Topological synthesis of compound geared differential mechanisms

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Abstract: In this article, a methodology on the topological synthesis of admissible n-terminal ($n \geq 4$) compound geared differential mechanisms is presented. Based on the characteristics of geared differential mechanisms, it is shown that a compound geared differential mechanism can be composed by adding a geared kinematic chain as the main component on a base geared differential mechanism. Admissible compound geared differential mechanisms with four and five terminals are enumerated systematically by selecting proper base geared differential mechanism and proper main component from the existing atlases and sharing the connecting links of each component as a common link. The decomposition-based methodology presented here is systematic and innovative, and can be applied to the enumeration of geared differential mechanisms with multiple terminals.

Keywords: differential mechanism, topological synthesis, geared mechanism, kinematic chain.

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1 Introduction

The standard two degree-of-freedom (dof) differential geared mechanism (DGM), a geared mechanism with one input that distributes the torque equally to its two outputs, has been widely known and used in automotive drive systems. By defining the input and outputs of the DGM as terminals, Hirose [5] introduced the general form of the two dof DGM and showed that torques on the two outputs are not necessarily equal, but the quotients of any two torques acting on terminals remain constant. The kinematic constraint equation and associated torque relations among terminals for the general DGM were also derived.

Yan and Hsieh [14] synthesized the atlas of two dof non-coupled DGMs as automotive gear differentials by assigning sun gears and planet gears on the admissible gear pair trees. Two dof coupled DGMs are constructed by joining the sun gears or carriers of two non-coupled DGMs into a set of coupling pairs. Hsu and Wu [7] identified the atlas of two dof DGMs with three terminals by adding a ground link to a set of coaxial links of one dof GKCs. However, their methodologies are all only applicable to 3-terminal DGMs. Chen and Yao [2] developed a decomposition-based methodology for the topological synthesis of admissible link-fractionated DGMs. They showed that an n-terminal link-fractionated DGM can be decomposed into an (n-2) dof GKC as the main component (MC) and a one dof mechanism as the input component (IC). Atlases of admissible ICs and MCs were identified from the atlases of non-fractionated GKCs [3,6,9,12]. Admissible link-fractionated DGMs can be enumerated systematically by selecting proper IC and proper MC and sharing the connecting links of each component as a common link. However, due to the dof and link limitations of the existing atlases of GKCs, Chen and Yao [2] enumerated only one 4-terminal link-fractionated DGM, and no admissible DGM with more than four terminals was found.

The application of multi-terminal DGMs is widely used in robotic grippers and all-wheel-drive vehicles. The method to obtain a multi-terminal DGM has been developed in the past by connecting a set of standard two dof geared differentials in a whiffletree configuration. However, the number of outputs is severely restricted to the power of two. Kota and Bidare [8] introduced a novel whiffletree-like configuration, in which the multi-terminal DGMs are created by connecting several differential building blocks. The number of outputs in the novel whiffletree-like DGM can be any integer greater than two. However, Kota and Bidare [8] allowed only two types of gear train arrangements in the DGMs, and hence only a very limited number of DGMs can be enumerated based on their methodology.

In this article, it will be shown that DGMs enumerated by Yan and Hsieh [14], Hsu and Wu [7], and Chen and Yao [2] are only a special case of DGMs. A compound DGM can be enumerated by adding a GKC as the MC on a base-DGM. Characteristics of compound DGMs are laid out based on the study of n-terminal DGMs. Admissible compound DGMs with four and five terminals are enumerated systematically by selecting proper base-DGM and MC followed by sharing the connecting links of each component as a common link. A gear-train-arrangement process for the equal-torque-distribution condition of DGMs is also presented. This systematic and innovative decomposition based methodology can be easily applied to the enumeration of DGMs with multiple terminals.

2 Characteristics of *n*-terminal differential geared mechanisms

Characteristics of DGMs with one input and (n-1) outputs, or n terminals ($n \geq 3$) are summarized as follows:

C1. There is one kinematic constraint equation among terminals of an n-terminal DGM. The kinematic constraint equation can be written as [2]

$$\sum_{i=1}^{n} a_i \cdot w_i = 0 \tag{1}$$

where a_i and w_i are the coefficient and the angular velocity associated with the i-th terminal.

C2. The n-terminal DGM is an (n–1) dof geared mechanism [2].

C3. The terminals in a DGM are adjacent to the ground link [10].

Any geared mechanism satisfying C1 to C3 is an admissible DGM. With the absence of losses, the power balance equation for an n-terminal DGM can be written as

$$\sum_{i=1}^{n} T_i \cdot w_i = 0 \tag{2}$$

where T_i is the torque associated with the i-th terminal.

