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Geared Robot Manipulators with a Jointed Unit: Topological Synthesis and Its Application

Abstract

An efficient and systematic methodology for the topological synthesis of geared robot manipulators (GRMs) with a jointed unit is developed. The approach is based on the idea that the kinematic structure of a GRM with a jointed unit is composed of an equivalent open-loop chain (EOLC), disjointed mechanical transmission lines (MTLs), and jointed MTLs. It is shown that the jointed MTLs can be decomposed as a two-DoF nonfractionated jointed unit and several one-DoF disjointed units, connected in series. The characteristics of the jointed unit are laid out, and admissible jointed units are enumerated from the existing atlas accordingly. A systematic methodology is developed to enumerate admissible GRMs with a jointed unit of preferred DoF and number of links. It is also shown that the concept of jointed MTLs leads to the design of GRMs with decoupled joint motion by proper choices of gear ratios in the drive train.

KEY WORDS—topological synthesis, geared robot manipulator, decoupled joint motion, drive train design

1. Introduction

The kinematic structure of a robot manipulator often takes the form of an open-loop configuration. An open-loop robot manipulator is mechanically simple and easy to construct. However, it does require the actuators to be located along the joint axes, which, in turn, degrades the dynamic performance of the system. Thus, many robot manipulators are constructed in a partially closed-loop configuration to reduce the inertia loads on the actuators. For the case of geared robot manipulators (GRMs), gear trains are used to transmit mechanical power to various joints of the manipulators and to permit the actuators to be located as close to the base as possible. However, in general, the motion of each joint in the GRM is associated with multiple actuators, which causes the difficulty in the control phase.

Several approaches for the topological synthesis of GRMs have been developed based on graph theory. Lin and Tsai (1989b), based on the results of two-degree-of-freedom (DoF) nonfractionated geared kinematic chains (GKCs), enumerated the atlas of bevel-gear-type spherical wrist mechanisms with up to eight links. Belfiore and Tsai (1991) used the concept of partial separation between structure and function to generate geared robotic wrists with grounded actuators. However, by comparing these graph theory–based works of Lin and Tsai (1989b) and Belfiore and Tsai (1991), inconsistent results are found. Figure 1(a) shows the graph representation of a GRM found in Belfiore and Tsai (1991) but not in Lin and Tsai (1989b).

Chang and Tsai (1990) introduced the concept of mechanical transmission lines (MTLs) to describe the relations between joint torques and actuator torques of a GRM. Admissible structure matrices were generated to describe the coupling relationships between MTLs. Chen and Shiue (1998) showed that an MTL can be decomposed as disjointed units including an input unit (IU) and several transmission units (TUs) connected in series. Atlases of admissible input and transmission units were enumerated (Chen and Shiue 1998). Based on these units, they developed an approach to compose these units into MTLs according to the preferred number of links. With the configuration of the structure matrix, admissible GRMs can be enumerated by selecting suitable MTLs. As an extension, Chen and Liu (1999) used the identified units as design primitives and proposed a symbolic decomposition scheme to derive the composition polynomial, which describes the topological structure of desired GRMs. However, the GRM in Figure 1(a) (Belfiore and Tsai 1991) cannot be obtained by applying the above methods.

In this paper, a systematic methodology for the topological synthesis of GRMs with a jointed unit is developed from both

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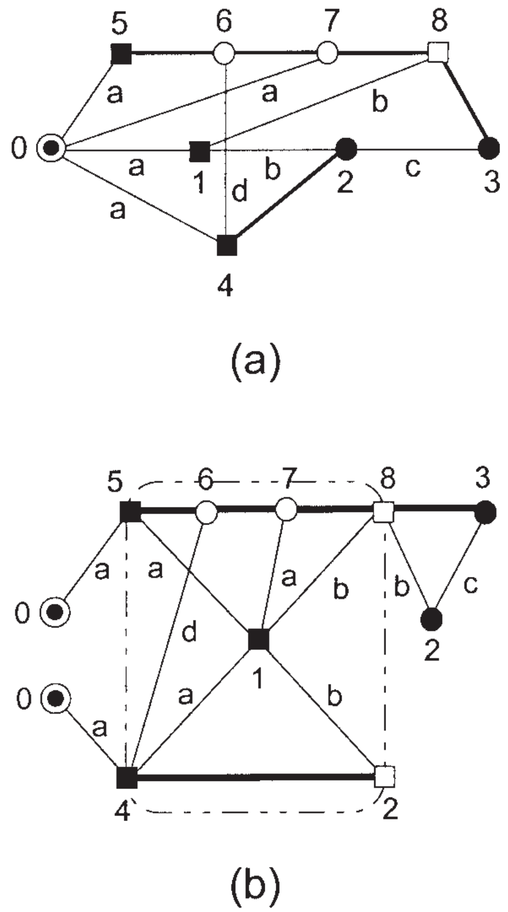


Fig. 1. (a) A three-DoF GRM. (b) A pseudo-isomorphic MTL subgraph.

a structure and mechanical coupling point of view. Admissible jointed units are first identified from the existing two-DoF nonfractionated GKCs (Lin and Tsai 1989a). It is shown that the construction of GRMs with a jointed unit can be treated as the creation of the jointed MTLs followed by adding preferred disjointed MTL(s). The minimum numbers of links of admissible two- and three-DoF ground-actuated GRMs with a jointed unit are derived, and the associated GRMs are enumerated accordingly. In contrast to coupled joint motions of GRMs with disjointed units only, it is shown that GRMs with a jointed unit can be designed to possess decoupled joint motions with proper selection of gear ratios in its drive train.

2. Geared Robot Manipulators

In graph representation, links are denoted by vertices and joints by edges, turning pairs by thin edges, and gear pairs by heavy edges. The thin edges are labeled according to their axis locations in space. By rearranging coaxial revolute joints, the canonical graph representation (Tsai 1988) can be obtained such that all edges lying on a thin-edge path traced from the base to any other vertex have different edge labels. Among these thin-edge paths, the linkage starts from the base link and ends at the output link and is defined as the equivalent open-loop chain (EOLC). Links in the EOLC are referred to as primary links while all others as secondary links (Chen and Tsai 1994).

