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On the Application of Kinematic Units to the Topological Analysis of Geared Mechanisms

Based on the concept of kinematic fractionation, a systematic method for the topological analysis of geared mechanisms is presented in this paper. It is shown that a geared kinematic chain (GKC) can be regarded as a combination of kinematic unit(s). By identifying the embedded kinematic units, kinematic insight in the GKC can be exposed. The disclosed information leads to straightforward and promising rules to prevent redundant links. These rules form the basis of a by-inspection procedure to determine admissible assignments of the ground, input and output links in a GKC. Admissible assignments of the ground, input and output links of one degree-of-freedom 5 link GKCs are determined as an illustrative example. [DOI: 10.1115/1.1352022]

1 Introduction

In the conceptual design phase, type synthesis is regarded as a sequential process consisting of topological synthesis and topological analysis [1]. Topological synthesis is referred to as a procedure to determine admissible topological structures of the desired mechanism, and topological analysis is aimed at determining the locations of ground, input and output links in a certain topological structure.

Numerous studies have been performed on the topological synthesis of non-fractionated epicyclic gear trains (EGTs). Freudenstein [2] enumerated admissible graphs of one-dof, up to 5-link EGTs. Tsai [3] presented a generic approach to enumerate admissible graphs of one-dof, up to 6-link EGTs. Tsai and Lin [4] modified the generic approach to enumerate admissible graphs of two-dof, up to 7-link EGTs.

In comparison with topological synthesis, research on topological analysis can be barely found. Olson, et al. [5] established the concept of coincident-joint graph and developed an exhaustive approach to determine possible locations of the ground, input and output links in one-dof, 5-link EGTs with both input and output links adjacent to the fixed link. However, additional redundancy check was required since the majority of their results contained redundant links. Hence, applications of the developed atlases from the topological synthesis are extensively limited.

For the design of geared mechanisms, avoiding redundant links is an important consideration to reduce the power loss and to enhance the compactness of the mechanism. Traditional approaches to identifying the existence of redundant link(s) rely on deriving the kinematic relation between input and output links. Those links, which do not appear in the derived kinematic equation, are determined as redundant links [5,6]. However, this redundancy check becomes laborious for a complicated mechanism.

Liu and Chen [7] proposed the concept of kinematic fractionation and a matrix-based method was developed to identify the type of fractionation of GKCs. They showed that over half of the one-dof, and all of the two-dof structurally non-fractionated GKCs can be fractionated via the concept of kinematic fractionation. However, this approach does not show the connecting condition among links and thus can not be applied directly to assign the locations of the ground, input and output links.

Based on displacement graphs [2], the concept of kinematic units will be introduced in this paper to represent the basic kinematic structure in a GKC. By identifying the kinematic units and the end vertices, admissible assignments of the ground, input and output links in a GKC can be obtained through a systematic approach without generating any invalid assignment containing redundant links. Since the redundancy check is not required to obtain admissible results, this approach is much more efficient than the former one developed by Olson, et al. [5]. As an illustrative example, admissible assignments of the ground, input and output links in one-dof, 5-link ground-actuated geared mechanisms will be determined. It is believed that the presented method acts as an efficient tool to harness the former developed atlases of EGTs, and can be applied to n-dof geared mechanisms as well.

2 Topological Structure of a GKC

Graph theory has been successfully used to represent the topological structure of a geared mechanism separated from its function. Some definitions are reviewed in the following:

(a) Structural graph: The structural graph shows the topological structure of a GKC. In a structural graph, links are represented by vertices, gear pairs are represented by heavy edges, and turning pairs are represented by thin edges, which is labeled according to its axis orientation in space. Figure 1(*b*) shows structural graph 1400-1-7 [2], which corresponds to a one-dof, 5-link gear train in Fig. 1(*a*).

(b) Displacement graph: A displacement graph expresses the topological structure of a GKC in an abbreviated form. The displacement graph of a GKC is obtained from its structural graph by deleting thin edges and transfer vertices, labeling each heavy edge with the corresponding carrier, and labeling each geared vertex according to the axis of the joint connecting the gear and the carrier [2]. Figure 1(*c*) shows the displacement graph associated with Fig. 1(*b*).

(c) Structurally non-fractionated GKC: A GKC without any cut vertex in its structural graph or its pseudo-isomorphic graph(s) [4] is referred to as a structurally non-fractionated GKC.

Since a structurally fractionated GKC can be decomposed into non-structurally fractionated sub-chains by separating the cut vertex, only non-structurally fractionated GKCs will be discussed in this paper.

