Dar-Zen Chen[1]
Professor
Department of Mechanical Engineering,
National Taiwan University,
Taipei, Taiwan, 10617
e-mail: dzchen@ntu.edu.tw

Win-Bin Shieh
Assistant Professor
Department of Mechanical Engineering,
Mingchi University of Technology,
Taipei, Taiwan, 24301

Yu-Ching Yeh
Graduate Student
Department of Mechanical Engineering,
National Taiwan University,
Taipei, Taiwan, 10617

Kinematic Characteristics and Classification of Geared Mechanisms Using the Concept of Kinematic Fractionation

A methodology based on the concept of kinematic fractionation for the revelation of kinematic characteristics and classification of geared mechanisms is presented. It is shown that structurally nonfractionated geared mechanisms can be considered as the combination of kinematic units (KUs). Each KU is considered as the basic motion transmission module inside a geared mechanism. Admissible connections of KUs are identified according to the structural characteristics of one- and two-degree-of-freedom geared mechanisms of up to four KUs. Graphs in the atlas of the geared mechanisms are classified based on the configurations of KUs. Such configurations are then used to construct possible propagation paths of motion via the assignments of input and output links. Since the propagation paths can be modeled by the control block diagram problems, the kinematic relations between input and output links are formulated to gain matrices. According to the types of entities in a gain matrix, various kinematic behaviors are disclosed. It is believed that such kinematic characteristics can be readily transformed into the functional requirements, and the synthesis of geared mechanisms of up to four KUs can be accomplished much easier. [DOI: 10.1115/1.2936894]

1 Introduction

Geared mechanisms have been widely used as power transmission and force amplification devices in machines and vehicles. The input power is transmitted to the output link through a power flow path made up of meshing pairs and their corresponding carriers. Through analysis, kinematic relations between input and output links of a geared mechanism are evaluated. Although the tabular and formula methods, etc., have been well developed and can solve for almost all gear train problems, such a procedure becomes tedious when gear trains are complex. In virtue of graph theory [1], the concept of fundamental circuits was applied to the kinematic analysis of geared mechanisms [2,3]. By solving a set of linear equations simultaneously, the kinematic relations can be obtained through a series of arithmetical manipulation, where kinematic insights are barely revealed. Chatterjee and Tsai [4] decomposed an epicyclic geared mechanism into several fundamental geared entities and applied the concept to the speed ratio analysis and power loss problems. However, this study is only applicable for the reverted type of epicyclic gear trains and for the analysis of kinematic relations among coaxial links.

Chen and Shiue [5] showed that a geared robotic mechanism can be fractionated into input units and transmission units. Chen [6] further verified the forward and backward gains of each unit and proposed a unit-by-unit evaluation procedure for the kinematic analysis of such geared robotic mechanisms. Although this approach is straightforward and provides clear kinematic insights in torque transmission problems, it is restricted to the analysis of geared robotic mechanisms.

Based on the concept of kinematic fractionation developed by Liu and Chen [7], motion transmission inside a geared mechanism can be considered as motion transmitted from input to output via a series of kinematic units (KUs). Liu et al. [8] further unveiled the topological structures among fractionated KUs, where two types of structures were identified. The local gain of each individual KU was formulated, and the input-output relations of a geared mechanism were established. However, how the global kinematic relations of geared mechanisms are related to the configurations of KUs is not discussed.

Rao [9] applied genetic algorithm for the design of a geared mechanism with high speed ratio or transmission efficiency. Based on the concept of fundamental KUs, Kahraman et al. [10] proposed a methodology for the determination of the speed ratio for each gearing pairs according to the configuration of the mechanism and the input and output requirements. Salgado and Del Castillo [11] established a power flow map by drawing the gearing power and transmission ratio curves for the design of a planetary gear train. Talpasanu et al. [12] developed structure matrices from the spanning tree and joint position of a geared mechanism for the analysis and synthesis of parallel axis epicyclic gear trains. Although the problems regarding the synthesis of geared mechanisms have been studied intensively and the admissible one and two degree-of-freedom (DOF) geared mechanisms up to six or higher links have been enumerated, few efforts have been focused on how these admissible structures can be applied to the real engineering world. The design of geared mechanisms based on the functional kinematic requirements still relies heavily on the expertise of engineers and the trial-and-error process.

