

Topological Synthesis of Fractionated Geared Differential Mechanisms

Dar-Zen Chen*

Kang-Li Yao

Department of Mechanical Engineering,
National Taiwan University,
Taipei, Taiwan 106 R.O.C

An efficient and systematic methodology for the topological synthesis of admissible fractionated geared differential mechanisms is presented. Based on the extension of the 2-dof automotive gear differential, it is shown that a fractionated geared differential mechanism can be decomposed into a main component and an input component. Characteristics of these two components are laid out, and the atlases of admissible input and main components are identified from the existing atlases of non-fractionated geared kinematic chains. With a systematic procedure to choose input and main components and select admissible connecting links, fractionated geared differential mechanisms with three and four input/output links are generated accordingly. [S1050-0472(00)00804-7]

1 Introduction

Figure 1(a) shows the functional representation of the 2 degree-of-freedom (dof) geared differential mechanism (GDM). The 1-input, 2-output GDM has been widely known and used in automotive drive systems. Hirose [1] introduced a general form and derived the kinematic constraint equation among input/output links of the 1-input, 2-output GDM. By assigning the function of one-degree vertices of admissible gear-pair-only tree, Yan and Hsieh [2] derived a methodology to synthesize the atlas of non-coupled and coupled 2-dof GDMs. However, structural isomorphism on GDMs enumerated by this method can not be averted and the methodology is restricted to GDMs with 1 input and 2 outputs. By adding a ground to a set of coaxial links of the 1-dof geared kinematic chain (GKC) and assigning three of the coaxial links as input/output links, Hsu and Wu [3] identified the atlas of 2-dof GDMs with 1 input and 2 outputs. However, their approach is only applicable to 2-dof GDMs and the atlas of 2-dof GDMs are lack of recognition of admissible input/output links. Kota and Bidare [4] used the automotive gear differential and a 5-link epicyclic gear train as building blocks a novel whiffletree-like multi-outputs GDMs. By adjusting the train values in the differential blocks, the number of outputs in the GDM can be any integer greater than two.

In this paper, a systematic methodology for the topological synthesis of admissible fractionated GDMs is developed. The characteristics of the 1-input, 2-output gear differential are clarified and extended to multi-output GDMs. It is shown that a GDM can be decomposed into an input component and a main component. Atlases of admissible input and main components are identified from the existing atlases of non-fractionated GDMs. With specified number of links and number of input/output links of the GDM, proper input and main components can be chosen from the atlases. Thus, admissible GDMs can be enumerated by joining the selected connecting links of the input and main components accordingly.

2 Fundamental Characteristics of Geared Differential Mechanisms

For the automotive gear differential shown in Fig. 1(a), link 1 is the input, links 2 and 3 are the outputs, and links 1, 2, and 3 are adjacent to link 0, the ground. In graph representation, links are denoted by vertices, gear pairs by heavy edges, revolute joints by thin edges, and the thin edges are labeled according to their axes

locations in space. Figure 1(b) shows a pseudo-isomorphic graph [5] of the gear differential shown in Fig. 1(a). In Fig. 1(b), the input and output links are denoted by rectangles. From Fig. 1(b), it can be seen that vertex 5 is a cut-vertex which divides the GDM into two components. The first component (0, 1, 5), a 1-dof geared mechanism using vertex 1 as the input and vertex 5 as the local output, is called the input component (IC) of the GDM. The second component (2, 3, 4, 5), a 1-dof GKC using vertex 5 as the local input and vertices 2 and 3 as the outputs, is called the main component (MC) of the GDM.