3 Concept of Kinematic Fractionation

Freudenstein [8] introduced the concept of fundamental circuit to represent the minimal kinematic structure in a structural graph. A fundamental circuit consists of two vertices representing a pair of meshing gears, a vertex representing the associated carrier, and the connecting edges. The kinematic relation in a fundamental

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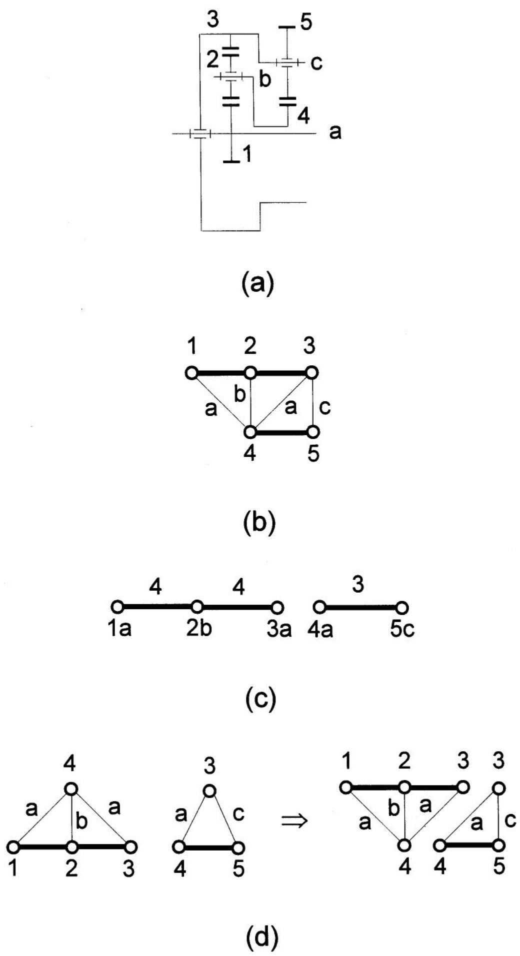


Fig. 1 Graph 1400-1-7. (*a*) the functional representation; (*b*) the structural graph; (*c*) the displacement graph; (*d*) the KUs

circuit can be described by a fundamental circuit equation. In Fig. 2, vertices i, $i+1$ and k form a fundamental circuit and the associated fundamental circuit equation can be expressed as

$$\theta_{i,k}=e_{i+1,i}\theta_{i+1,k} \tag{1}$$

where $\theta_{i,k}$ is the relative angular displacement between links i and k, and $e_{i+1,i}$ is the gear ratio between gears $i+1$ and i.

Also, vertices $i+1$, $i+2$ and k form another fundamental circuit and the associated fundamental circuit equation can be expressed as

$$\theta_{i+1,k}=e_{\mathrm{i+2,i+1}}\theta_{i+2,k} \tag{2}$$

Since gear vertex $i+1$, and the carrier vertex k are common to both fundamental circuits, Eqs. (1) and (2) have a common variable, $\theta_{i+1,k}$. Substituting Eq. (2) into Eq. (1) yields

$$\theta_{i,k}=e_{i+2,i+1}\cdot e_{i+1,i}\theta_{i+2,k} \tag{3}$$

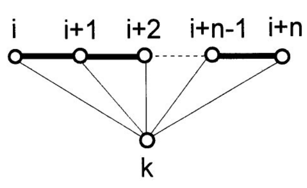


Fig. 2 A typical KU with connected heavy-edged path

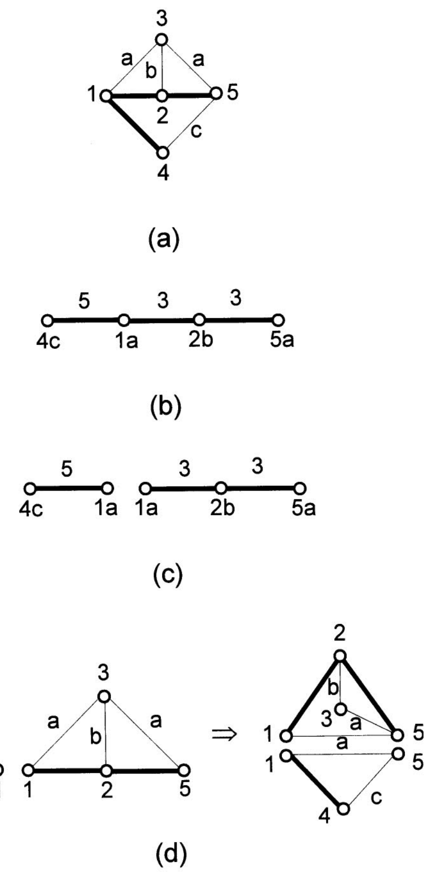


Fig. 3 Graph 1400-1-4. (*a*) the structural graph; (*b*) the displacement graph; (*c*) the separated displacement graph; (*d*) the KUs.

