2,4)}$ and $K_{3,4}^{S(2,4)}$, while $K_{2,3}^{S(2,4)}$ needs to be balanced. Therefore, *R4* is applied: $S_{2,3}$ with $(a_{S(2,3)}, b_{S(2,3)}) = (+,+)$ or $(-,-)$ or/and $S'_{2,4}$ with $(a_{S'(2,4)}, b_{S'(2,4)}) = (+,+)$ or $(+,-)$ should be installed.

Up to this point, there are no remaining components that need to be balanced in the system. All of them correspond to at least one balancing component, indicating that balance can be achieved. The example demonstrates that there are multiple possible spring configurations for a 4-link manipulator. Fig. 13 (a) and (b) are branch diagrams illustrating the choices of spring installation:

From the rules detailed above, the acceptable spring configurations for three-link and four-link spatial articulated manipulators are listed below; note that only the configurations with necessary springs are shown in Table 1. However, as illustrated in Fig. 13, there are actually more choices available to balance a 4-link manipulator by installing additional springs. Users have the flexibility to add extra springs to these configurations based on their design requirements and constraints. The manipulator can still be perfectly balanced as long as the balancing equations are satisfied.

In the configuration matrix shown in Table 1, the column and row number respectively denote the proximally attached link and the distally attached link of the springs; the symbol "0" indicates that there is no spring between the links; the symbol $1^{(+,+)}$ indicates that a spring $S_{i,j}$ with $(a_{S(i,j)}, b_{S(i,j)}) = (+,+)$ is attached; $1^{(+,+)/(-,-)}$ means that a spring attached between the links with $(a_{S(i,j)}, b_{S(i,j)}) = (+,+)$ or $(-,-)$ is allowed.

The installation rules and acceptable spring configurations for an arbitrary spatial articulated manipulator were developed in this