For the graph representation of the GRM shown in Figure 1(a), links 1, 4, and 5 are the input links and link 3 is the output link. Links 0, 1, 2, and 3 are primary links, and links 4, 5, 6, 7, 8 are secondary links. In Figure 1(a), primary links are denoted by solid nodes, input links by solid rectangles, and other secondary links by hollow nodes.

The displacement vector at the actuator-space, Φ, and the displacement vector at the joint-space, Θ, of a GRM can be related using fundamental circuit theory and coaxial conditions (Tsai 1988), as

$$\Phi = \mathbf{A}^T \Theta. \tag{1}$$

Torque vectors at joint-space, τ, and at actuator-space, ξ, are related by

$$\tau = \mathbf{A}\xi, \tag{2}$$

where $\mathbf{A}$ is called the structure matrix (Chang and Tsai 1990) with gear ratios as its elements.

The arrangement of secondary links, which describes where the input actuators are located and how the input torques are transmitted to various joints, forms the MTLs. Through the MTLs, which consist of spur or bevel gear trains, torques are transmitted to the end-effector. From eq. (2), it can be seen that the ith row of matrix $\mathbf{A}$ describes how the resultant torque at joint i is affected by the input actuators while the kth column of matrix $\mathbf{A}$ describes how the torque of the kth actuator is transmitted to various joints. In what follows, GRMs on which the number of MTLs equals its number of DoF are focused.

2.1. Disjointed Units

Figures 2(a) and 2(b) show the functional and canonical graph representations of a three-DoF GRM. By separating common vertices and joints in the EOLC and by rearranging the coaxial links, pseudo-isomorphic MTL subgraphs (Lin and Tsai 1989a) are obtained and shown in Figure 2(c). From Figure 2(c), it can be seen that MTLs share primary links as common links and each MTL can be viewed as a two-link unit and three-link units connected in series. Chen and Shiue (1998) showed that each MTL is composed of one-DoF GKCs called the disjointed units since each of them belongs to one and only one MTL. Figures 3(a) and 3(b) show the admissible disjointed IUs and TUs enumerated by Chen and Shiue (1998), and the unit with the asterisk denotes the unit that can be flipped to form a different unit. Figure 4(a) shows a typical

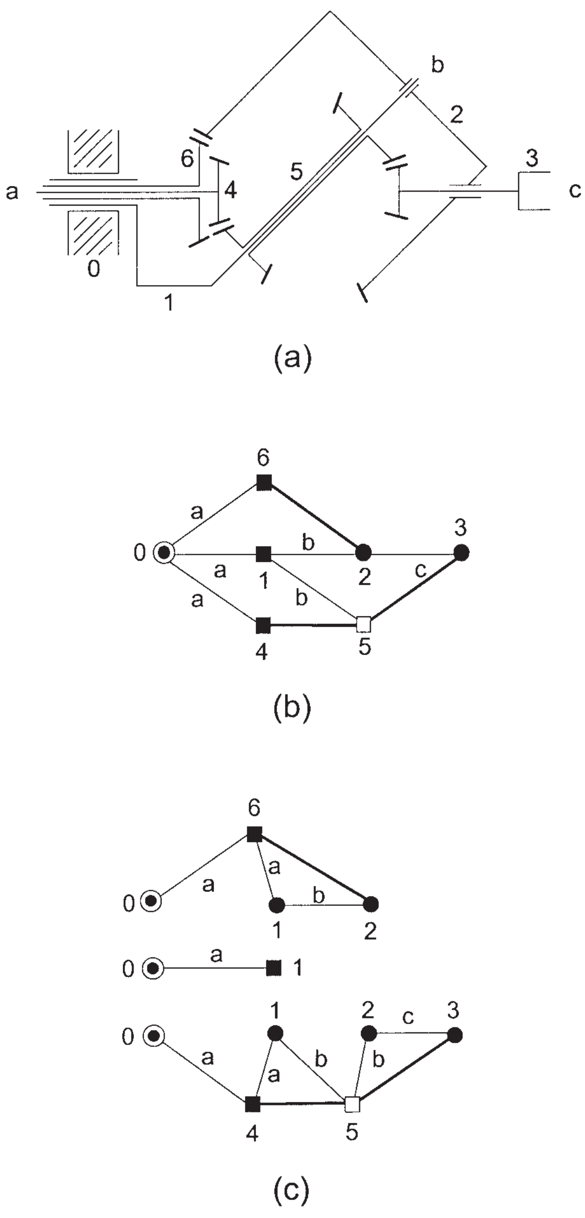


Fig. 2. A three-DoF disjointed GRM. (a) Functional representation. (b) Canonical graph representation. (c) Pseudo-isomorphic MTL subgraphs.

disjointed MTL. A disjointed unit is connected to its adjacent unit by sharing a common secondary link called a connecting link. A disjointed unit uses its postconnecting link to connect the preconnecting link of its succeeding disjointed unit. The common connecting links of adjacent disjointed units are coaxial with their associated primary links so that the articulated joint can be formed by rearranging the coaxial links. MTLs that consist of disjointed units only are called disjointed MTLs, and GRMs that consist of only disjointed MTLs are called disjointed GRMs. Since the torque transmission paths of a disjointed GRM are independent of each other, no gear ratio of the same gear pair appears in different columns of its structure matrix. In Figure 3, connecting links are denoted by hollow rectangles.

2.2. Jointed Units

By separating common vertices of the EOLC or part of the EOLC and their relative joints followed by rearranging the coaxial links, a pseudo-isomorphic MTL subgraph of the GRM in Figure 1(a) can be obtained and shown in Figure 1(b). From eq. (2), the torque vectors at joint-space and actuator-space of the GRM are related as

$$\begin{bmatrix} \tau_{1,0} \\ \tau_{2,1} \\ \tau_{3,2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & e_{24} & e_{65}e_{76}e_{87} + e_{24}(1 - e_{65}e_{76}) \\ 0 & 0 & e_{65}e_{76}e_{87}e_{38} \end{bmatrix} \begin{bmatrix} \xi_{1,0} \\ \xi_{4,0} \\ \xi_{5,0} \end{bmatrix}, \quad (3)$$

where $\tau_{1,0}$, $\tau_{2,1}$, and $\tau_{3,2}$ are the joint torques, $\xi_{1,0}$, $\xi_{4,0}$, and $\xi_{5,0}$ are the actuator torques applied at input links 1, 4, and 5, respectively, $e_{ij} = \pm N_i/N_j$ represents the gear ratio of the gear pair mounted on links i and j, positive or negative according to whether a positive rotation of gear i results in a positive or negative rotation of gear j about their predefined axes of rotation, and N_i is the number of teeth on gear i.