Similar substitutions can be repeated along the heavy-edged path. By repeating the substitution of fundamental circuit equations, kinematic relation between two ends of the heavy-edged path can be described by an augmented fundamental circuit equation. For instance, the relation between links i and $i+n$ in Fig. 2 can be written as

$$\theta_{i,k}=e_{i+n,i+n-1}\cdot e_{i+n-1,i+n-2}\cdots e_{i+1,i}\theta_{i+n,k} \tag{4}$$

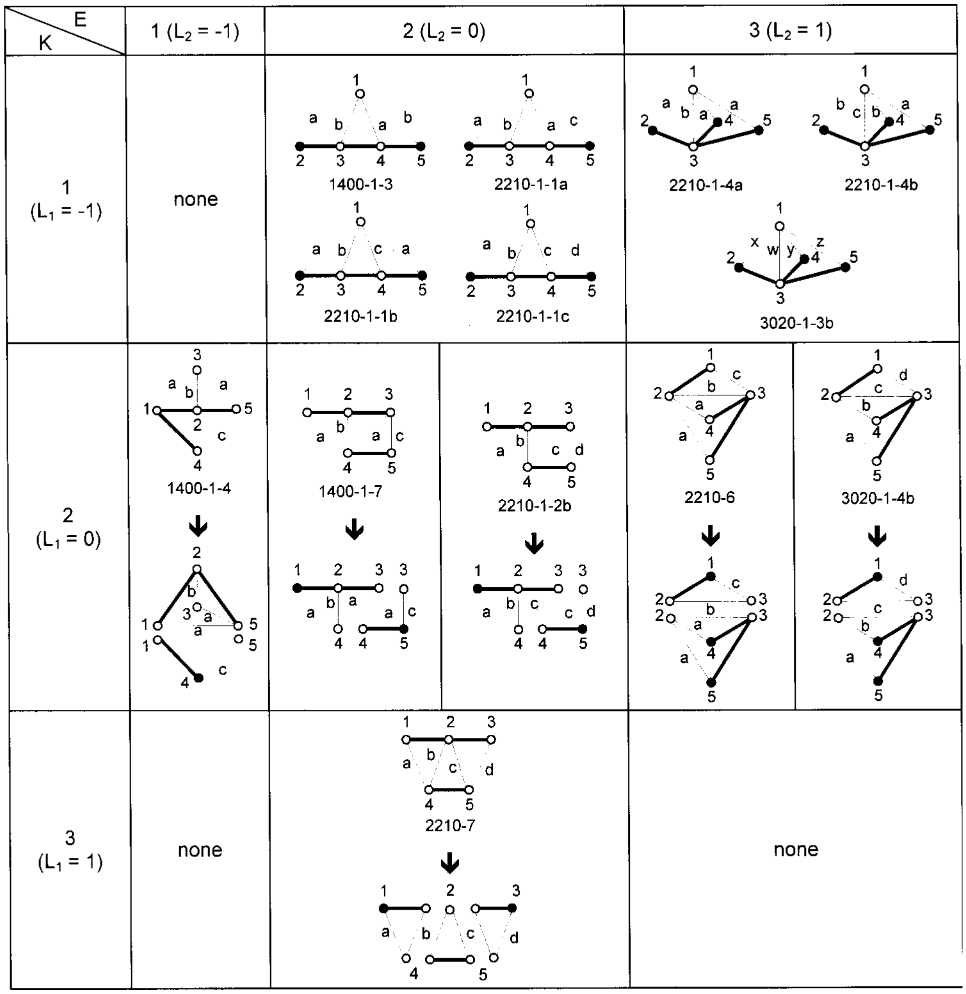
It can be seen that the gear ratios in Eq. (4) includes all the gears between link i and $i+n$, and thus it exposes the carrier, the ends and also the transmission path in an augmented fundamental circuit. In contrast to a fundamental circuit, which focused on the kinematic relation between only one gear pair, the concept of the augmented fundamental circuit provides a lucid way to group the links in a gear train and also provides a clear equation to lump the kinematic relations among the links. Hence, an augmented fundamental circuit can be regarded as the basic kinematic structure in a GKC.