Equations 1 and 2 can be written in matrix form as

$$\begin{bmatrix} T_1 & T_2 & \cdots & T_n \\ a_1 & a_2 & \cdots & a_n \end{bmatrix}_{2\times n} \cdot \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}_{n\times 1} = 0 \tag{3}$$

There are n variables in Equation 3 as w_1 to w_n. However, there are only (n–1) independent parameters within these variables since the n-terminal DGM is an (n–1) dof mechanism. Hence, the rank of the coefficient matrix in Equation 3 is one, and its row vectors are linear dependent. Thus, an extended characteristic of the torque distribution among the terminals can be obtained as:

C4. Under the no power loss assumption, the torque distribution among terminals of an n-terminal DGM is proportional to coefficients of the kinematic constraint equations among terminals as

$$\frac{T_1}{a_1} = \frac{T_2}{a_2} = \cdots = \frac{T_n}{a_n} \tag{4}$$

3 The link-fractionated differential geared mechanisms

In graph representation, links are denoted by vertices, joints by edges, revolute joints by thin edges, and gear pairs by heavy edges. Thin edges are labelled according to their axes in space. Figures 1(a) and 1(b) show the functional representation and a pseudo-isomorphic graph representation [9] of a standard 3-terminal link-fractionated DGM. Links 1, 2, and 3 are terminals of the standard DGM and are denoted by

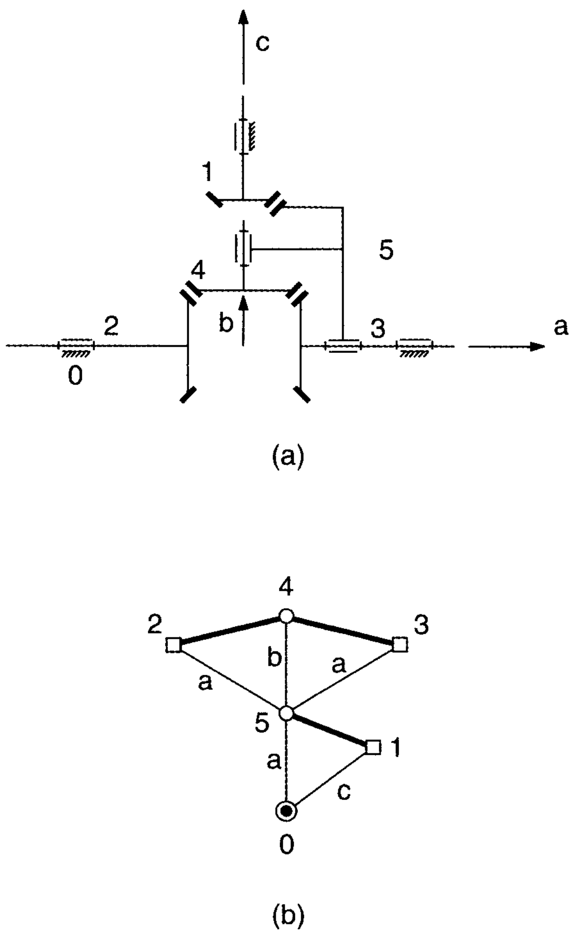


Figure 1 The standard 2-dof DGM: (a) Functional representation, (b) Pseudo-isomorphic graph representation.

rectangles. Chen and Yao showed that an n-terminal link-fractionated DGM can be decomposed into a one dof mechanism as the IC and a GKC as the MC by separating the cut-link [2]. In Figure 1(b), it can be seen that component (0, 1, 5) is the IC and component (2, 3, 4, 5) is the MC of the standard DGM. By defining local input/outputs of each component as ports, Chen and Yao laid out characteristics of ICs and MCs [2]. Figures 2(a), 2(b), and 2(c) show the admissible ICs with up to 3 links, 3-port MCs with up to 5 links, and 4-port MCs with 8 links, respectively [2]. In Figure 2, ports are denoted in rectangles, while an m_{ic}-link IC is coded as ICm_{ic}-# and an n-port m_{mc}-link MC is coded as MCn-m_{mc}-# with # as the series number.

Chen and Yao showed that link-fractionated DGMs can be enumerated by selecting connecting links in the IC and the MC and sharing them as a common link [2]. Figure 3 shows canonical graph representation [13] of admissible 3-terminal link-fractionated DGMs with up to 6 links and 4-terminal link-fractionated DGMs with up to 9 links [2]. In Figure 3, terminals are denoted by rectangles. An n-terminal m-link link-fractionated DGM is coded as SDn-m-# with # as the series number.

4 Decomposition of compound differential geared mechanisms

Figure 4(a) shows the functional representation of a 4-terminal 8-link DGM (U.S. Patents 5,423,726) applied to 3-wheel-drive automotive vehicles. Figure 4(b) shows a pseudo-isomorphic graph representation [9] of the 4-terminal DGM, with terminals