From eq. (3), it can be seen that gear ratio e_{24} appears in the second elements of the second and the third columns of the structure matrix. Thus, torque transmission paths of the second and third actuators are coupled at the second joint. The coupled MTLs are called jointed MTLs, and the two-DoF nonfractionated GKC enclosed by a dashed rectangle in Figure 1(b) is called a jointed unit, correspondingly. GRMs with at least one jointed unit are called jointed GRMs. In Figure 1(b), links 5 and 4 are the preconnecting links of the jointed unit, whereas links 8 and 2 are the postconnecting links. It can be seen that the postconnecting links 8 and 2 are coaxial with primary link 1 so that the joints between the postconnecting links and primary link 2 can be combined to form the second joint of the EOLC. Also, the preconnecting links 5 and 4 are coaxial with primary link 1 such that the

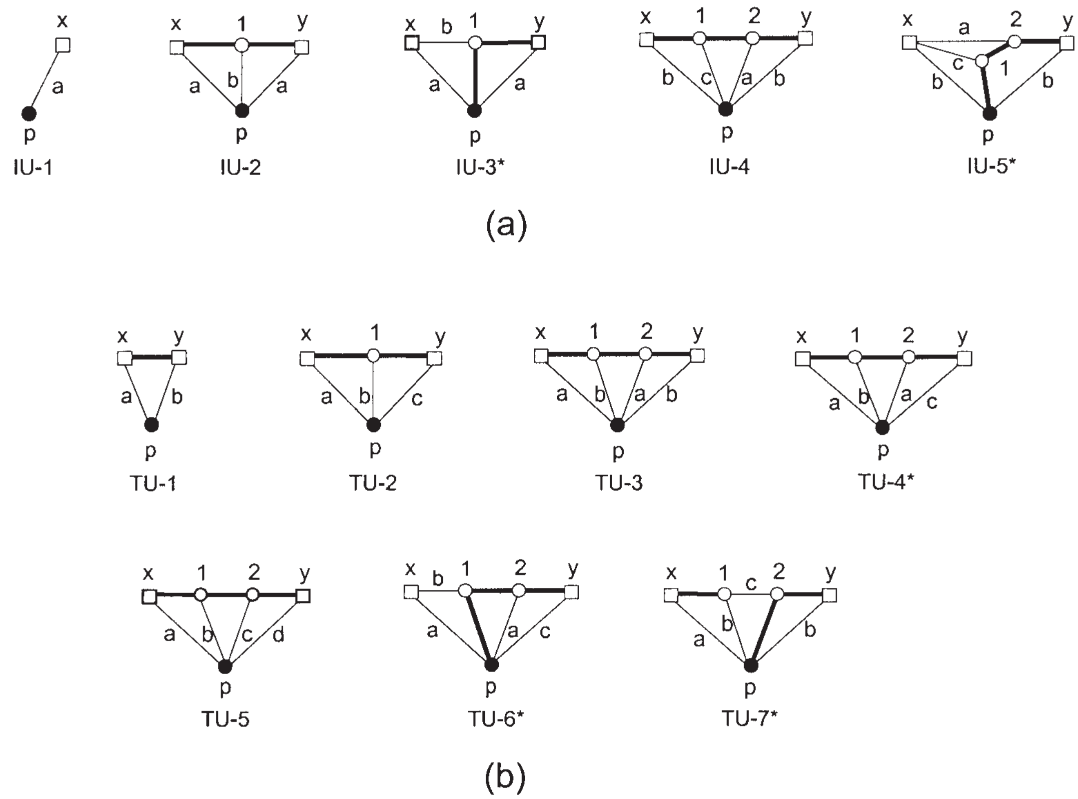


Fig. 3. (a) Admissible IUs. (b) Admissible TUs.

joints between the preconnecting links and primary link 1 can be combined to form the first joint of the EOLC.

3. Enumeration of Jointed Units

From Figure 1(b), it can be seen that the jointed unit is a two-DoF nonfractionated GKC with a primary link, two preconnecting links, and two postconnecting links. The jointed unit uses its postconnecting links to connect the preconnecting links of its succeeding disjointed units while it uses its preconnecting links to connect the postconnecting links of its preceding disjointed units. Figure 4(b) shows the typical jointed MTLs with a jointed unit. The characteristics of a jointed unit can be summarized as follows:

C1. A jointed unit is a two-DoF nonfractionated GKC.

C2. There is one and only one primary link.

C3. There are two links, called postconnecting links, which are incident and coaxial with the primary link.

C4. There are two links, called preconnecting links, which are incident and coaxial with the primary link.

C5. The preconnecting link, primary link, and postconnecting link of a jointed unit form a thin-edge path with two distinct edge labels.

Based on these characteristics, admissible jointed units can be systematically identified from the atlas of two-DoF nonfractionated GKCs (Lin and Tsai 1989a). The identification process can be broadly summarized as follows:

- Step 1. Determine the primary link: For a graph in the atlas, the vertex coaxial with at least two sets of vertices of distinct edge labels is presumed as the primary link.
- Step 2. Determine the postconnecting links and preconnecting links: From a set of coaxial links of the primary link, two of the vertices are assigned as the postconnecting links. From one of the other set of coaxial links of the primary link, two of the vertices are assigned as the preconnecting links.
- Step 3. Check redundancy: A link is considered to be a redundant link if removal of the link and its associated joints from the graph does not change the torque transmission route between pre- and postconnecting links. The graph with redundant links should be excluded.