From the distribution of carrier(s), basic kinematic structure(s) in a GKC can be identified from its associated displacement graph according to the following procedure:

Step 1. Separate the displacement graph into connected sub-graphs so that each sub-graph includes all the gear vertices associated with the same carrier.

In Fig. 1(*c*), each sub-graph contains only one carrier label and thus no further separation is required. Figures 3(*a*) and (*b*) show respectively the structural graph and the displacement graph of graph 1400-1-4 [2]. It can be seen that the displacement graph contains two carrier labels. Hence, Fig. 3(*b*) should be separated as shown in Fig. 3(*c*) in which each sub-graph has only one carrier label.

Table 1 KUs and end vertices of one-dof, 5-link GKCs



Step 2. Build gear-carrier pairs in each connected sub-graph by adding a vertex representing the carrier, connecting the gear and carrier vertices with thin edges and labeling each thin edge with the axis orientation.

Left-hand sides of Fig. 1(*d*) and Fig. 3(*d*) show the results obtained from Fig. 1(*c*) and Fig. 3(*c*) respectively.

Step 3. Obtain kinematic units (KUs) by connecting the common vertices in each sub-graph obtained from **Step 2** with a thin edge.

In Fig. 1(*d*), each sub-graph has already a thin edge connecting the common vertices 3 and 4. Hence, each connected sub-graph in Fig. 1(*d*) represents a KU which can be rearranged as shown on the right-hand side. On the left-hand side of Fig. 3(*d*), it can be seen that vertices 1 and 5 are common to both connected sub-graphs. One of the sub-graphs has already a thin edge between vertices 1 and 5 directly while another sub-graph uses a thin-edged path, 1-3-5, to connect vertices 1 and 5. It is known that the connection among vertices with thin edges having the same label can be rearranged arbitrarily without changing any kinematic characteristics [4]. The procedure is called coaxial rearrangement. Hence, a thin edge can be relocated between vertices 1 and 5 since the path 1-3-5 has a unique axis label. The rearranged result is shown on the right-hand side of Fig. 3(*d*) in which each connected sub-graph represents a KU.

A KU not only shows the topological structure of an augmented fundamental circuit but also highlights the interface between two augmented fundamental circuits. Hence, the kinematic structures and the connecting relations in a GKC can be clearly exposed by decomposing a GKC into KUs

The end of each motion transmission path of the entire GKC is referred to as an end vertex. Since motion is transmitted along the heavy-edged path, it is obvious that an end vertex should be on the open end of the heavy-edged path. As a GKC contains more than one KU, the end of motion transmission of the entire GKC should take the connecting condition between KUs into consideration, and the end vertices can be identified according to the following definition:

End vertex: An end vertex of a GKC is a vertex, which is on an open end of the heavy-edged path of a KU but is not common to another KU.

As shown in Fig. 1(*d*), vertices 1, 3, 4 and 5 are on the open ends of the heavy-edged paths. Since vertices 3 and 4 are the common vertices between the two KUs, only vertices 1 and 5 are end vertices.

According to the above procedure, all the KU(s) and the end vertices contained in a GKC can be identified. The KU(s) and the end vertices of the thirteen one-dof, 5-link structurally non-fractionated GKCs [2] are shown in Table 1 in which *K* represents the number of KUs, E represents the number of end vertices, and each dark vertex represents an end vertex. It can be seen that seven of the thirteen GKCs have only one KU and thus are classified as kinematically non-fractionated. For a kinematically non-fractionated GKC, the associated KU is identical to the GKC itself. The remaining six GKCs have more than one KU and are classified as kinematically fractionated.

4 Topological Requirements of a Geared Mechanism

The kinematic insight revealed by the KUs provides a substantial assistance to determine admissible locations of the ground, input and output links. In what follows, we shall confine the discussion to ground-actuated mechanisms. The assignment of the ground, input and output links is assumed to satisfy the following basic requirements:

1. The input link is adjacent to ground.
2. The motion of every driven link is properly constrained by the input(s).
3. No redundant links.

4.1 Mobility Requirement. The input link and its supporting link form an actuator pair [9]. From the first requirement, each actuator pair should be assigned on a thin edge incident to the ground. Hence, we have

Axiom 1: The number of thin edges incident to the ground is equal to or greater than the number of dof of the mechanism, *F*.