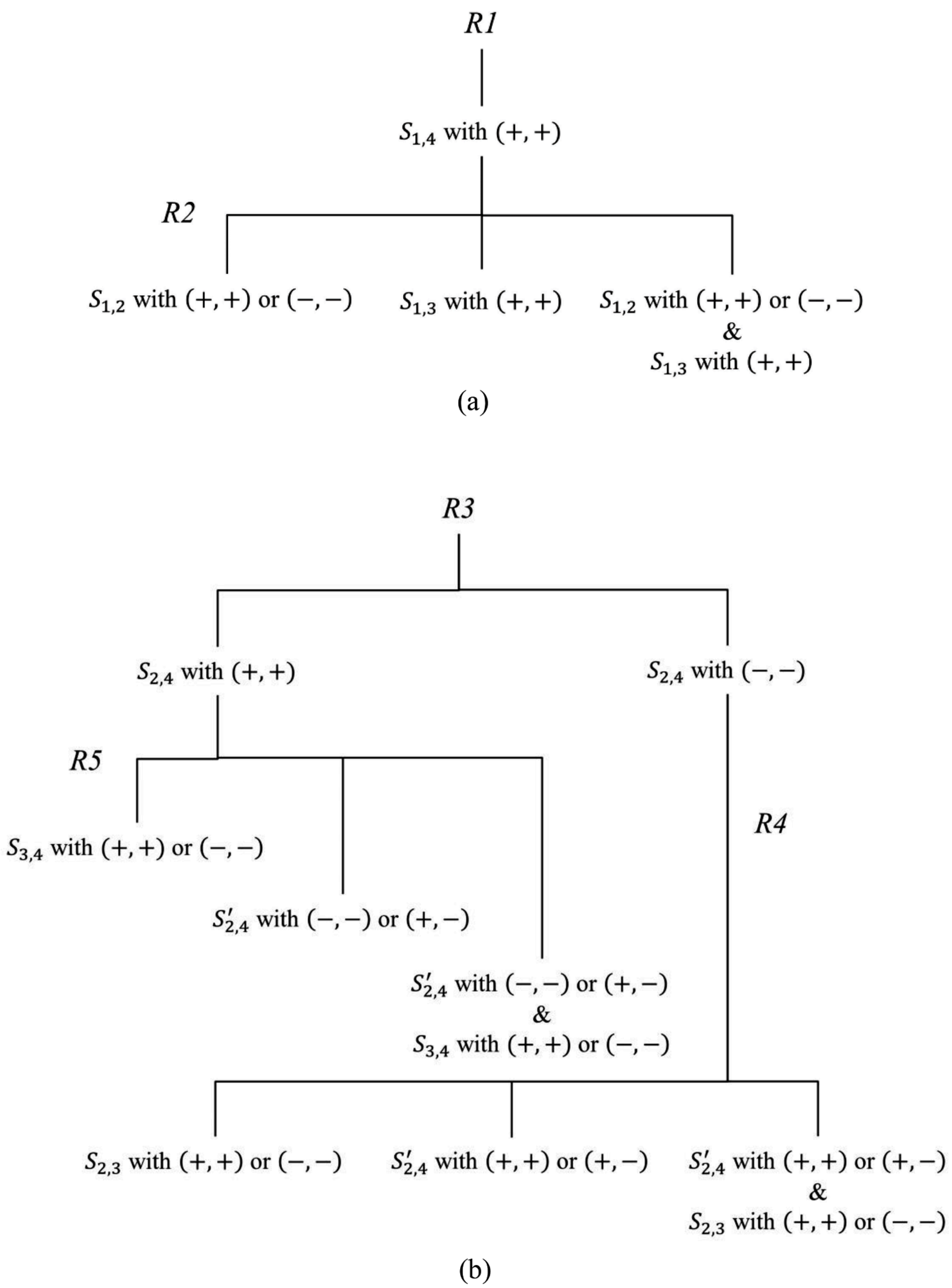

Fig. 13. The choices of spring installation for a 4-link spatial articulated manipulator.

Table 1
Acceptable spring configurations with necessary springs for three-link and four-link spatial articulated manipulators.

	Acceptable spring configurations with necessary springs
$n=3$	$\begin{bmatrix} - & 0 & 1^{(+,+)} \\ & - & 1^{(+,+)/(-,-)} \\ & & - \end{bmatrix}$
$n=4$	$\begin{bmatrix} - & 1^{(+,+)/(-,-)} & 0 & 1^{(+,+)} \\ & - & 1^{(+,+)/(-,-)} & 1^{(-,-)} \\ & & - & 0 \\ & & & - \end{bmatrix} \begin{bmatrix} - & 0 & 1^{(+,+)} & 1^{(+,+)} \\ & - & 1^{(+,+)/(-,-)} & 1^{(-,-)} \\ & & - & 0 \\ & & & - \end{bmatrix}$
	$\begin{bmatrix} - & 1^{(+,+)/(-,-)} & 0 & 1^{(+,+)} \\ & - & 0 & 1^{(+,+)} \\ & & - & 1^{(+,+)/(-,-)} \\ & & & - \end{bmatrix} \begin{bmatrix} - & 0 & 1^{(+,+)} & 1^{(+,+)} \\ & - & 0 & 1^{(+,+)} \\ & & - & 1^{(+,+)/(-,-)} \\ & & & - \end{bmatrix}$
	$\begin{bmatrix} - & 1^{(+,+)/(-,-)} & 0 & 1^{(+,+)} \\ & - & 0 & 1^{(+,-)}, 1^{(+,+)} \\ & & - & 0 \\ & & & - \end{bmatrix} \begin{bmatrix} - & 0 & 1^{(+,+)} & 1^{(+,+)} \\ & - & 0 & 1^{(+,-)}, 1^{(+,+)} \\ & & - & 0 \\ & & & - \end{bmatrix}$

section. In the following section, an example of a spring-gravity balanced four-link spatial articulated manipulator is shown.

4. Examples: gravity balancing of a four-link spatial articulated manipulator

A four-link spatial articulated manipulator with spherical joints only is given as an example. The dimensions and mass of the links are shown in Table 2.

The springs were attached to the manipulator according to one of the configurations in Table 1:

$$\begin{bmatrix} * & 1^{(+,+)} & 0 & 1^{(+,+)} \\ & * & 0 & 1^{(+,+)} \\ & & * & 1^{(-,-)} \\ & & & * \end{bmatrix} \tag{46}$$

Specifically, two ground-connected springs, $S_{1,2}$ and $S_{1,4}$ with $(a_{S(i,j)}, b_{S(i,j)}) = (+,+)$, and two non-ground-connected springs, $S_{2,4}$ with $(a_{S(i,j)}, b_{S(i,j)}) = (+,+)$ and $S_{3,4}$ with $(a_{S(i,j)}, b_{S(i,j)}) = (-,-)$ were attached.

Fig. 14 shows this example manipulator.