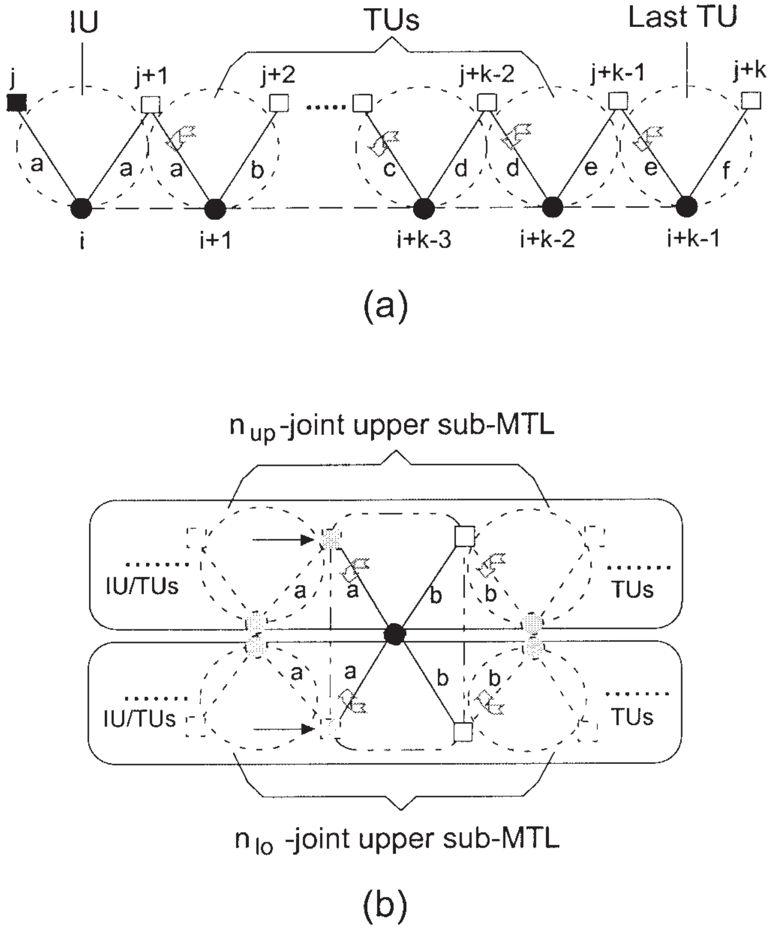


Fig. 4. (a) A typical disjointed MTL. (b) The typical jointed MTLs.

- Step 4. Check independence of preconnecting links: Preconnecting links are said dependent of each other if they are in the heavy-edge path with a common transfer vertex. The GKC with independent preconnecting links is identified as an admissible jointed unit.

- Step 5. Repeat steps 2 through 4 for each selected primary link until all possible assignments of post- and preconnecting links are found.

- Step 6. Repeat steps 1 through 5 until all possible assignments of primary link are examined.

Figure 5 shows the graph representations of admissible jointed units with up to seven links. The number in the parentheses is the identification code from Lin and Tsai (1989b). It can be seen that JU-2 and JU-4 are obtained by assigning the postconnecting links of JU-1 and JU-3 as preconnecting links and the preconnecting links of JU-1 and JU-3 as postconnecting links, respectively. Note that the jointed unit in Figure 1(b) is JU-1.

4. Decomposition of Number of Links

For an n-DoF jointed GRM with a jointed unit, there are $(n+1)$ primary links and n MTLs. Of these n MTLs, there are $(n-2)$ disjointed MTLs to be added to the jointed MTLs. There are $n_j + 1$ primary links for the m_j-link n_j-joint jointed MTLs while there are $n_d)_i + 1 (i = 1, \ldots, n-2)$ primary links for the $m_d)_i$-link $n_d)_i$-joint disjointed MTL. The number of links of an n-DoF jointed GRM, m, can be obtained as

$$m = (n+1) + [m_j - (n_j + 1)] + \sum_{i=1}^{n-2} \{m_d)_i - [n_d)_i + 1]\} \tag{4}$$

or

$$m = n + m_j - n_j + \sum_{i=1}^{n-2} \{m_d)_i - [n_d)_i + 1]\}. \tag{5}$$

From Figure 4(b), it can be seen that jointed MTLs can be separated as an upper sub-MTL and a lower sub-MTL,

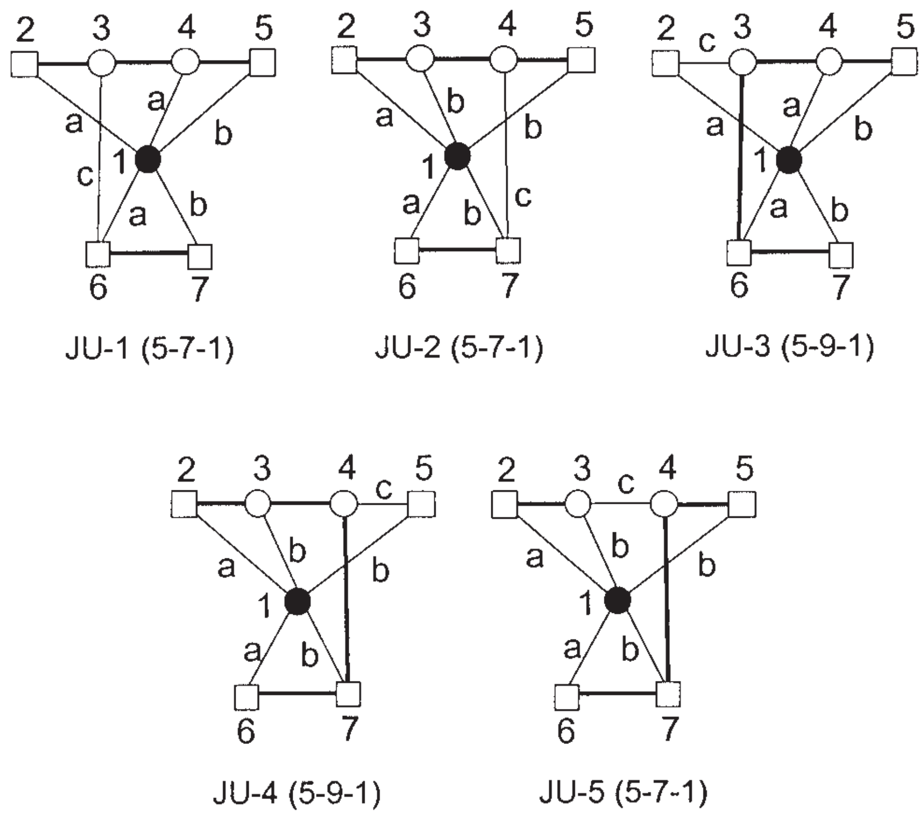


Fig. 5. Admissible jointed units.