The relative angular displacement between the actuator pair is used to determine the motion of other links. A link is said to be properly constrained if its motion can be determined according to the motion of actuator pair(s). With the concept of kinematic fractionation, the motion of a link is lumped with other links in the same KU. From Eq. (4), it can be seen that once the motion of an actuator pair is assigned in a KU, the motion of other links will be determined sequentially along the heavy-edged path. Note that as a KU has more than one actuator pair, the KU turns out to be overly constrained. Thus, we have

Axiom 2: A KU can contain at most one actuator pair.

With the single input, the motion of each link inside the KU is completely constrained. In addition, a mechanism with properly constrained motion should obey the following axiom:

Axiom 3: The total number of actuator pairs is equal to the number of dof of the mechanism, *F*.

4.2 Minimum Number of Output Link(s). Based on a GKC, a geared mechanism is developed such that the number of dof is equal to the number of inputs specified in advance. However, the number of outputs is usually not taken into account. Hence, the topological structure of the GKC only ensures to provide enough mobility but can not guarantee a proper kinematic structure to perform the specified function.

The ground, input and output links are said to be the ports of a mechanism. Let *M* be the number of ports in the mechanism. According to the statement in **Axiom 3**, *M* can be resolved as follows based on the fact that a mechanism can have only one ground link:

$$M=1+F+P \tag{5}$$

where *P* is the number of outputs.

By viewing each KU as a sub-structure of the GKC, a KU should contain at least one of the ports to avoid redundancy. Thus, we have:

Axiom 4: The number of ports in a mechanism is greater than or equal to the number of KUs in the associated GKC, i.e.

$$M\geqslant K \tag{6}$$

A link becomes redundant if it has no contribution to transmitting motion from the input link to the output link. Since the end vertex represents a terminal of a transmission path, every end vertex in the GKC needs to be assigned as one of the ports to avoid forming redundant links. Hence, the concept of end vertices leads to the following axiom:

Axiom 5: The number of ports in a mechanism is greater than or equal to the number of end vertices in the associated GKC, i.e.

$$M\geqslant E \tag{7}$$

By substituting Eq. (5) into Eqs. (6) and (7) respectively and since a mechanism should have at least one output, the number of outputs in a GKC can be derived as:

$$P\geqslant \text{Max.}\{1,L_1,L_2\} \tag{8}$$

where

$$L_1=K-(1+F) \quad \text{and} \tag{9}$$

$$L_2=E-(1+F) \tag{10}$$

By examining the one-dof, 5-link GKCs in Table 1, it can be determined according to Eq. (9) that

$$L_1=1-(1+1)=-1 \quad \text{for kinematically non-fractionated GKCs} \tag{11a}$$

$$L_1=2-(1+1)=0 \quad \text{for kinematically fractionated GKCs with 2 KUs} \tag{11b}$$

$$L_1=3-(1+1)=1 \quad \text{for kinematically fractionated GKCs with 3 KUs} \tag{11c}$$

From Eq. (10), it can be also determined that

$$L_2=1-(1+1)=-1 \quad \text{for GKC with 1 end vertex} \tag{12a}$$

$$L_2=2-(1+1)=0 \quad \text{for GKC with 2 end vertices} \tag{12b}$$

$$L_2=3-(1+1)=1 \quad \text{for GKC with 3 end vertices} \tag{12c}$$

By substituting the results of Eqs. (11) and (12) into Eq. (8), it can be concluded that the minimum number of outputs for all the one-dof, 5-link GKCs is one.

5 Procedure for Topological Analysis

Based on the requirements of admissible mechanisms, the procedure to determine the locations of ground, input and output links in a GKC is developed as follows. For the purpose of demonstration, graph 1400-1-7 is selected as an illustrative example by assuming the number of output equal to one, and thus the number of ports, *M*, is equal to 3.

5.1 Assignment of the Ground Vertex. From **Axioms 1** and **5**, the requirements of an admissible location of the ground can be derived as follows:

R1: *A vertex can be selected as the ground if the number of incident thin edges is equal to or more than the number of dof of the mechanism.*

R2: *A vertex is selected as the ground such that the number of end vertices, which are not used as the ground, is no more than* $M-1$.

According to **R1** and **R2**, it can be seen that every vertex in graph 1400-1-7 is eligible to be the ground.

5.2 Assignment of the Input Link. In a ground-actuated mechanism, the requirements of admissible locations of input link(s) can be derived from **Axiom 2** as follows:

R3: *A vertex can be selected as an input if it is adjacent to the ground vertex with a thin edge.*

R4: *As more than one input link is required, actuator pairs should be assigned in different KUs.*

In addition, another requirement for the input can be derived from **R2** as follows

R5: *The input vertices are selected such that the number of end vertices, which are not on the actuator pair(s), is no more than* M-1-F.