In this paper, structural characteristics of geared mechanisms based on the concept of kinematic fractionation are explored to obtain the configurations of KUs in geared mechanisms. According to the configurations, the propagation paths of motion and their corresponding global gains are formulated. The kinematic characteristics of geared mechanisms are revealed and classified. It is believed that the classification of geared mechanisms according to the types of kinematic characteristics based on the concept of kinematic fractionation provides more kinematic insights, and the design process of one- and two-DOF geared mechanisms with up to four KUs can be dramatically enhanced.

[1]Corresponding author.

Contributed by the Power Transmission and Gearing Committee of ASME for Publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received November 27, 2006; final manuscript received September 13, 2007; published online July 10, 2008. Review conducted by Teik C. Lim.

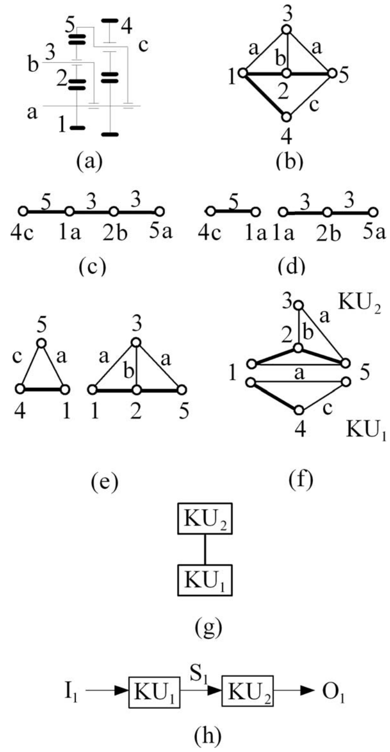


Fig. 1 One-DOF, five-link GKC of graph 1400-1-4 [1]. (*a*) Functional schematic. (*b*) Graph representation. (*c*) Displacement graph. (*d*) Disconnected displacement graph. (*e*) Disconnected KUs. (*f*) Fractionated KUs. (*g*) Block configuration of KUs. (*h*) Propagation path of KUs.

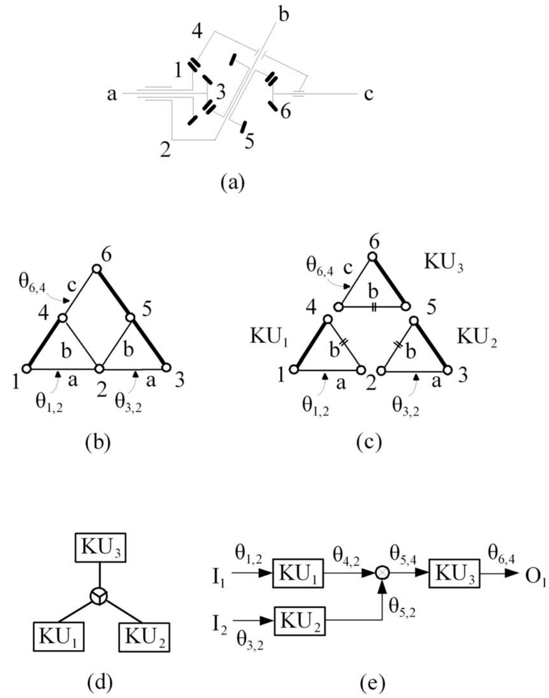


Fig. 2 Two-DOF, six-link GKC of graph 4-1-1 [14]. (*a*) Functional schematic. (*b*) Graph representation. (*c*) Fractionated KUs with coaxial-triangle type common linkages. (*d*) Block configuration of KUs. (*e*) Propagation path of KUs.