To achieve balance, the components of the springs must satisfy the Eqs. (39a, b) as follows:

$$G_{1,4} + K_{1,4}^{S(1,4)} = 0 \tag{47a}$$

$$G_{1,3} + K_{1,3}^{S(1,4)} = 0 \tag{47b}$$

$$G_{1,2} + K_{1,2}^{S(1,4)} + K_{1,2}^{S(1,2)} = 0 \tag{47c}$$

$$K_{2,4}^{S(1,4)} + K_{2,4}^{S(2,4)} = 0 \tag{47d}$$

$$K_{2,3}^{S(1,4)} + K_{2,3}^{S(2,4)} = 0 \tag{47e}$$

$$K_{3,4}^{S(1,4)} + K_{3,4}^{S(2,4)} + K_{3,4}^{S(3,4)} = 0 \tag{47f}$$

And according to Eq. (18) and Eqs. (23a~d), they can be expanded as

$$k_{S(1,4)} a_{S(1,4)} b_{S(1,4)} = m_4 g d_4 \tag{48a}$$

$$k_{S(1,4)} a_{S(1,4)} r_3 = m_3 g d_3 + m_4 r_3 g \tag{48b}$$

Table 2
Dimension and mass of the example spatial articulated manipulator.

Link's number	length	Mass center	mass
2	$r_2 = 0.30$ (m)	$d_2 = 0.15$ (m)	$m_2 = 4.0$ (kg)
3	$r_3 = 0.24$ (m)	$d_3 = 0.12$ (m)	$m_3 = 3.2$ (kg)
4	$r_4 = 0.20$ (m)	$d_4 = 0.10$ (m)	$m_4 = 2.8$ (kg)

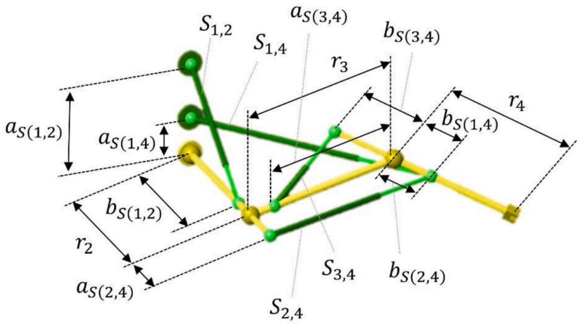

Fig. 14. A four-link spatial manipulator with springs $S_{1,2}$, $S_{1,4}$, $S_{2,4}$ and $S_{3,4}$.

$$k_{S(1,4)}a_{S(1,4)}r_2 + k_{S(1,2)}a_{S(1,2)}b_{S(1,2)} = m_2gd_2 + (m_3 + m_4)r_2g \tag{48c}$$

$$k_{S(1,4)}r_2b_{S(1,4)} - k_{S(2,4)}a_{S(2,4)}b_{S(2,4)} = 0 \tag{48d}$$

$$k_{S(1,4)}r_2r_3 - k_{S(2,4)}a_{S(2,4)}r_3 = 0 \tag{48e}$$

$$k_{S(1,4)}r_3b_{S(1,4)} + k_{S(2,4)}r_3b_{S(2,4)} - k_{S(3,4)}a_{S(3,4)}b_{S(3,4)} = 0 \tag{48f}$$

Solving, the attachment parameters of the springs were determined and are shown in Table 3.

With the springs thus applied, the gravitational force of the example manipulator is theoretically balanced throughout its entire workspace. To prove this, an arbitrary manipulator trajectory was simulated and is shown in Fig. 15. The initial posture and rotation of each joint are listed in Table 4Table 5, where θ_i, ϕ_i and ψ_i represent the angular displacement of the links with respect to the $\hat{x}_{i-1}$, $\hat{y}_{i-1}$ and $\hat{z}_{i-1}$ coordinates, respectively. And the manipulator's end effector can be seen to move from the starting point to the end point in 30 seconds.

Fig. 16 shows the gravitational energy of the manipulator, the spring elastic energy and the total energy during the trajectory.

As mentioned in the previous section, our approach is applicable to both spatial and planar articulated manipulators. Here, we provide another example trajectory that exhibits only planar motion in the $\hat{x}_0$- $\hat{z}_0$ plane, considering only the rotation around the $\hat{y}_i$ axis (i.e., ϕ_i). The trajectory of the articulated manipulator is given as follows:

Fig. 17 illustrates the variation of the gravitational energy, springs' elastic energy, and total energy of the manipulator throughout the planar trajectory.