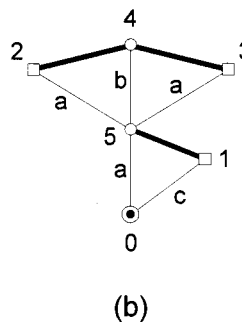
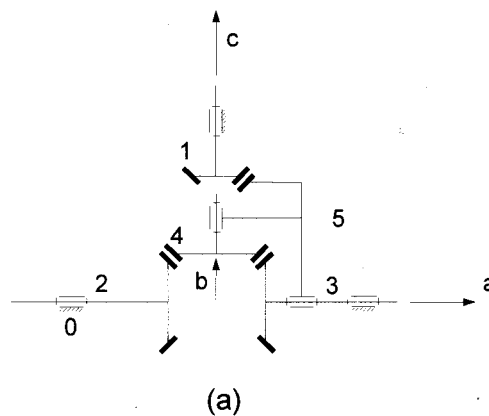


Fig. 1 The standard 2-dof GDM. (a) Functional representation; (b) pseudo-isomorphic graph representation.

*Corresponding author, e-mail: dzchen@ccms.ntu.edu.tw

Contributed by the Mechanisms Committee for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received November 1998. Associate Technical Editor: C. Innocenti.

In Fig. 1(a), fundamental circuits are (2, 4)(5), (3, 4)(5), and (1, 5)(0). The associated fundamental circuit equations [6] can be written as

$$\begin{bmatrix} 0 & 1 & 0 & -\gamma_{4,2} & \gamma_{4,2}-1 \\ 0 & 0 & 1 & -\gamma_{4,3} & \gamma_{4,3}-1 \\ 1 & 0 & 0 & 0 & -\gamma_{5,1} \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} = 0 \quad (1)$$

where w_i denotes the angular velocity of link i , $\gamma_{i,j} = \pm N_i/N_j$ denotes the gear ratio of the gear pair composed of links i and j and the sign of the gear ratio is determined according to the positive rotation of link i results in a positive or negative rotation of link j along their pre-defined axes of rotation, and N_i is the teeth number of gear i .

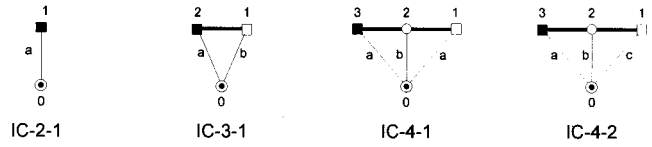
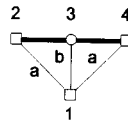


Fig. 2 Admissible ICs with up to 4 links

By eliminating w_4 and w_5 in Eq. (1), kinematic constraint equation among angular velocities of the input and output links, w_1 , w_2 , and w_3 , can be obtained as

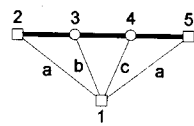
$$(\gamma_{3,2}-1)w_1 + \gamma_{5,1}w_2 - \gamma_{5,1}\gamma_{3,2}w_3 = 0 \quad (2)$$

where

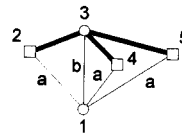


MC-3-4-1

(a)

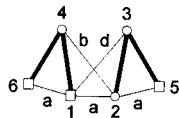


MC-3-5-1

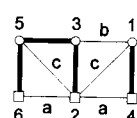


MC-3-5-2

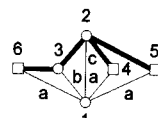
(b)