2 Concept of Kinematic Fractionation

In the graph representation of geared mechanisms, links are represented by vertices, gear pairs by heavy edges, and turning pairs by thin edges; each thin edge is labeled according to its associated axis location. Liu and Chen [13] defined the KU as a basic kinematic structure in geared mechanisms. Each KU is one-DOF and is composed of a carrier and all gears on it. The identification of KUs in a geared mechanism [7] is briefly described as follows with an illustration of a five-link gear kinematic chain (GKC) of graph 1400-1-4 [1] in Fig. 1. The functional schematic and the graph representation of the five-link GKC are shown in Figs. 1(*a*) and 1(*b*), respectively. The procedure starts with the construction of the displacement graph [1] formed by all gearing pairs, as shown in Fig. 1(*c*). In the displacement graph, the meshing gears and their associated axes are labeled under the vertices, and carriers are labeled above the heavy edges. Then, the displacement graph is segmented into subgraph(s), each with only one carrier label. Figure 1(*d*) shows the disconnected displacement graph of Fig. 1(*c*). Add a carrier vertex to each segment of the disconnected displacement graph and connect the gear-carrier pairs by thin edges labeled with axis location. Each subgraph is referred to a disconnected KU, as shown in Fig. 1(*e*). Since vertices 1 and 5 are coaxial and common to both subgraphs, a thin edge can be formed by coaxial rearrangement without changing the kinematic characteristics of the mechanism [8]. As a result, the geared mechanism of Fig. 1(*a*) is fractionated into two resultant KUs, as shown in Fig. 1(*f*).

3 Topological Characteristics of a Kinematic Unit

3.1 Common Linkage of KUs. The topological connection of KUs is determined by the structural characteristics of geared mechanisms. Since a KU does not exist alone within a geared mechanism, unless the mechanism itself is a single KU, it is connected to other KUs through certain common vertices and thin edges. These common vertices and thin edges shared by the adjacent KUs are referred to common linkages of KUs. Two types of common linkages among KUs are identified in all GKCs of one-DOF up to six links and two-DOF up to seven links [8]:

(1) two-link chain type common linkage: the common linkage that is shared exclusively by two KUs is referred to the two-link chain type, as shown in Fig. 1(*f*), where the motion between KU_1 and KU_2 is transmitted through a common linkage of 1–5. Such a two-link chain type common linkage of KUs can be represented by the block configuration shown in Fig. 1(*g*).

(2) coaxial-triangle type common linkages: the common linkages that are shared by three KUs are referred to the coaxial-triangle type. Figure 2(*a*) shows a functional schematic of a two-DOF, six-link GKC of graph 4-1-1 [14], and its graph representation is shown in Fig. 2(*b*). The common linkages among KUs, as shown in Fig. 2(*c*), is referred to a coaxial-triangle type, where the output motions of KU_1 and KU_2 are transmitted through the common linkages of 2-4, 2-5, and 4-5 to KU_3. Because of this, the block configuration of the coaxial-triangle type common linkages is represented by a node connecting to three KUs, as shown in Fig. 2(*d*).

3.2 Rule of Connection of KUs. Based on the structural characteristics of geared mechanisms, the DOF of a geared mechanism can be expressed as

$$F = e - h \tag{1}$$

where e and h are the numbers of turning-pair and gearing-pair edges in the graph representation of a geared mechanism, respectively.

Similar to Eq. (1), since the DOF of each KU is equal to 1, the numbers of turning-pair edges and gearing-pair edges of the ith KU in a geared mechanism can be related by

$$e_i = h_i + 1 \tag{2}$$

For a geared mechanism containing u fractionated KUs, the total number of the thin edges possessed by all fractionated KUs can be summed up as

$$\sum_{1}^{u} e_i = \sum_{1}^{u} (h_i + 1) = \sum_{1}^{u} h_i + u = h + u \tag{3}$$

It is observed from Eq. (3) that when a geared mechanism is fractionated into KUs, the total number of the thin edges increases, depending on the number of KUs, while the total number of heavy edges remains unchanged. Referring to the two-KU example of Fig. 1(*f*) and the three-KU graph of Fig. 2(*c*), both graphs contain three heavy edges, while the total numbers of thin edges possessed by the two-KU and three-KU graphs are five and six, respectively.

Alternatively, according to the Grübler–Kutzbach criterion, the DOF of a geared mechanism can also be represented as

$$F = 3(v - 1) - 2e - h \tag{4}$$

where v is the number of links of a geared mechanism or the number of vertices of a graph.