which are able to influence different numbers of joints. The upper and lower sub-MTLs can be further decomposed to disjointed units with a common jointed unit. The numbers of components in the upper and lower sub-MTLs are equal to the number of joints they influence. Hence, an n_{up}-joint upper sub-MTL has one jointed unit and $(n_{up} - 1)$ disjointed units, $(n_{up} - 1)$ common connecting links, and $(n_{up} + 1)$ primary links. Similarly, an n_{lo}-joint upper sub-MTL has one jointed unit and $(n_{lo} - 1)$ disjointed units, $(n_{lo} - 1)$ common connecting links, and $(n_{lo} + 1)$ primary links. Since the primary link is counted in both upper and lower sub-MTLs, the number of links of the jointed MTLs, m_j, can be expressed as

$$m_j = m_{iu/up} + \sum_{j=1} m_{tu/up})_j - (n_{up} - 1) - (n_{up} + 1) + m_{iu/lo} + \sum_{j=1} m_{tu/lo})_j - (n_{lo} - 1) - (n_{lo} + 1) + (n_j + 1) + m_{ju} + 1 \quad (6)$$

or

$$m_j = m_{iu/up} + \sum_{j=1} m_{tu/up})_j + m_{iu/lo} + \sum_{j=1} m_{tu/lo})_j - 2n_{up} - 2n_{lo} + n_j + m_{ju} + 2, \quad (7)$$

where m_{ju} is the number of links of the jointed unit, $m_{iu/xx}$ and $m_{tu/xx})_j$ are the number of links of the IU and the jth TU of upper and lower sub-MTLs, respectively.

5. The Creation of Jointed GRMs

The creation of admissible jointed GRMs with a jointed unit of preferred DoF and number of links can be treated as the problem of adding disjointed MTL(s) to the jointed MTLs. The enumeration process can be broadly summarized as follows:

- Step 1: Determine the number of links of the jointed MTLs. The number of links of the jointed MTLs can be determined from the preferred DoF and number of links of the jointed GRM, the number of joints influenced by the jointed MTLs, and the number of links and joints influenced by each disjointed MTL. From eq. (5), the number of links of the jointed MTLs, m_j, can be written as

$$m_j = m - n + n_j - \sum_{i=1}^{n-2} \{m_d)_i - [n_d)_i + 1]\}. \quad (8)$$

- Step 2: Determine the arrangement sequence of the units in the jointed MTLs. The distribution of m_j to the upper and lower sub-MTLs can be determined by eq. (7) by specifying the numbers of joints that are influenced by the upper and lower sub-MTLs. Hence, the arrangement sequence of the units in the jointed MTLs can be evaluated by properly choosing the disjointed units and jointed units from the atlas.

- Step 3: Connect the units and construct the joints by rearranging the coaxial links. A jointed unit is selected from Figure 5 and connected with the selected disjointed units by sharing the common connecting links between adjacent units according to the arrangement sequence. By rearranging the coaxial links between connecting links and primary links, articulated joints are formed.

- Step 4: Assign the last postconnecting link(s) as primary link(s). Assigning the postconnecting link(s) of the last unit in the arrangement sequence as the primary link completes the formation of jointed MTLs. In cases where only one postconnecting link of the jointed unit is succeeded with a disjointed unit, the other postconnecting link is assigned as the primary link of the succeeding disjointed unit. Note that by assigning postconnecting link 7 as a primary link, jointed units JU-2, JU-4, and JU-5 as shown in Figure 5 become disjointed. Hence, JU-2, JU-4, and JU-5 are not valid jointed units if the postconnecting link in their lower sub-MTL has to be assigned as a primary link. In cases where the jointed unit is the last unit in the arrangement sequence, both postconnecting links are assigned the last primary link. Note that JU-2, JU-4, and JU-5 are not valid if they are used as the last unit in the arrangement sequence since their postconnecting links are in a same fundamental circuit. Hence, only JU-1 and JU-3 can be the last unit in the arrangement sequence.

- Step 5: Permute the IU and TU and repeat steps 3 and 4 until all the possible arrangement sequences are accessed.

- Step 6: Permute the JU, repeat steps 3 through 5 until all valid jointed units are accessed.

- Step 7: Add the disjointed MTLs to the jointed MTLs. The disjointed MTL(s) are added to the jointed MTLs according to the specified configuration of the jointed GRM by sharing the common primary links.

For the purpose of demonstration, two- and three-DoF ground-actuated jointed GRMs with the simplest configurations will be used for the following discussion. A jointed GRM is referred to with the simplest configuration if it has the least number of units. For two-DoF ground-actuated jointed GRMs, the jointed unit is either located at the first or the second joint. In cases where the jointed unit is at the first joint, the succeeding TU(s) can be located in the upper sub-MTL, the lower sub-MTL, or both upper and lower sub-MTLs. Among these configurations, the two with only one TU are the simplest configurations. In cases where the jointed unit is at the second joint, the configuration with two preceding IUs located in both upper and lower sub-MTLs to ensure grounded actuators is the simplest configuration. Figure 6(a) shows the simplest configurations of two-DoF ground-actuated jointed GRMs. For three-DoF ground-actuated jointed GRMs, a direct drive disjointed MTL located at the first joint is added since it has the minimum number of links and joints. The simplest configurations can be obtained with the jointed unit located at the first, the second, or the third joint. Figure 6(b) shows the simplest configurations of three-DoF ground-actuated jointed GRMs. Note that the thin lines in Figure 6(b) denote the direct drive disjointed MTLs.