The simulation results show that in both spatial or planar motions the total energy remained constant during the trajectory, i.e., the energy change due to gravity is perfectly eliminated. Note that, the springs used in the example are simplified as ideal zero-free-length springs. Previous studies [26,27] have proposed the methods to achieve zero-free-length spring by arrangement of cable-pulley. By utilizing a spring-cable-pulley system, the spring balance method can be implemented in practical application. In addition, it's important to note that springs spanning over links may potentially introduce interference into the manipulator's workspace. Theoretically, the utilization of more non-adjacent springs leads to a broader span of space they cover. Consequently, there is a possibility of interference occurring. In our example, we opted for the configuration with the fewest non-adjacent springs in Table 1, and no interference was encountered during the target trajectory. In practical applications, it is crucial for users to carefully assess potential interference issues based on the manipulator's trajectory requirements and dimensions. By conducting a thorough kinematic analysis of mechanisms or utilizing simulations, users can select an appropriate spring configuration in Table 1, and adjust the spring attachment parameters according to balancing equations to ensure the manipulator operates within the desired workspace without any interference from the springs.

5. Conclusion

This paper developed a novel spring-gravity balancing method for spatial articulated manipulators. It differs from past studies because neither auxiliary links nor any additional devices are required in this approach. The elimination of auxiliary devices avoids the introduction of extra inertia and complexity, therefore simplifies the design and implementation process associated with gravity balance systems for manipulators in practical applications.

Furthermore, this paper derived the energy representation in quadratic form, which was previously used for planar manipulators only, and extended it to spatial articulated manipulators. And the quadratic form of the planar and the spatial manipulators are unified in this paper. Based on this representation, the relationship between energy and link posture was clearly shown. It was also found that ground-connected springs can be used to cancel out the gravitational forces. At the same time, they contribute redundant energy that must be balanced by non-ground-connected springs. The requirement of spring attachment were derived, i.e., that the springs must be attached on the connecting line of the link joints. In addition, a completed discussion to the balance theory of planar and spatial articulated manipulator was conducted. It was found that the use of a spring on a planar manipulator is more flexible, as it allows for

Table 3
Spring parameters to achieve gravity balance for the example manipulator.

	$a_{S(i,j)}$	$b_{S(i,j)}$	$k_{S(i,j)}$
$S_{1,4}$	0.060 (m)	0.064 (m)	718.67 (N/m)
$S_{1,2}$	0.150 (m)	0.240 (m)	294.00 (N/m)
$S_{2,4}$	0.100 (m)	0.064 (m)	2156.00 (N/m)
$S_{3,4}$	− 0.200 (m)	− 0.100 (m)	2195.20 (N/m)

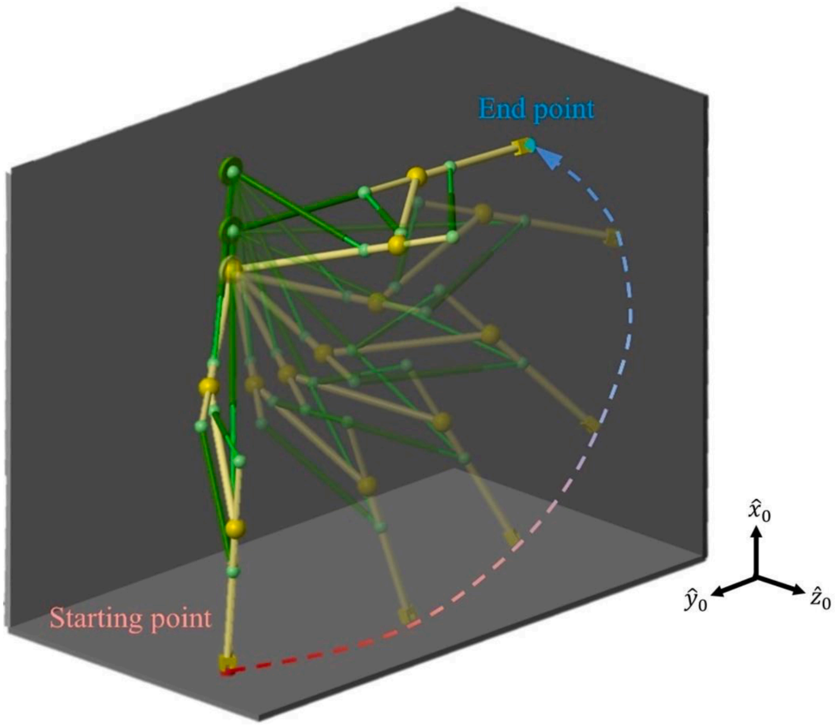

Fig. 15. Trajectory of the example manipulator.

Table 4
Initial posture and rotation of the links.