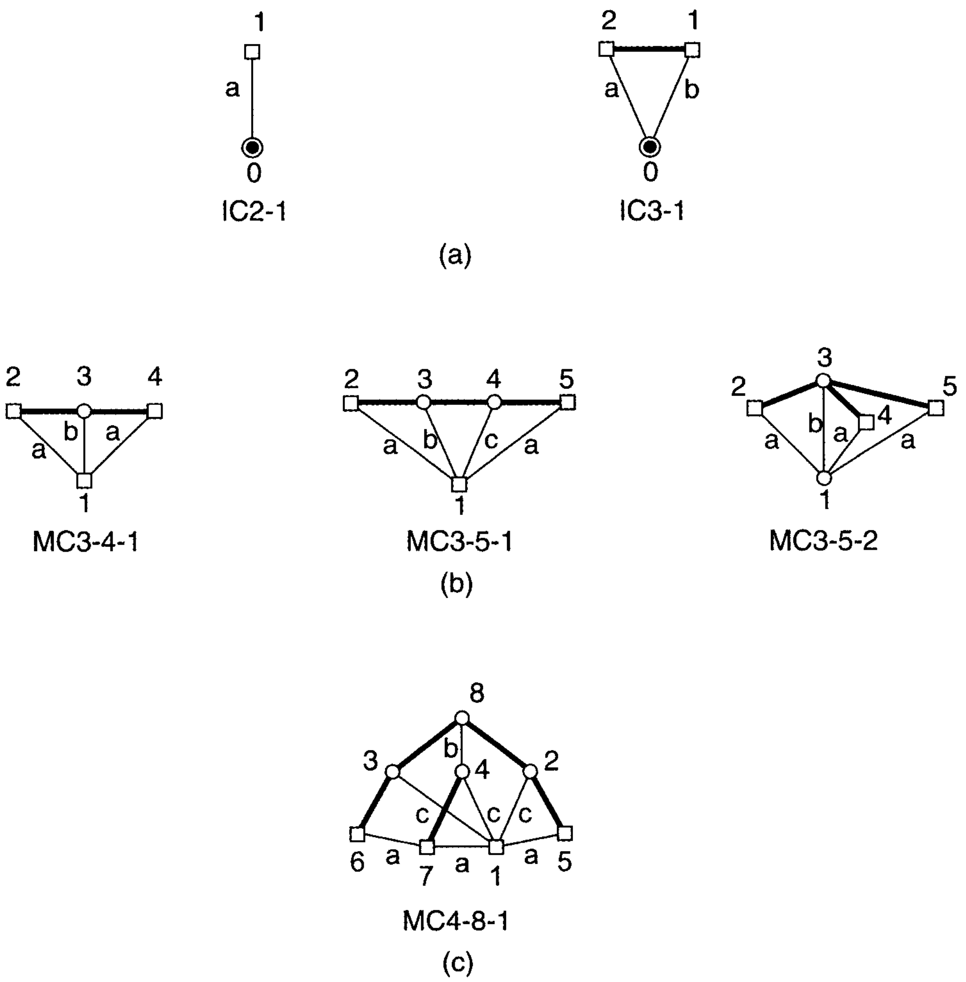


Figure 2 Graph representation: (a) ICs with up to 3 links, (b) 3-port MCs with up to 5 links, (c) 4-port MC with 8 links.

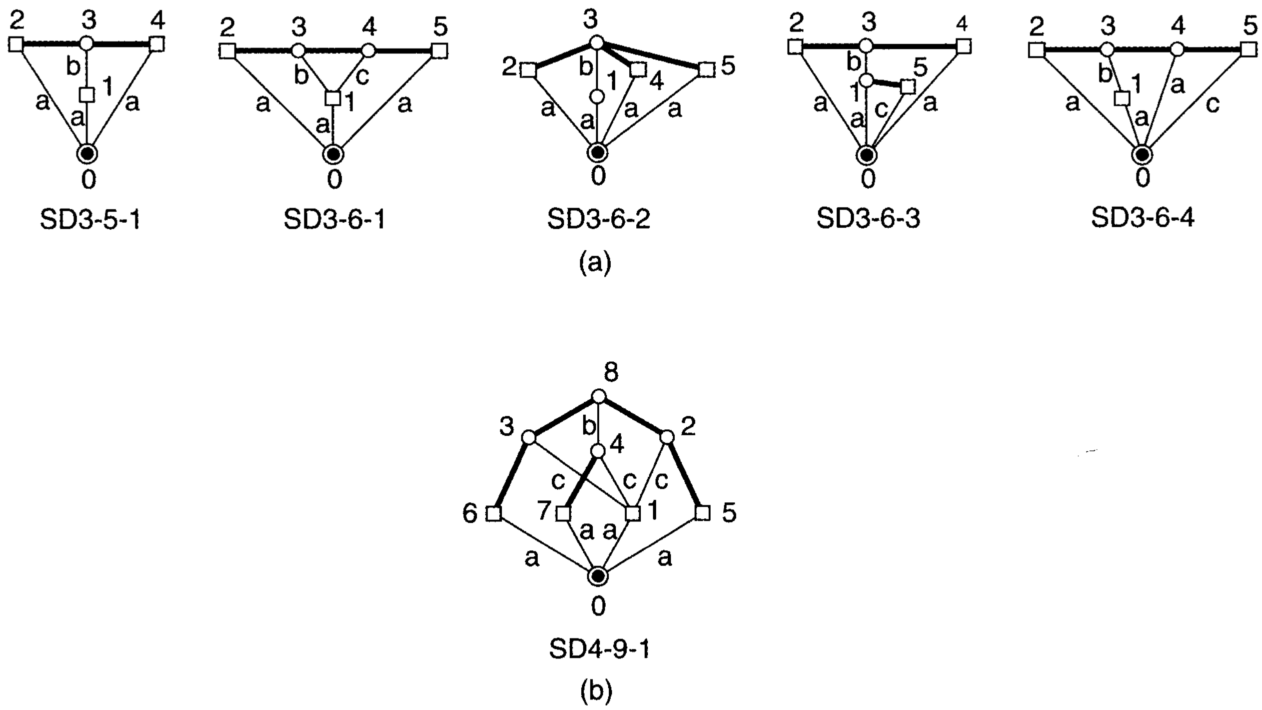


Figure 3 Admissible link-fractionated DGMs: (a) 3-terminal, (b) 4-terminal.