Since all admissible jointed units are seven-link, the minimum number of links of each configuration in Figure 6 can be evaluated by choosing the IU-1 and TU-1 as the disjointed units that have the least number of links within admissible IUs and TUs. For the configurations shown in Figure 6(a), since two-DoF jointed GRMs are composed of only jointed MTLs, the minimum numbers of links, $m)_{\min}$, of the C-1, C-2, and C-3 configurations can be found as 8, 8, and 7, respectively, by eq. (7). Hence, the C-3 configuration has the minimum number of links within the simplest configurations of two-DoF ground-actuated jointed GRMs. For the configurations shown in Figure 6(b), since the direct drive disjointed MTLs, which have two links ($m_d = 2$) and one joint ($n_d = 1$), are added to the jointed MTLs, the minimum number of links of each configuration can be related to the minimum number of links of the jointed MTLs, $m_j)_{\min}$, by eq. (5).

$$m)_{\min} = 3 + m_j)_{\min} - 3 + [2 - (1 + 1)] = m_j)_{\min}. \quad (9)$$

Thus, the minimum numbers of links of the C-4, C-5, C-6, C-7, and C-8 configurations can be found as 10, 10, 9, 9, and 10, respectively, by eq. (7). Hence, the C-6 and C-7 configurations have the minimum number of links within the simplest configurations of three-DoF ground-actuated jointed GRMs.

Table 1 shows the combinations of jointed and disjointed units of admissible two- and three-DoF ground-actuated

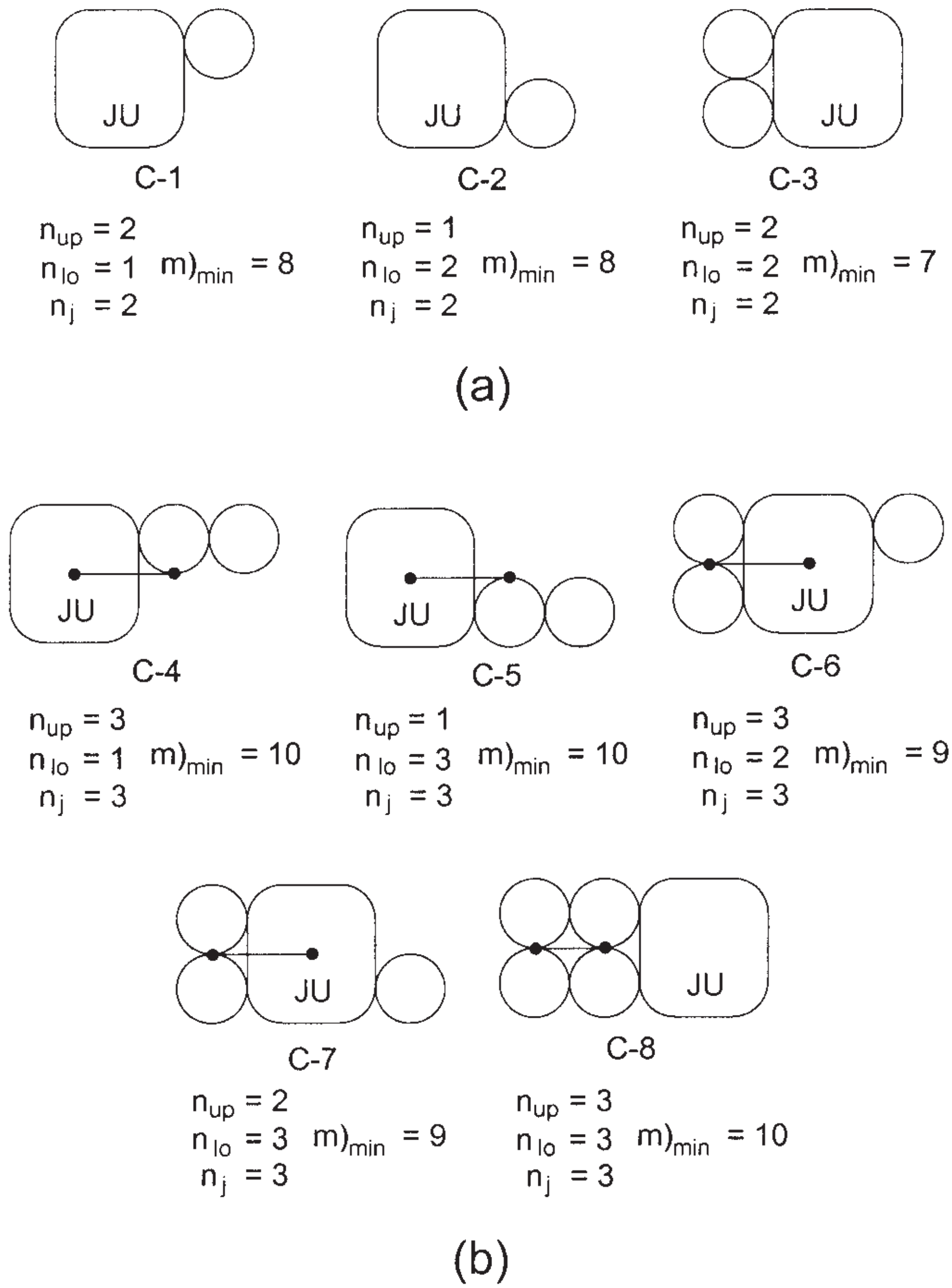
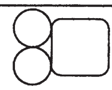
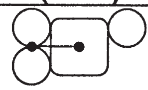
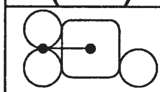


Fig. 6. Admissible two- and three-DoF ground-actuated jointed GRMs with simplest configurations.

Table 1. Admissible Two- and Three-DoF Ground-Actuated Jointed GRMs of Minimum Number of Links

n	m	configuration	No. of links of units / No. of admissible units			subtotal	total
2	7	 (C-3)	2/1	7/2		2	2
			2/1				
3	9	 (C-6)	2/1	7/2	3/1	2	7
			2/1		0		
		 (C-7)	2/1	7/5	0	5	
			2/1		3/1		

jointed GRMs with the minimum number of links. For the two-DoF configuration C-3 in Table 1, since IU-1 is the only two-link IU within admissible IUs and only JU-1 and JU-3 can be used as the jointed unit, two admissible jointed GRMs are obtained and Figure 7(a) shows their canonical graph representations. Similarly, for the three-DoF configurations in Table 1, since only JU-1 and JU-3 can be used as the jointed unit with the C-6 configuration while all JUs can be used as the jointed unit with the C-7 configuration, two and five admissible jointed GRMs with each configuration are obtained and Figure 7(b) shows their canonical graph representations. Note that in Figure 7, the jointed GRM of C-X-# indicates the C-X configuration and # is its series number.