According to **R3**, **R4** and **R5**, the input link in graph 1400-1-7 can be selected according to different selections of ground vertices:

(1) Vertex 1 is the ground: It can be found that vertex 4 is adjacent to vertex 1 with a thin edge. In addition, a thin edge can be formed between vertices 3 and 1 by coaxial rearrangement. Since both of the two vertices satisfy **R5**, two admissible combinations of the ground and input links are obtained and can be expressed as $[G; I]=[1; 3]$ and [1; 4].

(2) Vertex 2 is the ground: Vertex 4 is the only one vertex that is connected to vertex 2 with a thin edge. However, if vertex 4 is assigned as the input link, **R5** can not be satisfied. Thus, no admissible results can be obtained by choosing vertex 2 as the ground.

(3) Vertex 3 is the ground: After coaxial rearrangement, it can be found that vertices 1, 4 and 5 are adjacent to vertex 3 with a thin edge. However, only vertices 1 and 5 satisfy **R5**. Thus, the admissible combinations of the ground and input links are obtained as [3; 1] and [3; 5].

(4) Vertex 4 is the ground: Vertices 1, 2 and 3 are found to be adjacent to vertex 4 with a thin edge. However, only vertex 1 satisfies **R5**. Thus, the only one admissible combination of the ground and input links is obtained as [4; 1].

(5) Vertex 5 is the ground: The only one vertex connected to vertex 5 with a thin edge is vertex 3, which also satisfies **R5**. Thus, one admissible combination of the ground and input links is obtained as [5; 3].

5.3 Assignment of the Output Vertex. According to the requirement described in Eq. (7), admissible locations of output link(s) should meet the following requirements:

R6: *Each end vertex, which is not on the actuator pair, is assigned as the output link.*

R7: *In the case that all end vertices are on the actuator pair, any one of the other vertices can be assigned as an output link.*

R8: *The output link(s) is selected such that each KU contains at least one of the ground, input and output links.*

The output link in graph 1400-1-7 can be determined as follows:

Case 1. $[G; I]=[1; 3]$, $[1; 4]$, $[3; 1]$ or $[4; 1]$: The remaining end vertex is vertex 5. According to **R6** and **R8** admissible combinations of the ground, input and output links are determined as $[G; I; O]=[1; 3; 5]$, [1; 4; 5], [3; 1; 5], and [4; 1; 5].

Case 2. $[G; I]=[3; 5]$ or $[5; 3]$: The remaining end vertex is vertex 1. Thus, admissible combinations of the ground, input and output links are determined as [3; 5; 1], and [5; 3; 1].

From Cases 1 and 2, six sets of admissible arrangements of the ports are obtained as listed in Table 2(*b*). Since these arrangements have more than one carrier, the associated mechanisms have at least one floating carrier. Hence, these configurations are all classified as EGTs.

Based on the above procedure, admissible assignments of the ground, input and output links for one-dof, 5-link kinematically non-fractionated and kinematically fractionated GKCs are obtained as shown in Tables 2(*a*) and (*b*) with the number of output equal to one. It can be seen that only four ordinary gear trains (OGTs) are obtained in which the unique carrier vertex is used as the ground link. It is also found that no ground-actuated mechanism with one output can be obtained from graph 3020-1-4b.

In Tables 2(*a*) and (*b*), seven EGT configurations are marked with an asterisk. The input and output links in these configurations are both adjacent to the ground. In such configurations, the distinction between the input and output links is unnecessary since the input and output links are topologically identical. By disregarding the distinction between input and output links, four distinct EGTs are obtained, which can be expressed as $[G; (I/O)]=[2; (1/5)]$ for graph 2210-1-1b, [2; (4/5)] for graph 2210-1-4a, [5; (3/4)] for graph 1400-1-4 and [3; (1/5)] for graph 1400-1-7.