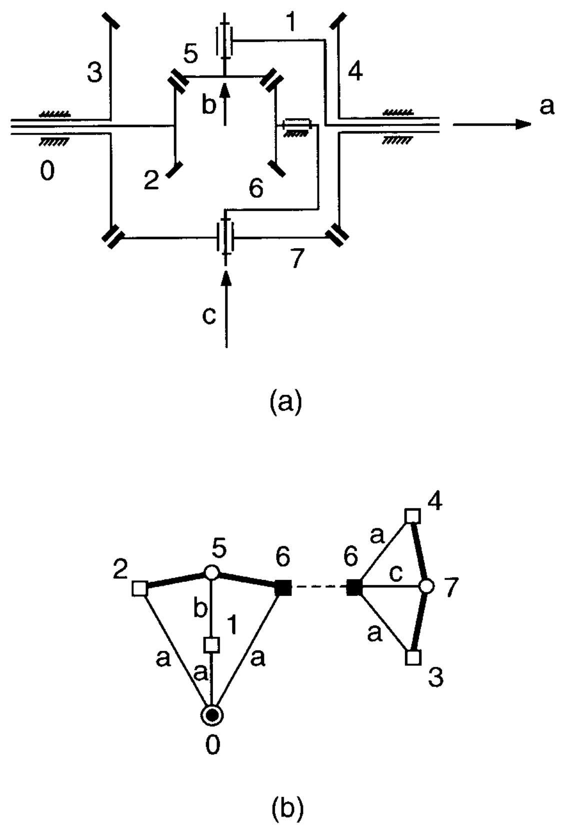


Figure 4 A 4-terminal 8-link DGM: (a) Functional representation, (b) Pseudo-isomorphic graph representation.

represented by hollow rectangles. By separating the cut-link 6, it can be seen that the 4-terminal DGM can be decomposed into two components. In Figure 4(b), the cut-link is separated into two solid rectangles and a dashed line, where the solid rectangle denotes the connecting link in each component and the dashed line shows the connecting relation. In each component, terminals and the connecting link are defined as ports. From the atlas of link-fractionated DGMs shown in Figure 3(a), it can be seen that the first component (0, 1, 2, 5, 6) is the link-fractionated DGM SD3-5-1 with links 1, 2, and 6 as the ports. From the atlases of MCs shown in Figure 2(b), it can be seen that the second component (3, 4, 6, 7) is the MC3-4-1 with links 3, 4, and 6 as the ports.

Hence, the 4-terminal DGM shown in Figure 4 can be decomposed into the SD3-5-1 as the base-DGM and the MC3-4-1 as the MC by separating link 6, and is defined as a compound DGM. As an extension, an m-link compound DGM with n terminals is assumed to be decomposed into an m_{bd}-link base-DGM with n_{bd} ports, and an m_{mc}-link MC with n_{mc} ports by separating a cut-link. Thus, the number of links among the compound DGM, its decomposed base-DGM, and the MC must satisfy the following equation:

$$m = m_{\mathrm{bd}} + m_{\mathrm{mc}} - 1 \tag{5}$$

The connecting links of the base-DGM and the MC are ports of each of the two components but not terminals of the compound DGM. Hence,

$$n = n_{\mathrm{bd}} + n_{\mathrm{mc}} - 2 \tag{6}$$

Equations 5 and 6 lead to the following characteristic:

R1. An m-link compound DGM with n terminals can be decomposed into an m_{bd}-link DGM with n_{bd} ports as the base-DGM and an m_{mc}-link MC with n_{mc} ports, while

$$m_{bd} + m_{mc} = m + 1 \tag{7}$$

$$n_{bd} + n_{mc} = n + 2 \tag{8}$$

From the 4-terminal compound DGM shown in Figure 4(b), it can be seen that those terminals located in the MC, as links 3 and 4, and the ground link are in different components. Since all terminals must be adjacent to the ground link via revolute joints [10], the common link of the two components, terminals in the MC, and the ground link must be coaxial. It leads to the following characteristic:

R2. In a compound DGM, terminals in the MC, the common link of the base-DGM and the MC, and the ground are coaxial.

From Equation 6, since the minimum numbers of ports in the base-DGM and the MC are both equal to three, the minimum number of terminals of a compound DGM can be derived as

$$n_{\min} = n_{(bd)\min} + n_{(mc)\min} - 2 = 3 + 3 - 2 = 4 \tag{9}$$

From Figure 3(a) and Figure 2(b), the minimum number of links of a 3-port base-DGM is equal to five and the minimum number of links of a 3-port MC is equal to four. Hence, the minimum number of links of a 4-terminal compound DGM can be derived from Equation 5 as

$$m_{\min} = m_{(bd)\min} + m_{(mc)\min} - 1 = 5 + 4 - 1 = 8 \tag{10}$$

Similarly, it can be seen that the minimum number of links of a 5-terminal compound DGM is eleven.