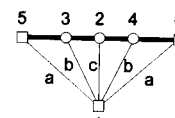
MC-3-6-1



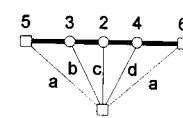
MC-3-6-2



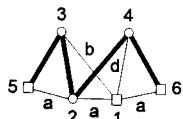
MC-3-6-3



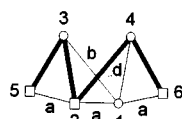
MC-3-6-4



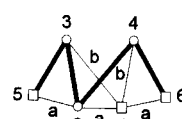
MC-3-6-5



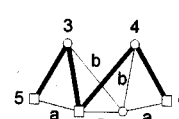
MC-3-6-6



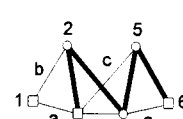
MC-3-6-7



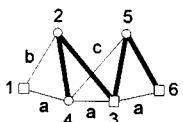
MC-3-6-8



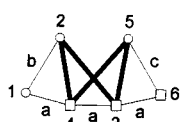
MC-3-6-9



MC-3-6-10



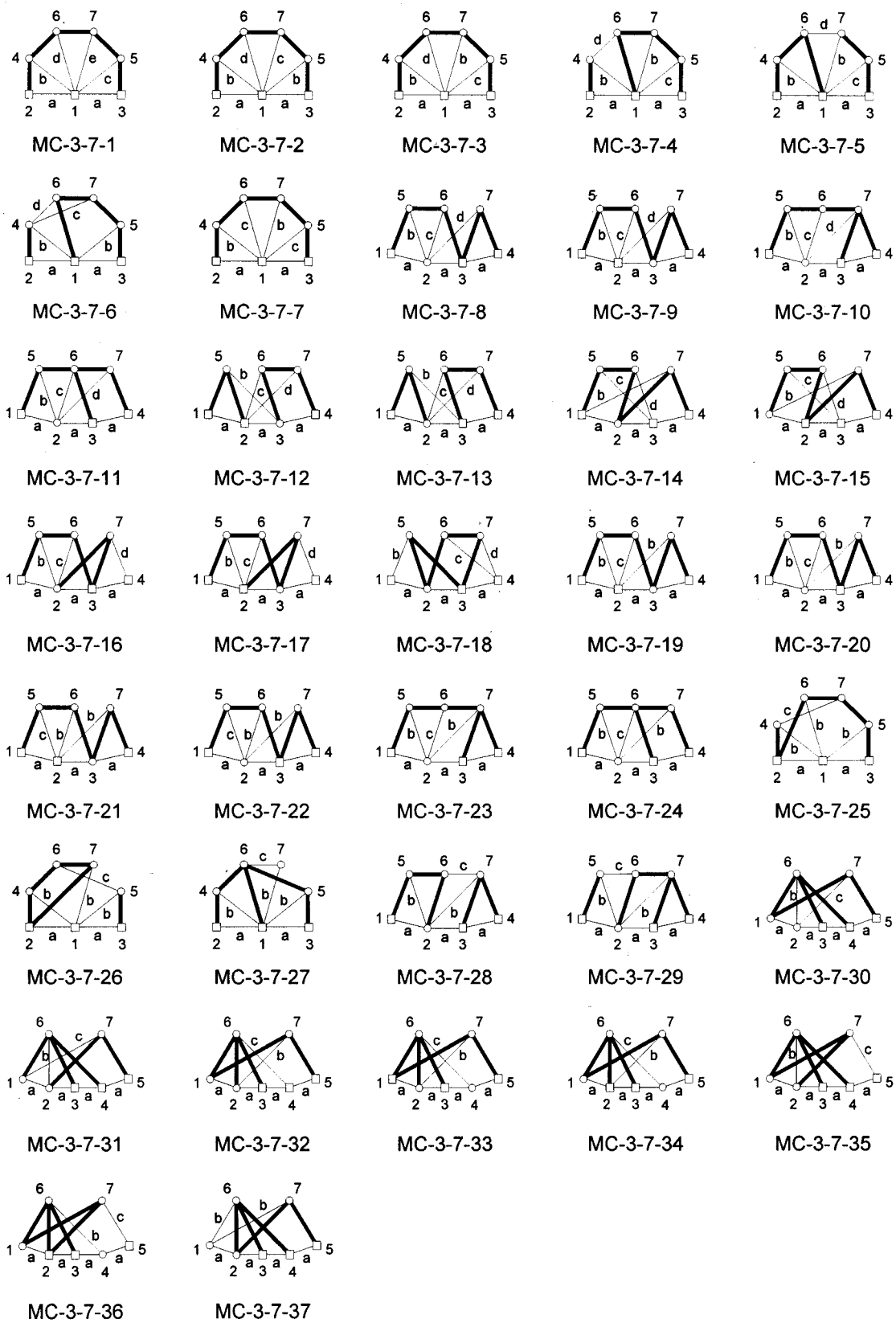
MC-3-6-11



MC-3-6-12

(c)

Fig. 3 Admissible 3-port MCs with up to 7 links. (a) 4-link; (b) 5-link; (c) 6-link; (d) 7-link.