Table 2 Admissible [*G*; *I*; *O*] assignment of one-dof, 5-link GKCs. (*a*) kinematically non-fractionated GKCs; (*b*) kinematically fractionated GKCs

	G	I	O	E/OGT
1400-1-3	1	2	5	OGT
	2	1	5	EGT
		4	5	EGT
	3	5	2	EGT
2210-1-1a	1	2	5	OGT
	2	1	5	EGT
		4	5	EGT
	4	2	5	EGT
2210-1-1b	1	2	5	OGT
	2	1	5	EGT*
		5	1	EGT*
			3	EGT
2210-1-1c	1	2	5	OGT
	2	1	5	EGT
2210-1-4a	2	4	5	EGT*
2210-1-4b	2	4	5	EGT
3020-1-3b	2	4	5	EGT

(a)

	G	I	O	E/OGT
1400-1-4	1	3	4	EGT
	2	3	4	EGT
	3	1	4	EGT
		2	4	EGT
		5	4	EGT
	4	5	2	EGT
			3	EGT
	5	3	4	EGT*
		4	2	EGT
			3	EGT*
1400-1-7	1	3	5	EGT
		4	5	EGT
	3	1	5	EGT*
		5	1	EGT*
	4	1	5	EGT
	5	3	1	EGT
2210-1-2b	1	4	5	EGT
	3	5	1	EGT
	4	1	5	EGT
	5	3	1	EGT
2210-6	4	5	1	EGT
2210-7	1	4	3	EGT
	4	1	3	EGT
3020-1-4b	1	×	×	×
	4			
	5			

(b)

By comparing the above four EGT configurations with those results obtained by Olson, et al. [5], it can be found that the results of both approaches come into agreement. However, the coincident-joint-graph approach uses an exhaustive method to generate fifty-four possible configurations, in which fifty configurations are found containing redundant link(s) after deriving the kinematic relation between input and output links. Hence, it is much more time-consuming with the coincident-joint graph approach [5].

6 Examples

6.1 A Mechanism with Multiple Outputs. Figures 4(*a*) and (*b*) show respectively the structural graph and KUs of the one-dof GKC, graph 6201-2 [3]. It can be observed that the GKC contains two KUs and four end vertices. From Eqs. (9) and (10), it can be determined that

$$L_1=2-(1+1)=0 \tag{13a}$$

$$L_2=4-(1+1)=2 \tag{13b}$$

By substituting L_1 and L_2 into Eq. (8), it is found that at least two output links should be assigned in the mechanism to prevent redundancy. By requesting 2 outputs, that is $M=1+1+2=4$, admissible assignments for the ports can be obtained according to the procedure in the previous section.

According to **R2**, it is known that the ground vertex should be selected such that the number of end vertices, which are not assigned as the ground, is no more than $M-1=3$. Hence, the ground vertex can only be selected from vertex 3, 4, 5 or 6. However, from the symmetry of the structural graph, it can be found that vertices 3, 4, 5 and 6 are topologically equivalent. Thus, only one admissible location for the ground is obtained.

As vertex 3 is used as the ground, it can be seen that vertex 1 is connected to vertex 3 with a thin edge, and vertex 6 can also have a thin edge to connect with vertex 3 after coaxial rearrangement. However, from **R5**, it is known that the number of end vertices, which are not on the actuator pair should be no more than $M-1-F=2$. Thus, the input link can only be assigned on vertex 6.

With $[G;I]=[3;\ 6]$, the remaining two end vertices should be used as the output links to prevent redundancy. Thus, a set of admissible assignment of the ground, input and output links are obtained as $[G;I;O]=[3;6;(4,\ 5)]$. Figure 4(*c*) shows an admissible functional representation of the mechanism.

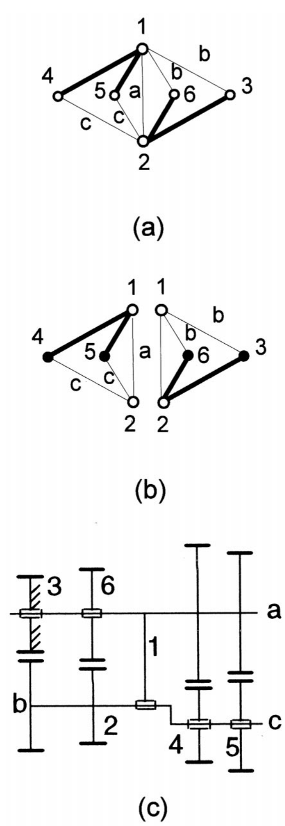


Fig. 4 Graph 6201-2. (*a*) the structural graph; (*b*) the KUs; (*c*) an admissible mechanism configuration

6.2 A Two-dof Mechanism. Figures 5(*a*) and (*b*) show respectively the structural graph and KUs of the two-dof GKC, graph 4-1-1 [4]. It can be observed that the GKC contains three KUs and three end vertices. From Eqs. (9) and (10), it can be determined that

$$L_1=3-(1+2)=0 \tag{14a}$$

$$L_2=3=-(1+2)=0 \tag{14b}$$

According to Eq. (8), the mechanism requires at least one output link. By requesting one output, that is $M=1+2+1=4$, admissible assignments of the ports can be obtained as follows.