5 Composition of *n*-terminal *m*-link compound differential geared mechanisms

The creation of an n-terminal m-link compound DGM can be treated as a matter of determining proper base-DGM and proper MC, and joining the selected connecting links from these components into a cut-link. The composition process is presented in the following steps with the enumeration of admissible 4-terminal 8-link compound DGMs as an illustrative example.

Step 1: Determine the number of ports of the base-DGM and the MC.

In this step, the number of terminals of the compound DGM is specified. Equation 8 shows the relation among the number of terminals of the DGM with the number of ports in the base-DGM and the MC. From the atlas of admissible MCs shown in Figures 2(b) and 2(c), the range of n_{mc} is from three to four, and only two admissible sets of n_{bd} and n_{mc} can be derived from Equation 8 as (a) $n_{bd} = (n - 1)$ and $n_{mc} = 3$, and (b) $n_{bd} = (n - 2)$ and $n_{mc} = 4$.

For the 4-terminal 8-link compound DGM, since n is equal to four and the minimum n_{bd} is equal to three, only one admissible set of n_{bd} and n_{mc} can be derived as $n_{bd} = 3$ and $n_{mc} = 3$.

Step 2: Determine the numbers of links of the base-DGM and the MC.

In this step, the number of links of the compound DGM is first specified. Equation 7 shows the relation among the number of links of the DGM, the base-DGM, and the MC. For the first set of $n_{bd} = (n-1)$ and $n_{mc} = 3$ in Step 1, since the range of m_{mc} of 3-port MCs is from four to five from Figure 2(b), two admissible sets of m_{bd} and m_{mc} can be derived from Equation as (a) $m_{bd} = (m-3)$ and $m_{mc} = 4$, and (b) $m_{bd} = (m-4)$ and $m_{mc} = 5$. For the second set of $n_{bd} = (n-2)$ and $n_{mc} = 4$, since m_{mc} of 4-port MCs is equal to eight from Figure 2(c), the m_{bd} and m_{mc} can be derived as $(m-7)$ and 8, respectively.

For the 4-terminal 8-link compound DGM, with m equal to eight and the minimum m_{bd} equal to five, the admissible set of m_{bd} and m_{mc} corresponding to $n_{bd} = 3$ and $n_{mc} = 3$ is derived as $m_{bd} = 5$ and $m_{mc} = 4$, respectively.

Step 3: Determine the base-DGM and the MC.

In Steps 1 and 2, several sets of admissible base-DGM and its corresponding MC can be derived. The base-DGM and the MC of each set can then be selected from the atlases of DGMs and MCs, respectively. For the 4-terminal 8-link compound DGM, the 3-port 5-link base-DGM is selected from Figure 3(a) as the SD3–5–1, and the corresponding 3-port 4-link MC is selected from Figure 2(b) as MC3–4–1.

Step 4: Determine the connecting link of the base-DGM and the MC.

One of the ports in the base-DGM and one of the ports in the MC are selected as the connecting links. In the base-DGM SD3–5–1, links 1, 2, and 4 can be chosen as the connecting link to the MC. In the MC3–4–1, links 1, 2, and 4 can be chosen as the connecting link to the base-DGM.

Step 5: Join the connecting links in the base-DGM and MC as the cut-link. Ports in the base-DGM and the MC not being selected as connecting links in Step 4 are assigned as terminals of the DGM.

Step 6: Assign terminals in the MC, the cut-link, and the ground as coaxial links.

In Steps 5 and 6, with different combinations of connecting links in the base-DGM and the MC, there are nine possible configurations of the 4-terminal 8-link compound DGM. The nine possible configurations are shown in Table 1.

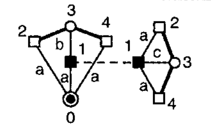
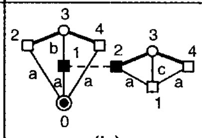
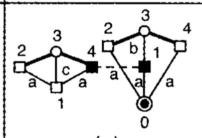
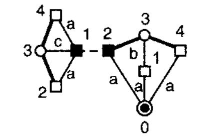
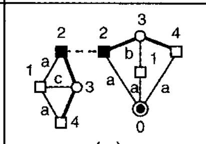
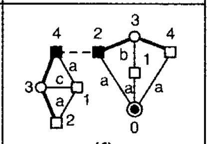
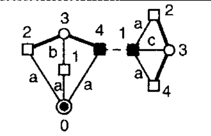
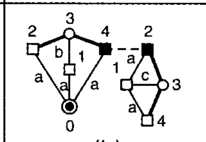
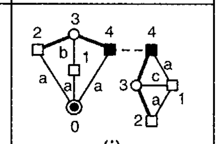
Step 7: Exclude the isomorphic compound DGMs with the isomorphic detection techniques such as the Max. and Min. Code [1] or the Degree Code [11].