Equating Eq. (1) with Eq. (4) yields the relation between the numbers of vertices and turning-pair edges,

$$e = v - 1 \tag{5}$$

Similarly, the numbers of turning-pair edges e_i and vertices v_i of the ith KU in a geared mechanism can be related as

$$v_i = e_i + 1 \tag{6}$$

Collecting all u fractionated KUs in a geared mechanism and making use of Eqs. (2) and (6) yield the total number of vertices of gear mechanisms

$$\sum_{1}^{u} v_i = \sum_{1}^{u} (e_i + 1) = \sum_{1}^{u} (h_i + 2) = \sum_{1}^{u} h_i + 2u = h + 2u \tag{7}$$

Substituting Eq. (5) in Eq. (1), number of vertices in the graph representation of geared mechanisms can be expressed as

$$v = F + h + 1 \tag{8}$$

Subtracting Eq. (7) from Eq. (8) yields the difference between the numbers of vertices of the graph representation and fractionated KU graph of gear mechanisms

$$R = \sum_{1}^{u} v_i - v = (h + 2u) - (F + h + 1) = 2u - F - 1 \tag{9}$$

Equation (9) indicates that the difference between the numbers of vertices in these two graphs is determined by the number of fractionated KUs and the DOF of geared mechanisms. Normally, the number of vertices for all fractionated KUs is more than that of a graph representation due to the common linkages, where some common vertices are counted repeatedly. This implies that the number of repeated vertices is determined by the number and the types of common linkages of KUs. Hence, the topological configuration of KUs in a geared mechanism would be discussed from the perspective of common linkages as follows.

3.3 Configuration of KUs. Since a KU is formed by at least one fundamental circuit, the maximum number of KUs in a geared mechanism is limited by the number of gearing-pair edges of the mechanism. Besides, since a nonfractionated geared mechanism itself is a KU [7], the minimum number of KUs is equal to 1. In this paper, the structural characteristics of geared mechanisms of up to four KUs will be explored as follows. Note that, as observed from Eq. (8), since the minimum numbers of links in one- and two-DOF geared mechanisms of up to four KUs are six and seven, one- and two-DOF geared mechanisms of up to six and seven

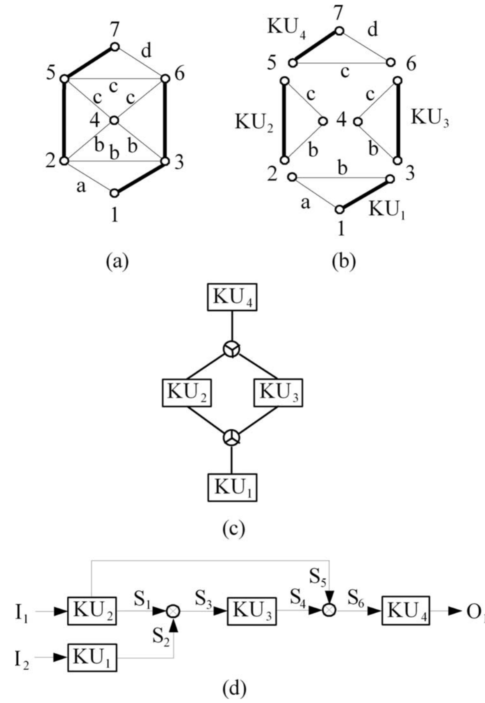


Fig. 3 Two-DOF, seven-link GKC of graph 5-17-1 [15]. (*a*) Graph representation. (*b*) Fractionated KUs with two coaxial-triangle type common linkages. (*c*) Block configuration of KUs. (d) Kinematic propagation path VII(a) of KUs.

links, respectively, are concerned.

Consider the two-link chain type connection of KUs, as shown in Fig. 1(*f*) in which two common vertices 1 and 5 shared by the two KUs are counted twice. Since the two repeated vertices are accountable for each two-link chain type connection, the total number of repeated vertices in a fractionated KU graph can be written as

$$R = 2C_2 \tag{10a}$$

where C_2 denotes the number of two-link chain type common linkages appearing in a fractionated geared mechanism.