(d)

Fig. 3 (Continued)

$$\gamma_{3,2} = \frac{\gamma_{4,2}}{\gamma_{4,3}} \quad (3)$$

In the following discussion, GDMs with one input and $(n-1)$ outputs ($n \geq 3$) will be focused on. As an extension, the fractionated GDM is assumed to be decomposed into a geared mechanism as the IC and a GKC as the MC while the two components are connected by a cut-link. By extending Eq. (2), it is assumed that the kinematic constraint equations among the input and $(n-1)$ outputs of a GDM can be written as

$$\sum_{i=1}^n a_i \cdot w_i = 0 \quad (4)$$

where a_i and w_i are the proportional constant and angular velocity associated with the i -th input or output link.

It can be seen that the number of independent parameters in Eq. (4) is $(n-1)$ since there is one equation with n variables. This is the case of an $(n-1)$ -dof geared mechanism, since $(n-1)$ independent parameters are needed to specify its motion. From Fig. 1(b), it can be seen that the outputs and the ground of the standard GDM are in opposite components. Since there is only one input in the $(n-1)$ -dof GDM, the IC is a 1-dof geared mechanism while the MC is an $(n-2)$ -dof GKC. In addition, it is observed that the cut-link that connects the two components, the outputs, and the ground are coaxial. Thus, we have

C1. A fractionated GDM with n terminal links is an $(n-1)$ -dof geared mechanism, where a terminal link is defined as an input or output link. The terminal links in a GDM are adjacent to the ground link.

C2. There is one kinematic constraint equation among the terminal links of the GDM. The kinematic constraint equation can be written as Eq. (4).

C3. An $(n-1)$ -dof fractionated GDM can be decomposed into a 1-dof geared mechanism as the IC and an $(n-2)$ -dof GKC as the MC. The IC and MC share a common link but no common joint. This common link is called the connecting link.

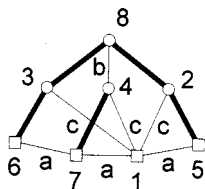
C4. The ground link, the connecting link and the output links of a fractionated GDM are coaxial.

3 Admissible Input/Main Components

With the concept of decomposing GDMs, characteristics of IC and MC can be further established. The 1-dof IC includes the input and the ground of the GDM. A link, which is adjacent to the ground in the IC, is used as the local output of the component and as the connecting link to the MC. Thus, we have

C5. The IC of a fractionated GDM is a 1-dof geared mechanism with its input and local output adjacent to ground.

Figure 2 shows admissible ICs with up to 4 links. In Fig. 2, the connecting link is represented by a solid rectangle, input link by a hollow rectangle and each m -link IC is coded as IC- m -# with # as its series number. Note that the two-link chain, IC-2-1, is treated as a degenerate geared mechanism in which the connecting link is used as the input link.



MC-4-8-1

Fig. 4 Admissible 4-port MC with up to 8 links

The $(n-2)$ -dof MC includes $(n-1)$ outputs of the GDM. A link of the MC, which is coaxial to the outputs, is used as the local input and as the connecting link to the IC. This leads to the following characteristic:

C6. The MC of an n -terminal fractionated GDM is an $(n-2)$ -dof GKC with n ports, where a port is defined as an output or local input link. All ports in the MC are coaxial.

Since the connecting link of the MC and the connecting link of the IC are to be combined as the cut-link of the GDM, kinematic property of the input can be expressed in terms of that of the connecting link. Hence, Eq. (4), the kinematic constraint equation among the terminals of the GDM, can be rewritten as the kinematic constraint equation among the ports of the MC. Thus, we have

C7. There is one kinematic constraint equation among the ports of a MC. The kinematic constraint equation can be written as

$$\sum_{i=1}^{n-1} a_i \cdot w_i + b_c \cdot w_c = 0 \quad (5)$$

where b_c and w_c are the proportional constant and angular velocity associated with the connecting link.