In Table 1, DGMs (c), (d), and (g) are isomorphic with (b), and DGMs (f), (h), and (i) are isomorphic with (e). Hence, three non-isomorphic 4-terminal 8-link compound DGMs can be found from the nine configurations.

Figure 5 shows the 4-terminal compound DGMs with up to 9 links in canonic form [13]. Figure 6 shows the 5-terminal compound DGMs with 11 links. An n-terminal m-link compound DGM is coded as CDn-m-#, with # as the series number. In Figures 5 and 6, all 4-terminals compound DGMs except CD4–8–2 and all 5-terminal compound DGMs except CD5–11–5 are believed to be new as compared with the results of Kota and Bidare [8].

Note that the two compound DGMs CD4–8–1 and CD4–8–3 shown in Figure 5 were taken as invalid cases by Kota and Bidare [8]. They concluded that CD4–8–1 and CD4–8–3 could not properly distribute the input torque to the three outputs. However, this conclusion is based on an assumption that Kota and Bidare [8]

Table 1 Composition of the 4-terminal 8-link compound DGMs.

		Connecting Link in MC3-4-1		
		Link 1	Link 2	Link 4
Connecting Link in base-DGM SD3-5-1	Link 1	 (a)	 (b)	 (c)
	Link 2	 (d)	 (e)	 (f)
	Link 4	 (g)	 (h)	 (i)

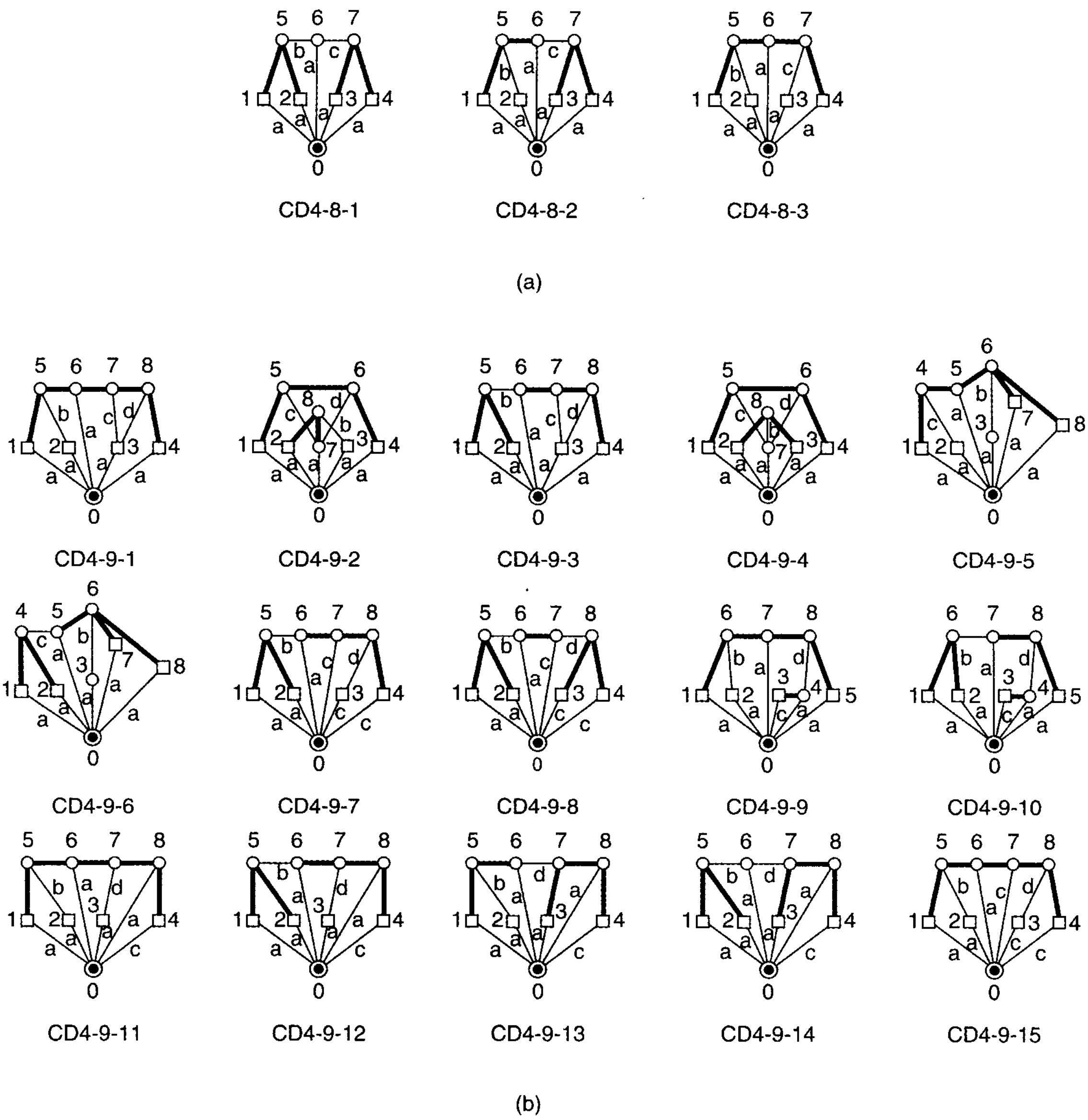


Figure 5 Admissible 4-terminal compound DGMs: (a) 8-link, (b) 9-link.