In a similar manner, consider the coaxial-triangle type connection, as shown in Fig. 2(*c*), where three common vertices 2, 4, and 5 shared by the three KUs are counted twice. Hence, there are three repeated vertices for each coaxial-triangle type connection. However, if any two coaxial-triangle type common linkages are connected, such a relation has to be modified accordingly. Consider the two-DOF, seven-link GKC of graph 5-17-1 [15], as shown in Fig. 3(*a*). This geared mechanism is kinematically fractionated into four KUs, forming two coaxial-triangle type common linkages connected in series, as shown in Figs. 3(*b*) and 3(*c*). The number of repeated common vertices is equal to three times of the number of coaxial-triangle type common linkages subtracted by the number of any two coaxial-triangle type common linkages connected in series. Thus, the number of repeated vertices can be expressed as

$$R = 3C_3 \quad \text{for } C_3 < 2 \tag{10b}$$

and

$$R = 3C_3 - (C_3 - 1) = 2C_3 + 1 \quad \text{for } C_3 \geqq 2 \tag{10c}$$

where C_3 denotes the number of coaxial-triangle type connection in a fractionated geared mechanism.

For geared mechanisms composed of any combination of two-link chain type and/or coaxial-triangle type common linkages, R can be rewritten as

$$R = 2C_2 + 3C_3 \quad \text{for } C_3 < 2 \tag{11a}$$

and

$$R = 2C_2 + 2C_3 + 1 \quad \text{for } C_3 \geqq 2 \tag{11b}$$

Equating Eqs. (11*a*) and (11*b*) with Eq. (9) yields

$$2C_2 + 3C_3 = 2u - F - 1 \quad \text{for } C_3 < 2 \tag{12a}$$

and

$$2C_2 + 2C_3 = 2u - F - 2 \quad \text{for } C_3 \geqq 2 \tag{12b}$$

Equations (12*a*) and (12*b*) show that if the number of KUs and the DOF of a geared mechanism are given, the numbers of two-link chain type and coaxial-triangle type common linkages can be determined. As a consequence, seven admissible block configurations I–VII of one- and two-DOF geared mechanisms of up to four KUs are obtained and tabulated in the second column of Table 1. As observed from the table, the coaxial-triangle type common linkages are only found in two-DOF geared mechanisms. This can be easily realized as follows. Considering a one-DOF geared mechanism, i.e., $F=1$, the values on both sides of Eq. (12*a*) have to be odd and those of Eq. (12*b*) have to be even. In order for the left-hand-side values of these two equations satisfy the conditions, C_3 has to be zero for $C_3 < 2$, and there is no possible solution of C_3 for $C_3 \geqq 2$.

According to the configurations of Table 1, previously enumerated geared mechanisms of one-DOF up to six links and two-DOF up to seven links are examined and categorized. The result is shown in Table 2, with geared mechanisms represented by their graph numbers. Such classification of geared mechanisms not only reveals the structural characteristics of the enumerated geared mechanisms; it also benefits the disclosure of transmission of motion inside the mechanisms.

It is worthy to note that the graph numbers of the geared mechanisms of one-DOF up to five links are referred to the atlas of epicyclic gear trains of Freudenstein [1], the graph numbers of one-DOF six-link geared mechanisms are referred to Tsai [14], and those of two-DOF up to seven links are referred to Tsai and Lin [15].

4 Kinematic Propagation Paths

A kinematic propagation path of motion represents the transmission of motion between KUs within a geared mechanism. Based on the configurations of KUs shown in Table 1, possible kinematic propagation paths can be established once input and output links are adequately selected.

In order to maintain the mobility of the geared mechanisms, the following general rules have to be fulfilled:

R1: Each KU has one local input. Since the DOF of a KU is equal to 1, any KU possesses only one local input.

R2: The number of global inputs is equal to the DOF of a geared mechanism. Therefore, geared mechanisms pertaining to configurations I–IV have one global input, while those belonging to configurations V–VII possess two global inputs. Although the global input(s) of a geared mechanism can be designated at any KU, kinematic propagation paths with certain arrangements have to be excluded. Referring to the GKC of graph 4-1-1in Fig. 2(*b*), the block configuration of the graph is formed by KU_1, KU_2, and KU_3, as shown in Figs. 2(*c*) and 2(*d*). According to R2, this two-DOF geared mechanism has two global inputs. Any two of the three KUs can be used as inputs. However, inputs at KU_1-KU_2, KU_2-KU_3, or KU_3-KU_1 are considered to be identical to each other due to the symmetry of configuration with regard to the topological perspective. Hence, we have the following.