Let g be the number of gear pairs, and m be the number of links of the MC. The fundamental circuit equations associated with the MC can be written in matrix form as

$$\mathbf{B}_{g \times m} \cdot \mathbf{w}_{m \times 1} = 0 \quad (6)$$

Note that, by eliminating angular velocities of the links except the ports, Eq. (5) can be deduced from Eq. (6). Hence, columns associated with the links except the ports in matrix \mathbf{B} must contain at least two non-zero elements such that the associated items in matrix \mathbf{B} can be eliminated. Thus, we have

C8. All links except the ports in an MC must be associated with at least two fundamental circuits.

Based on these characteristics, steps for identifying MCs from existing atlas of non-fractionated GKC can be broadly summarized as follows:

Step 1. Choose an admissible graph: A graph with at least n coaxial vertices from the atlas of $(n-2)$ -dof GKC is selected as the admissible graph.

Step 2. Determine the admissible set of coaxial links: From the admissible graph, the coaxial vertices with at least n members are presumed as the admissible set of coaxial links.

Step 3. Determine the ports: From the admissible set of coaxial links, n of the vertices are assigned as ports. The graph is eligible to be a candidate MC if all links except the ports in the graph are associated with at least two fundamental circuits.

Step 4. Check redundancy: A link is redundant if all the gear ratios of its associated gear pairs do not appear in the kinematic constraint equation. A candidate MC with no redundant links is identified as an admissible MC.

Step 5. Repeat steps 3 to 4 for each admissible set of coaxial links until all possible assignments of ports and non-ports are found.

Step 6. Repeat steps 2 to 5 until every admissible set of coaxial links is examined.

With this procedure, admissible n -port, m -link MCs can be identified from the atlases of $(n-2)$ -dof non-fractionated GKC. Figure 3 shows the graphs of admissible 3-port MCs with up to 7 links identified from the atlas of 1-dof non-fractionated GKC [7,8]. Figure 4 shows the graphs of admissible 4-port MCs with up to 8 links identified from the atlas of 2-dof non-fractionated GKC [5,9]. In Figs. 3 and 4, ports are represented by rectangles, and each n -port m -link MC is coded as MC- n - m -# with # as its series number. Note that, for the case that the number of coaxial links is greater than the number of ports, several sets of ports may be selected in Step 3. Hence, a GKC may lead to several admissible MCs based on different port assignments, such as MC-3-6-6 and MC-3-6-7, MC-3-6-8 and MC-3-6-9, MC-3-6-10 and MC-3-6-11 in Fig. 3.