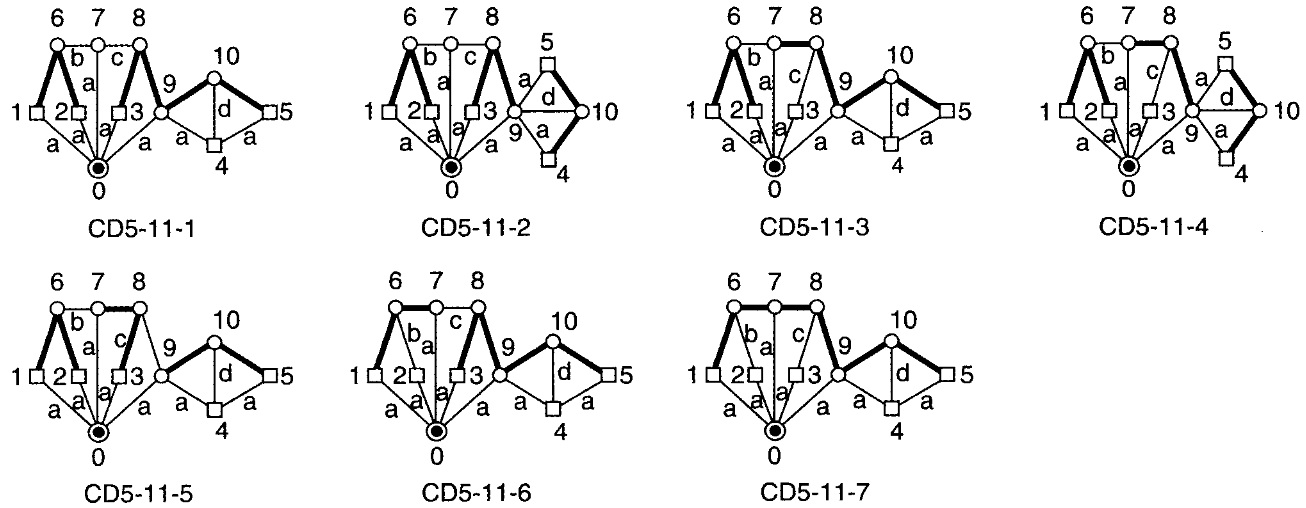


Figure 6 Admissible 5-terminal compound DGMs with 11 links.

allowed only two types of gear train arrangements, which are defined as differential building blocks shown in Figure 7, in DGMs they enumerated. In Figures 7(a) and 7(b), it can be seen that train values of outputs, as links 2 and 3, of differential building blocks are all restricted to negative. This limitation excludes many admissible gear train arrangements on DGMs, and thus makes the two compound DGMs CD4–8–1 and CD4–8–3 unavailable to properly distribute the input torque to the three outputs.

The gear-train-arrangement process of the 4-terminal compound DGM CD4–8–1 in equal-torque-distribution condition is shown as follows. By eliminating the non-terminals in fundamental circuit equations [4] of the CD4–8–1, the kinematic constraint equation among terminals 1, 2, 3, and 4 can be derived as:

$$(\gamma_{4,3}-1)w_1-\gamma_{2,1}(\gamma_{4,3}-1)w_2-(\gamma_{2,1}-1)w_3+\gamma_{4,3}(\gamma_{2,1}-1)w_4=0 \tag{11}$$

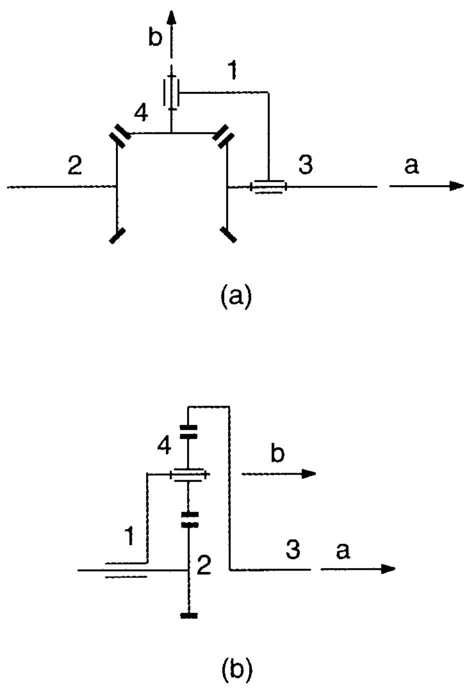


Figure 7 Two types of differential building blocks: (a) A standard differential, (b) A simple EGT.

where

$$\gamma_{2,1} = \frac{\gamma_{5,1}}{\gamma_{5,2}} \tag{12}$$

and

$$\gamma_{4,3} = \frac{\gamma_{7,3}}{\gamma_{7,4}} \tag{13}$$

where $\gamma_{ij} = \pm N_i/N_j$ denotes the gear ratio of the gear pair composed of links i and j, the sign of the gear ratio is determined according to the positive rotation of link i results in a positive or negative rotation of link j along their pre-defined axes of rotation, and N_i is the teeth number of gear i.