R3: Assignment of global input(s) at topologically symmetric KU(s) has to be avoided. Followed by the determination of global input(s), global output(s) of a geared mechanism has to be chosen. Consider a six-link GKC of graph 6205-1 [14] of Fig. 4(*a*). If vertex 2 (or 6) is selected as the input, as indicated in Fig. 4(*b*), the adequate output will be vertex 6 (or 2). This way, transmission path of motion propagates from the input to pass through entire GKC, i.e., from KU_1 to KU_3 or vice versa. Since the kinematic relation relies on all KUs, there is no redundant link in the geared mechanism. On the other hand, if a vertex other than 6 (or 2), say, 4, is selected as the output, transmission of motion does not pass through all KUs, leaving 6 (or 2) as a redundant link. Note that since vertices 2 and 6 in Fig. 4(*b*) are on the open ends of the heavy-edged path of KU_1 and KU_3, respectively, but are not common to KU_2, they are referred to end vertices [7]. Hence, to prevent redundancy of links, we have the following.

R4: KU(s) with end vertices not utilized as the global input(s) are bound to be the global output(s). Although the number of local inputs for each KU is 1, the number of local outputs of a KU is not limited to 1. If more than one KU is adjacent to and driven by a KU, the motion of the driving KU is transmitted to its following adjacent KUs simultaneously. Hence, we have the following.

R5: The number of local outputs of a KU depends on the number of its following KUs. Referring to propagation path IV(b) of Table 1, KU_2 has two local outputs: One is via the first propagation path of KU_2 to KU_1, and the other is through the second propagation path of KU_2 to KU_3. As previously stated, there are two types of KU connections in geared mechanisms, and the manners for the transmission of motion of these two types are different in nature.

R6: In the two-link chain type configuration, the local output of a KU is the local input of its next KU, while in the coaxial-triangle type configuration, the local input of a KU is determined by local outputs of its two preceding KUs. Referring to the two-link chain type configuration of Fig. 1(*g*), the local input of KU_2 is the local output of KU_1, as shown in Fig. 1(*h*), while, referring to the coaxial-triangle type configuration of Fig. 2(*d*), the local input of KU_3 is summed by both local outputs of KU_1 and KU_2 and hence is represented by an addition node, as shown in Fig. 2(*e*).

Applying R1–R6 to the block configurations I–VII in Table 1, six admissible propagation paths of motion are obtained for the one-DOF configurations as shown in and seven paths are identified for the two-DOF configurations, as shown in Table 1.

5 Gain of Kinematic Paths

The kinematic propagation path in a geared mechanism represents the motion transmitted from one KU to another, resembling a signal processing problem where the overall system is made up of control unit blocks. In a geared mechanism, KUs are considered as basic blocks, while angular displacement or angular velocity is considered as a signal. Based on this concept, the global gain between input and output links of geared mechanisms can be derived in virtue of block diagram algebra.

Referring to the system in Fig. 1(*h*), signal S_1 is generated after input I_1 passes KU_1. Then,

$$S_1 = g_1 I_1 \tag{13}$$

where g_1 is the local gain of KU_1. Output O_1 is generated after signal S_1 goes through KU_2 and becomes

$$O_1 = g_2 S_1 = g_1 g_2 I_1 \tag{14}$$

where g_2 is the local gain of KU_2 and $g_1 g_2$ is the global gain of the geared mechanism in Fig. 1(*h*).