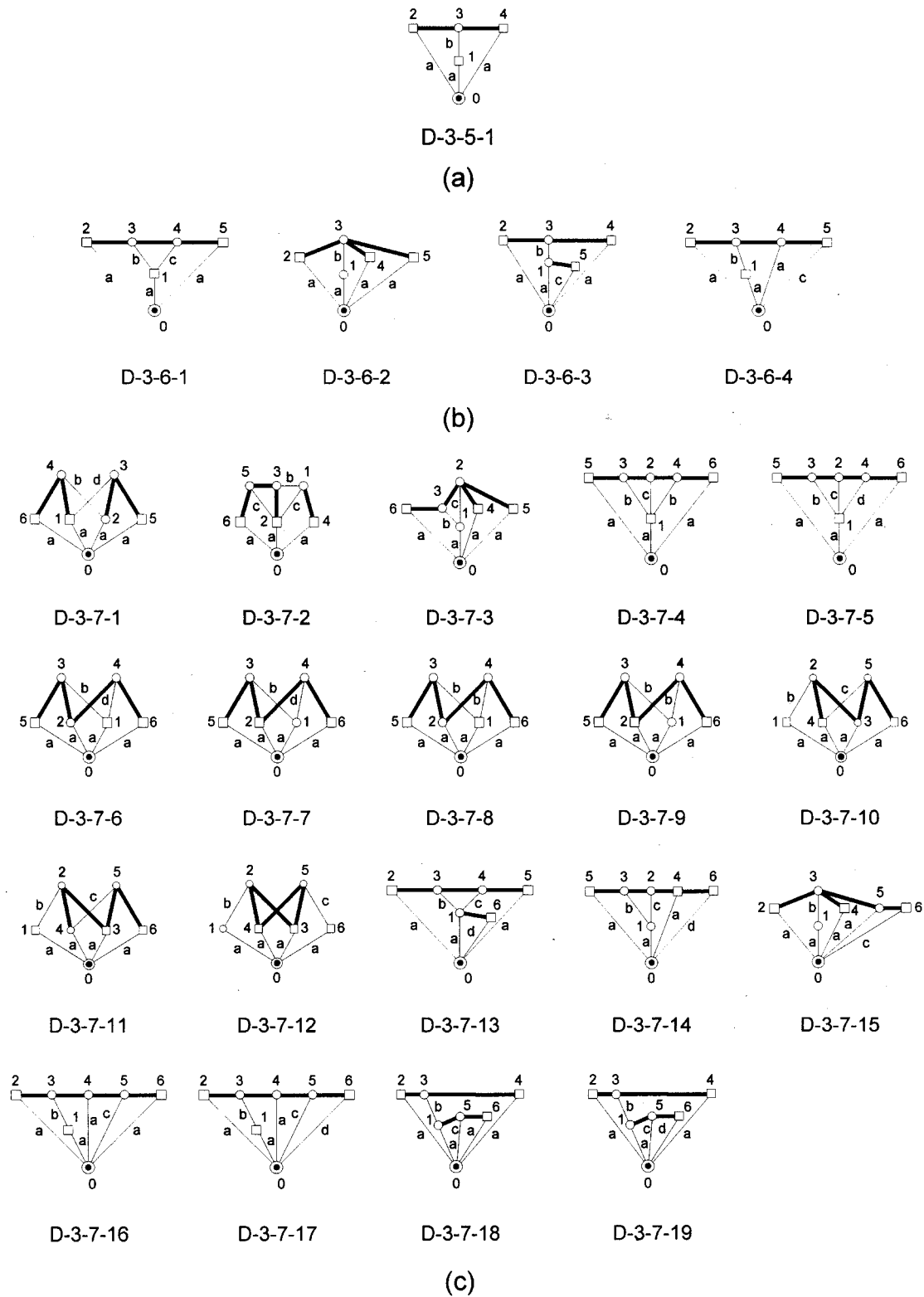


Fig. 5 Admissible 3-terminal fractionated GDMs with up to 7 links. (a) 5-link; (b) 6-link; (c) 7-link.

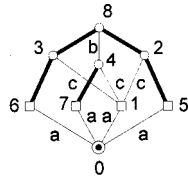
4 Composition of Geared Differential Mechanisms

The creation of an n -terminal, m -link fractionated GDM can be treated as a matter of selecting the proper IC and MC and joining the admissible connecting links together. The process can be broadly stated as:

Step 1. Determine the number of links of the IC and MC: Since

the connecting links in the IC and MC are joined together as a cut-link, by the specified number of links of the GDM, m , the number of links of IC, m_{ic} , and the number of links of MC, m_{mc} , can be related as:

$$m_{ic} + m_{mc} = m + 1 \quad (7)$$



D-4-9-1

Fig. 6 Admissible 4-terminal fractionated GDM with up to 9 links

Table 1 Compositions of 3-terminal GDMs

m	IC	MC	No. of non-isomorphic connecting links in MC	Sub-Total	Total
5	2-1	3-4-1	1	1	1
6	2-1	3-5-1 to -2	1	2	4
	3-1	3-4-1	2	2	
7	2-1	3-6-1 to -12	1	12	19
	3-1	3-5-1	2	3	
		3-5-2	1		
	4-1	3-4-1	2	2	
	4-2	3-4-1	2	2	
8	2-1	3-7-1 to -37	1	37	71
	3-1	3-6-1	3	28	
		3-6-2	3		
		3-6-3	2		
		3-6-4	2		
		3-6-5	2		
		3-6-6	2		
		3-6-7	2		
		3-6-8	2		
		3-6-9	2		
		3-6-10	3		
		3-6-11	3		
		3-6-12	2		
	4-1	3-5-1	2	3	
3-5-2		1			
4-2	3-5-1	2	3		
	3-5-2	1			

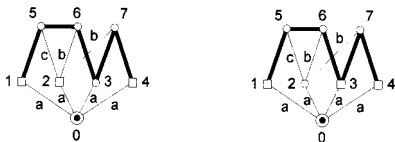


Fig. 7 Innovative 3-terminal link-fractionated GDMs.

By limiting the IC to those shown at Fig. 2, the corresponding m_{mc} can be determined from Eq. (7), since the range of m_{ic} is within the limits of two to four.

Step 2. Choose the IC and MC: Admissible IC with m_{ic} links can be selected from Fig. 2 and a corresponding n -port, $(m+1-m_{ic})$ -link MC can be selected from Figs. 3 or 4.

Step 3. Choose the connecting links: The connecting link in the IC is already specified in Fig. 2. One of the ports of the MC can be selected as the connecting link. An isomorphic check may be required to determine the nonisomorphic connecting link of the MC for each selected IC.

Step 4. Join the connecting links in MC and IC as a cut-link.

Step 5. Assign the outputs, cut-link, and ground as coaxial links.

Step 6. Rearrange the coaxial revolute joints to the canonical form [10].

With these steps, n -terminal, m -link fractionated GDMs can be created systematically. Figures 5 and 6 show admissible 3-terminal fractionated GDMs with up to 7 links and 4-terminal fractionated GDM with up to 9 links in canonical form. In Figs. 5 and 6, terminal links are denoted by rectangles, and an n -terminal, m -link GDM is coded as D- n - m -# with # as its serial number. Table 1 shows the compositions of admissible 3-terminal fractionated GDMs with up to 8 links.

For the purpose of demonstration, the composing process of the 3-terminal, 7-link fractionated GDMs is shown as follows. From Eq. (7), we have as

$$m_{ic} + m_{mc} = 7 + 1 = 8 \quad (8)$$

Since the range of m_{ic} is from two to four, three sets of m_{ic} and m_{mc} are possible:

- $m_{ic}=2$ and $m_{mc}=6$: IC-2-1 is selected as the IC, and one of the twelve 3-port, 6-link MCs can be selected from Fig. 3 as the MC. Note that since ports of MC are coaxial, for using IC-2-1 as the IC, all ports are isomorphic connecting links. Hence, twelve 3-terminal, 7-link GDMs, D-3-7-1 to D-3-7-12 can be formed and shown in Fig. 5(c).
- $m_{ic}=3$ and $m_{mc}=5$: IC-3-1 is selected as the IC, and one of the two 3-port, 5-link MCs from Fig. 3 can be selected as the MC. For MC-3-5-1, since ports 2 and 5 are symmetric, only ports 1 and 2 are the non-isomorphic connecting links. For MC-3-5-2, it can be seen that ports 2, 4, and 5 are isomorphic connecting links. Hence, three 3-terminal, 7-link GDMs, D3-7-13, D-3-7-14 and D-3-7-15 can be formed and shown in Fig. 5(c).
- $m_{ic}=4$ and $m_{mc}=4$: One of IC-4-1 and IC-4-2 can be selected as the IC, and only MC-3-4-1 is eligible for the MC. For MC-3-4-1, ports 2 and 4 are isomorphic connecting links. Hence, four 3-terminal, 7-link GDMs, D-3-7-16 to D-3-7-19 can be formed and shown in Fig. 5(c).