Comparison among graph theory–based works done by Lin and Tsai (1989b), Belfiore and Tsai (1991), and this paper shows the following results. It can be seen that all graphs in the atlas enumerated by Lin and Tsai (1989b) are composed of disjointed units only, while seven of the nine-link graphs with grounded actuators in the atlas enumerated by Belfiore and Tsai (1991) consist of a jointed unit. However, the graphs of 9-2-3-06* and 9-2-3-08* in Belfiore and Tsai (1991) should be excluded since the preconnecting links of the jointed units in them are dependent. In Figure 7(b), the graphs of C-6-2 and C-7-4 containing JU-3 as the jointed unit are precluded by Belfiore and Tsai (1991) since JU-3 has one preconnecting link transmitting power by revolute joint while the other by gear pair.

6. An Application

Admissible jointed GRMs with the C-1 configuration in Figure 6(a) of minimum number of links are used as illustrative examples. Figure 8(a) shows the canonical graph representations of the eight-link jointed GRMs, with JU-1 and JU-3 as the jointed unit, respectively, and TU-1 as the disjointed unit. For configuration C-1-1 in Figure 8(a), the input angular displacements, $q_{3,0}$ and $q_{7,0}$, can be described as the function of the joint angular displacements, $\theta_{1,0}$ and $\theta_{2,1}$, by eq. (1).

$$\begin{bmatrix} q_{7,0} \\ q_{3,0} \end{bmatrix} = \begin{bmatrix} e_{17} & 0 \\ (1 - e_{43}e_{54})e_{17} + e_{43}e_{54}e_{65} & e_{43}e_{54}e_{65}e_{26} \end{bmatrix} \begin{bmatrix} \theta_{1,0} \\ \theta_{2,1} \end{bmatrix}. \quad (10)$$

By applying eq. (2), the joint torques, $\tau_{1,0}$ and $\tau_{2,1}$, and the actuator torques applied at the input links 7 and 3, ξ_7 and ξ_3, can be related by structure matrix as

$$\begin{bmatrix} \tau_{1,0} \\ \tau_{2,1} \end{bmatrix} = \begin{bmatrix} e_{17} & (1 - e_{43}e_{54})e_{17} + e_{43}e_{54}e_{65} \\ 0 & e_{43}e_{54}e_{65}e_{26} \end{bmatrix} \begin{bmatrix} \xi_7 \\ \xi_3 \end{bmatrix}. \quad (11)$$

By making the off-diagonal element of the structure matrix in eq. (11) equal to zero, we have

$$e_{34}e_{45} = 1 - \frac{e_{65}}{e_{17}} \quad (12)$$

where e_{34} and e_{45} are the reciprocals of e_{43} and e_{54}.

Thus, eq. (11) can be rewritten as

$$\begin{bmatrix} \tau_{1,0} \\ \tau_{2,1} \end{bmatrix} = \begin{bmatrix} e_{1,7} & 0 \\ 0 & e_{4,3}e_{5,4}e_{6,5}e_{2,6} \end{bmatrix} \begin{bmatrix} \xi_7 \\ \xi_3 \end{bmatrix}. \quad (13)$$

For the condition in eq. (12) to hold, one of the following relations between these gear ratios must be retained:

- e_{34} and e_{45} are of the opposite sign: e_{65} and e_{17} are of the same sign and their absolute values satisfy the inequality $|e_{65}| > |e_{17}|$, or