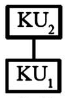
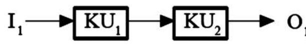
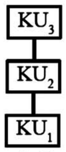
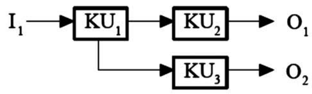
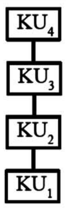
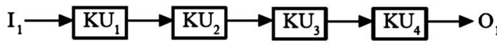
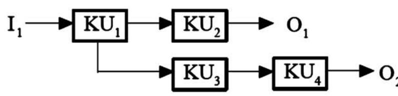
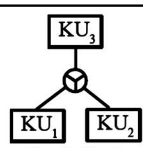
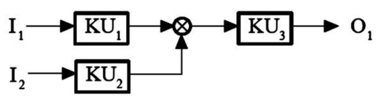
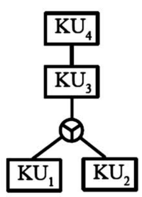
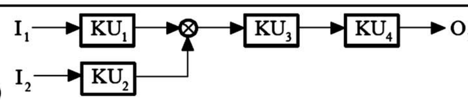
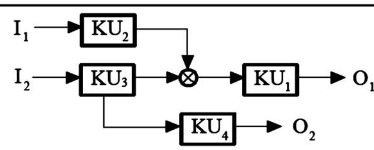
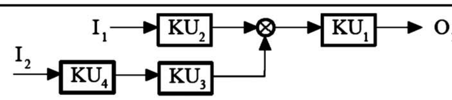
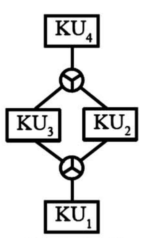
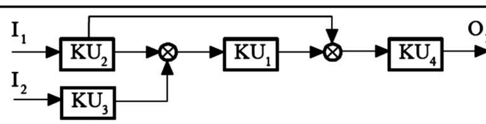
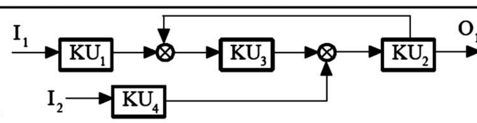
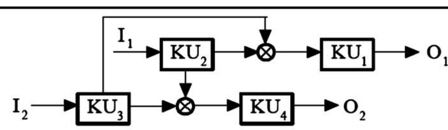
For convenience, the gain of an output and its associated input in a multi-input and multi-output (MIMO) system are expressed in a matrix form as

$$[\mathbf{O}]_{n\times 1} = [\mathbf{G}]_{n\times m}[\mathbf{I}]_{m\times 1} \tag{15}$$

where n and m, respectively, are the numbers of outputs and inputs, G is an $n \times m$ gain matrix, I is an $m \times 1$ input column matrix, and O is an $n \times 1$ output column matrix.

Referring to the propagation path in Fig. 2(*e*), the global output of the GKC of graph 4-1-1 can be easily derived in a similar

Table 1 Block configurations of KUs and the kinematic propagation paths and gains of geared mechanisms. One-DOF: I–IV. Two-DOF: V–VII

No.	Configuration	Propagation path	Gain
I	 $C_2=0, C_3=0$	(a) 	$[g_1]$
II	 $C_2=1, C_3=0$	(a) 	$[g_1 g_2]$
III	 $C_2=2, C_3=0$	(a) 	$[g_1 g_2 g_3]$
		(b) 	$\begin{bmatrix} g_2 g_1 \\ g_2' g_3 \end{bmatrix}$
IV	 $C_2=3, C_3=0$	(a) 	$[g_1 g_2 g_3 g_4]$
		(b) 	$\begin{bmatrix} g_2 g_1 \\ g_2' g_3 g_4 \end{bmatrix}$
V	 $C_2=0, C_3=1$	(a) 	$[g_1 g_3 \quad g_2 g_3]$
VI	 $C_2=1, C_3=1$	(a) 	$[g_1 g_3 g_4 \quad g_2 g_3 g_4]$
		(b) 	$\begin{bmatrix} g_2 g_1 & g_3 g_1 \\ 0 & g_3' g_4 \end{bmatrix}$
		(c) 	$[g_2 g_1 \quad g_4 g_3 g_1]$
VII	 $C_2=0, C_3=2$	(a) 	$[g_2 g_3 g_4 + g_2' g_4 \quad g_1 g_3 g_4]$
		(b) 	$\dfrac{[g_1 g_3 g_2 \quad g_4 g_2]}{1 - g_3 g_2'}$
		(c) 	$\begin{bmatrix} g_2 g_1 & g_3 g_1 \\ g_2' g_4 & g_3' g_4 \end{bmatrix}$

manner to Eqs. (13) and (14). Since there are two inputs in the geared mechanism, the output can be represented as the matrix multiplication of a gain row matrix by an input column matrix as

$$[O_1] = [g_1 g_3 I_1 + g_2 g_3 I_2] = [g_1 g_3 \quad g_2 g_3][I_1 \quad I_2]^T \qquad (16)$$

where g_1, g_2, and g_3 are the local gains of KU_1, KU_2, and KU_3.