Comparing among the atlas of 3-terminal, 7-link fractionated GDMs shown in Fig. 5 with the kinematic graphs derived by Yan and Hsieh [2] and Hsu and Wu [3] shows the following results. In the case of Yan and Hsieh [2], their methodology fails to enumerate the GDMs with an isolated gear pair, such as D-3-7-2, and those with multiple sets of coaxial links, such as D-3-7-4. Also, the 3-terminal, 7-link GDMs, D-3-7-13 to 19 are believed to be new by comparing the results of Yan and Hsieh [2] and Hsu and Wu [3].

By comparing the 3-terminal, 8-link GDMs composed of IC-2-1 and MC-3-7-# with those identified by Hsu and Wu [3], the GDMs composed of IC-2-1 and MC-3-7-21, and IC-2-1 and MC-3-7-22 as shown in Fig. 7 are believed to be new.

We believe that this decomposition based method of enumeration, which uses the atlas of GKC developed by earlier investigators as foundation, is more straightforward, more efficient, and more reliable than those approaches that started from scratch. Although we have used the enumeration of 3-terminal and 4-terminal GDMs as examples, the methodology presented here is completely general and can be applied to the enumeration of n -terminal fractionated GDMs.

5 Conclusion

This paper describes a systematic methodology for the topological synthesis of admissible n -terminal fractionated GDMs. It is shown that a fractionated GDM can be decomposed into a GKC as the MC and a 1-dof geared mechanism as the IC. The characteristics of fractionated GDM and those of the IC and MC are addressed. A set of identification steps to identify admissible MCs from existing atlases of nonfractionated GKC is developed. The procedure to select proper IC and MC to form the fractionated GDM is described, and atlases of admissible fractionated GDMs with 3 and 4 terminals are developed.

References

- [1] Hirose, S., 1986, "Connected Differential Mechanism and Its Applications," *Robot Grippers*, Bedford, IFS, pp. 142–153.
- [2] Yan, H. S., and Hsieh, L. C., 1994, "Conceptual Design of Gear Differentials for Automotive Vehicles," *ASME J. Mech. Des.*, **116**, pp. 565–570.
- [3] Hsu, C. H., and Wu, Y. C., 1998, "A Methodology for Systematic Synthesis of Two-DOF Gear Differentials," *JSME Int. J., Ser. C*, **41**, No. 2, pp. 299–306.
- [4] Kota, S., and Bidare, S., 1997, "Systematic Synthesis and Applications of Novel Multi-Degree-of-Freedom Differential Systems," *ASME J. Mech. Des.*, **119**, pp. 284–291.
- [5] Lin, C. C., and Tsai, L. W., 1989, "The Creation of Non-Fractionated Two Degree-of Freedom Epicyclic Gear Train," *ASME J. Mech., Transm., Autom. Des.*, **111**, pp. 524–529.
- [6] Freudenstein, F., 1972, "Kinematics and Statics of a Coupled Epicyclic Spur-Gear Train," *Mech. Mach. Theory*, **7**, pp. 263–275.
- [7] Freudenstein, F., 1971, "An Application of Boolean Algebra to the Motion of Epicyclic Drives," *ASME J. Eng. Ind., Series B*, **93**, pp. 176–182.
- [8] Tsai, L. W., 1987, "An Application of the Linkage Characteristics Polynomial to the Topological Synthesis of Epicyclic Gear Trains," *ASME J. Mech., Transm., Autom. Des.*, **109**, No. 3, pp. 329–336.
- [9] Hsu, C. H., 1992, "An Application of Generalized Kinematic Chains to the Structural Synthesis of NonFractionated Epicyclic Gear Trains," *ASME Pro. Mechanical Design and Synthesis*, DE-Vol. 46, pp. 451–458.
- [10] Chatterjee, G., and Tsai, L. W., 1994, "Enumeration of Epicyclic-Type Automatic Transmission Gear Trains," *SAE Paper 941012*.