Under the equal-torque-distribution condition, the torque distribution among terminals of the CD4–8–1 can be related by selecting link 1 as the input and links 2, 3, and 4 as outputs as:

$$\frac{T_1}{-3} = \frac{T_2}{1} = \frac{T_3}{1} = \frac{T_4}{1} \tag{14}$$

With the no power loss assumption, torques among the four terminals are proportional to the coefficients of the kinematic constraint equation. Hence, Equations 11 and 14 lead to

$$\frac{(\gamma_{4,3} - 1)}{-3} = \frac{-\gamma_{2,1}(\gamma_{4,3} - 1)}{1} = \frac{-(\gamma_{2,1} - 1)}{1} = \frac{\gamma_{4,3}(\gamma_{2,1} - 1)}{1} \tag{15}$$

The gear ratios $\gamma_{2,1}$ and $\gamma_{4,3}$ can be derived from Equation 15 as 1/3 and −1. From Equations 12 and 13, gear ratios of the four gear pairs as $\gamma_{5,1}$, $\gamma_{5,2}$, $\gamma_{7,3}$, and $\gamma_{7,4}$ are assigned as $-1/3, -1, -1$, and 1 respectively. Figure 8(a) shows a possible gear train arrangement of the CD4–8–1. Note that the train value $\gamma_{2,1}$ of the gear train composed of links 1, 5, and 2 is assigned as positive, which is different from the two differential building blocks Kota and Bidare [8] derived. Hence, it can be seen that the compound DGM CD4–8–1 is eligible to equally distribute the input torque to the three outputs. Based on a similar derivation from Equations 11 to 15, Figure 8(b) shows a possible gear train arrangement of CD4–8–3 in the equal-torque-distribution condition.

We believe that the decomposition-based method of enumeration of n-terminal compound DGMs, which uses the atlas of DGMs and MCs developed by earlier investigators as foundation, is more systematic, more straightforward, and more complete than the previous approaches, such as the work by Kota and Bidare [8]. A gear-train-arrangement process is also presented, in which the equal-torque-distribution condition can be achieved in DGMs. On the other hand, it is shown in this article that a link-fractionated DGM not only can be composed of an IC and an MC by Chen and Yao [2], but also can be composed of a base-DGM and an MC. Since an n-terminal compound DGM is composed of an MC and a base-DGM with ports fewer than n, and the n-terminal compound DGM can be selected as a new base-DGM with n ports, compound DGMs with any number of terminals can be enumerated by repeating the procedure. Hence, although only the compound DGMs

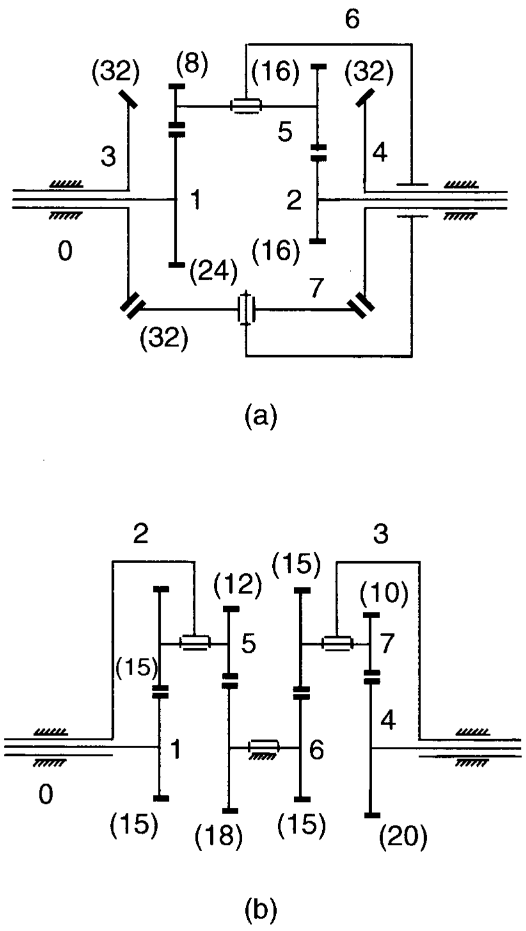


Figure 8 A possible gear train arrangement of: (a) CD4–8–1, (b) CD4–8–3.

with four and five terminals are enumerated as examples, the methodology presented here is completely general and innovative, and can be easily applied to the enumeration of DGMs with multiple terminals.

6 Conclusion

This article describes a systematic methodology for the topological synthesis of admissible *n*-terminal compound DGMs. In this approach, it is shown that compound DGMs can be composed by adding an MC on a base-DGM. Characteristics of the compound DGM are addressed, and the procedure to select proper base-DGM and MC to form compound DGMs is described. Atlases of admissible 4-and 5-terminal compound DGMs are developed. The decomposition-based methodology presented here is completely general, and can be easily applied to the enumeration of DGMs with multiple terminals.

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