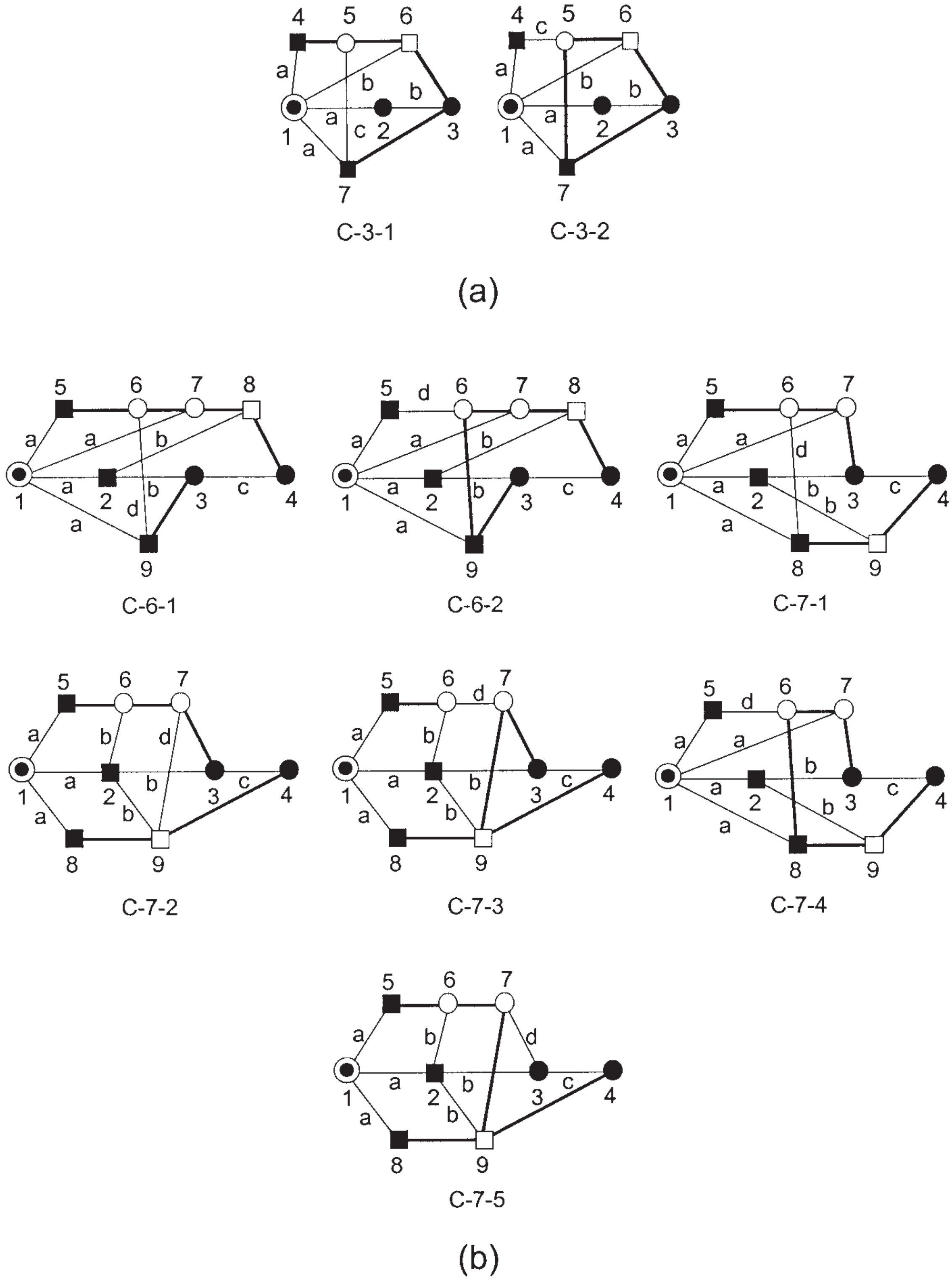


Fig. 7. Admissible ground-actuated jointed GRMs of minimum number of links. (a) Two-DoF case. (b) Three-DoF case.

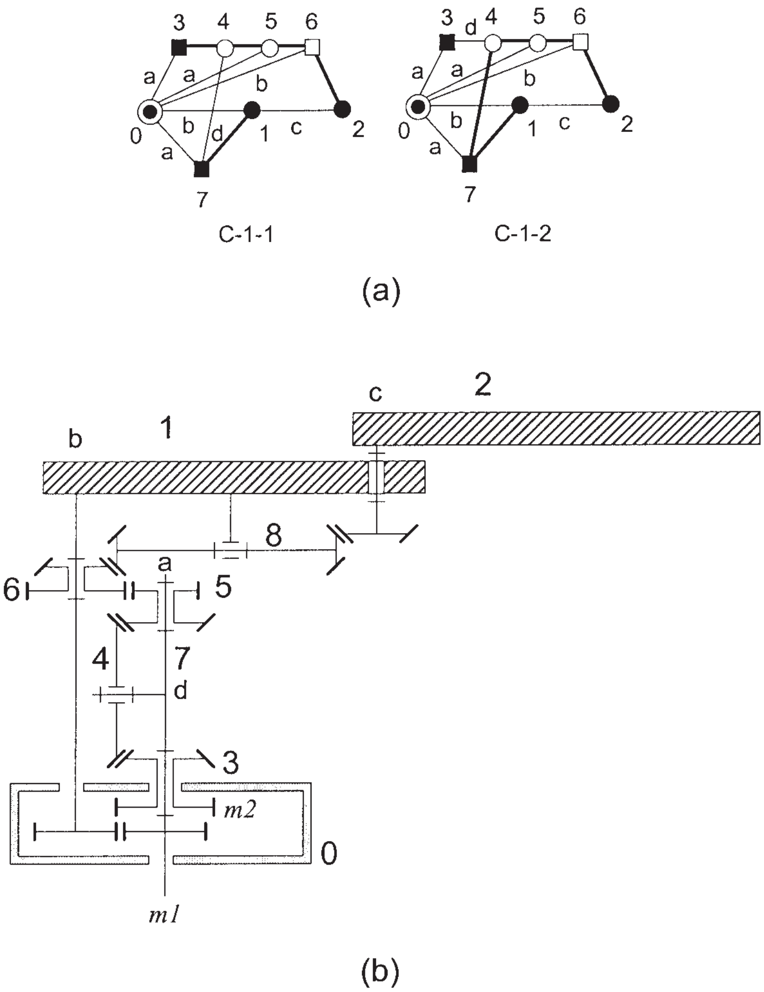


Fig. 8. (a) Admissible jointed GRMs with C-1 configuration. (b) The functional representation.

- e_{34} and e_{45} are of the same sign: (i) e_{65} and e_{17} are of the same sign and their absolute values satisfy the inequality $|e_{65}| < |e_{17}|$, or (ii) e_{65} and e_{17} are of the opposite sign.

Figure 8(b) shows one of the possible functional schematic drive train arrangements of case (a). It can be seen that bevel gear 4 meshes with gears 3 and 5 simultaneously, which ensures the opposite signs of e_{34} and e_{45}, and the external gear pairs mounted on gears 1, 7, and gears 6, 5 ensure the negative values of e_{65} and e_{17}. Note that in Figure 8(b), link 8 is used as an idler gear to extend the center distance between links 2 and 6. Since link 8, which is attached to the arm, is the only floating gear traveling in the space with the arm movement, the mechanical structure of moving components of the arm mechanism shown in Figure 8(b) is compact to avoid the degradation of dynamic performance. From eq. (13), it can be seen that the coupled effect of joint motions of GRMs is eliminated since the resultant joint torques, $\tau_{1,0}$ and $\tau_{2,1}$, are only influenced, respectively, by actuator torques, ξ_7 and ξ_3. Hence, the jointed GRMs with remotely located actuators can be designed to possess independent jointed motions through gear ratio determination in its drive train design.

7. Conclusion

In this paper, kinematic structures of jointed GRMs are investigated. It is shown that the kinematic structure of a jointed GRM can be viewed as a combination of the EOLC and several MTLs including jointed and disjointed MTLs. It is also shown that jointed MTLs can be viewed as a jointed unit and several disjointed units connected together. Rules for the enumeration of jointed units are outlined, and admissible jointed units are obtained accordingly. With the atlases of admissible jointed and disjointed units, a systematic methodology for the enumeration of jointed GRMs with preferred DoF and the number of links is developed. The methodology has been demonstrated by the enumeration of admissible two- and three-DoF ground-actuated jointed GRMs with minimum number of links. It is shown that a jointed GRM with remotely located actuators can be designed to possess decoupled joint motions with proper choices of gear ratios on the drive train.

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