Initial posture			
	θ_i	ϕ_i	ψ_i
$i = 2$	0°	-50°	-20°
$i = 3$	0°	+20°	-20°
$i = 4$	+90°	-2°	-50°
Rotation of links			
$i = 2$	+21°	+60°	+36°
$i = 3$	±0°	+96°	+30°
$i = 4$	− 75°	-30°	+30°

mono-articulated springs (i.e., a spring attached between two adjacent links) to be attached to links with an angular offset. Based on the criteria of spring attachment, the rules for spring application to achieve the gravity balance of an arbitrary n-link spatial articulated manipulator were proposed, and the acceptable spring configurations for three- and four-link spatial manipulators were derived. At the end of the paper, a four-link spatial articulated manipulator was provided as an illustrative example. The simulation results showed that by following the method proposed in this paper the gravitational forces can be perfectly balanced in a manipulator's entire workspace. Note that this paper is based on a simplified model that considers only the gravity of links and the static motion of the manipulator. Consequently, in this idealized scenario, the theoretical required torque of actuators is zero, as the spring-manipulator system's energy remains constant. However, it is important to acknowledge that in real-world scenarios, other factors come into play, such as the weight and friction of actuators, as well as the dynamic behavior of the manipulator. These factors are crucial in practical applications and must be taken into account to obtain realistic torque values and to assess the method's effectiveness in real-world

Table 5
The trajectory of the example manipulator in the $\hat{x}_0$- $\hat{z}_0$ plane.

Initial posture			
	θ_i	ϕ_i	ψ_i
$i = 2$	0°	-40°	0°
$i = 3$	0°	-30°	0°
$i = 4$	0°	-30°	0°
Rotation of links			
$i = 2$	±0°	+120°	±0°
$i = 3$	±0°	+15°	±0°
$i = 4$	±0°	+15°	±0°

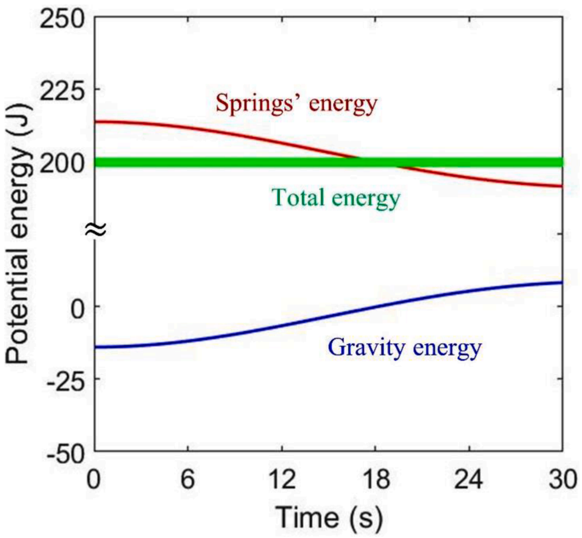

Fig. 16. Energy of the example manipulator during the simulated trajectory.

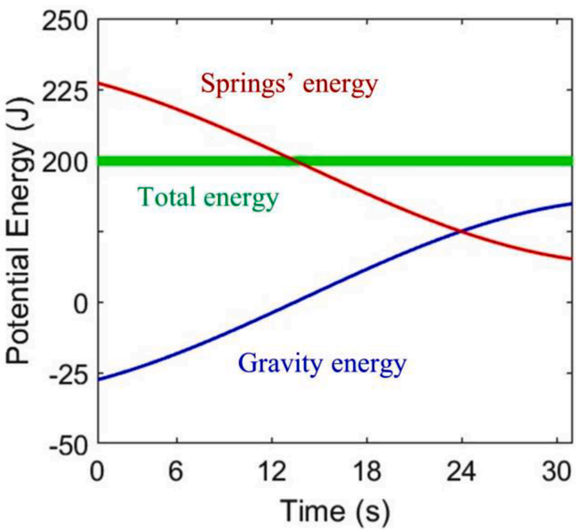

Fig. 17. Energy of the example manipulator during the planar trajectory.

scenarios.

Overall, this research provides a novel approach for achieving balance in spatial articulated manipulators, with potential applications in various fields such as the design of industrial robot arms or exoskeletons. By effectively counteracting the effect of gravity, it can reduce the loading on actuators and enhance machine efficiency. This has significant implications for improving the performance and overall functionality of manipulator systems in practical applications. This research can contribute to advancements in industrial automation, robotics, and other fields where precise and efficient manipulation is required.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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