From **R1**, it is known that the ground should be located on a vertex, which is incident with at least two thin edges. According to **R2**, it is known that the ground vertex should be selected such that the number of end vertices, which are not assigned as the ground, is no more than $M-1=3$. **R1** and **R2** indicate that the ground vertex can be selected from vertex 2, 4 or 5. The locations of input and output links are determined based on different selections of the ground vertex:

Case 1. Vertex 2 is the ground: The input links can only be placed on vertices 4 and 5, which are connected to vertex 2 with a thin edge. However, both vertices are not valid locations for the input links according to **R5**. Thus, no admissible configurations can be obtained by selecting vertex 2 as the ground.

Case 2: Vertex 4 is the ground: Vertices 1, 2, 5 and 6 are connected to vertex 4 with a thin edge. According to **R4** and **R5**, only

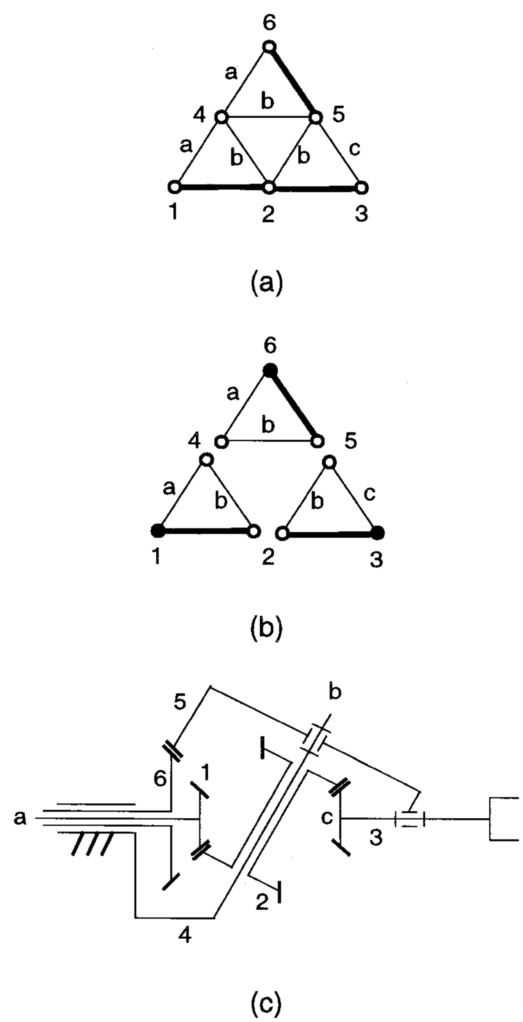


Fig. 5 Graph 4-1-1. (*a*) the structural graph; (*b*) the KUs; (*c*) an admissible mechanism configuration

vertices 1 and 6 are valid locations for the inputs. Based on $[G; I]=[4; (1, 6)]$, the remaining end vertex, vertex 3, is selected as the output according to **R6**. Thus, a set of admissible combination of the ground, input and output links is obtained as $[G; I; O]=[4; (1, 6); 3]$.

Case 3: Vertex 5 is the ground: Vertices 2, 3 and 4 are connected to vertex 5 with a thin edge. However, no admissible locations for the inputs can be obtained according to **R4** and **R5**. Thus, no admissible configurations can be obtained by selecting vertex 5 as the ground.

From the above discussion, it is found that only one set of admissible assignment of the ground, input and output links can be obtained for graph 4-1-1. Fig. 5(*c*) shows an admissible functional representation of the mechanism.

Through the process of assigning the locations of the ground, input and output links, it can be found that the concept of KU and the locations of end vertices provide a designer with clear comprehension of possible redundancy.

7 Conclusion

The concept of kinematic unit is introduced to determine the basic kinematic structures in a GKC. The identified kinematic units and end vertices depict the kinematic insight in the GKC and provide a clear guidance to prevent inducing redundant links. A systematic approach for the topological analysis of geared mechanisms is developed accordingly. From the approach, admissible assignments of the ground, input and output links can be obtained directly by inspection without deriving the kinematic relations between input and output links. Admissible assignments for one-dof, 5-link GKCs are obtained as an illustrative example.

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