Table 2 Classification of geared mechanisms with graph numbers

No.	u	v	Graph number [1,14,15]
I	1	3	3000
		4	2200-2a, 2200-2b
		5	1400-1-3, 2210-1-1a, 2210-1-1b, 2210-1-1c, 2210-1-4a, 2210-1-4b, 3020-1-3b
		6	6401-1, 6401-2, 6401-3, 6401-4, 6401-5, 6401-6, 6401-7, 6401-8, 6401-9, 6401-10, 6401-11, 6503-1, 6503-2, 6503-3, 6503-4, 6503-5, 6503-6, 6503-7, 6503-8, 6503-9, 6503-10, 6503-11, 6601-1, 6601-2, 6601-3, 6601-4, 6601-5
II	2	4	2200-1
		5	1400-1-4, 1400-1-7, 2210-1-2b, 2210-6, 3020-1-4b
		6	6101-1, 6101-2, 6101-3, 6103-1, 6103-2, 6103-3, 6103-4, 6201-1, 6201-2, 6201-3, 6203-1, 6203-2, 6203-3, 6203-4, 6206-1, 6206-2, 6206-3, 6301-1, 6301-2, 6301-3, 6301-4, 6301-5, 6305-1, 6305-2, 6305-3, 6305-4, 6305-5, 6402-1, 6402-2, 6403-1, 6403-2, 6404-1, 6404-2, 6501-1, 6501-2, 6502-1, 6506-1, 6506-2
III	3	5	2210-7
		6	6102-1, 6102-2, 6202-1, 6202-2, 6205-1, 6205-2, 6302-1, 6303-1, 6303-2, 6304-1, 6306-1, 6306-2, 6504-1, 6505-1,
IV	4	6	6204-1
V	3	6	4-1-1, 4-1-2, 4-2-1
		7	5-1-1, 5-1-2, 5-1-3, 5-1-4, 5-2-1, 5-2-2, 5-2-3, 5-2-4, 5-3-1, 5-3-2, 5-4-1, 5-4-2, 5-4-3, 5-4-4, 5-5-1, 5-5-2, 5-5-3, 5-5-4, 5-5-5, 5-6-1, 5-6-2, 5-6-3, 5-6-4, 5-6-5, 5-7-1, 5-7-2, 5-7-3, 5-7-4, 5-8-1, 5-9-1, 5-9-2, 5-19-1, 5-19-2, 5-20-1
VI	4	7	5-10-1, 5-10-2, 5-11-1, 5-11-2, 5-12-1, 5-12-2, 5-13-1, 5-13-2, 5-14-1, 5-14-2, 5-14-3, 5-15-1, 5-15-2, 5-18-1
VII	4	7	5-16-1, 5-17-1

As mentioned earlier, certain KUs can have more than one local output. Therefore, additional local gain due to various propagation paths of the same KU is required. Let g_1', g_2', g_3', etc., denote the additional local gains of KU_1, KU_2, and KU_3, etc., respectively. Referring to the kinematic gain path VII(a) in Fig. 3(*d*), signal S_1 is generated after input I_1 passes KU_2 and becomes

$$S_1 = g_2 I_1 \tag{17}$$

where g_2 is the local gain of the first propagation path of KU_2.

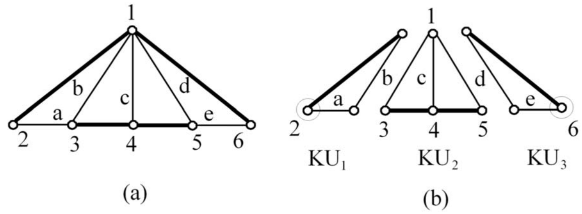


Fig. 4 One-DOF, six-link GKC of graph 6205-1 [14]. (*a*) Graph representation. (*b*) Fractionated KUs and end vertices.

S_2 is the signal after input I_2 passes KU_1 and S_3 equals the summation of S_1 and S_2; thus,

$$S_2 = g_1 I_2 \tag{18}$$

where g_1 is the local gain of KU_1, and

$$S_3 = S_1 + S_2 = g_2 I_1 + g_1 I_2 \tag{19}$$

Similarly, the output signal of KU_3, S_4, can be written as

$$S_4 = g_3 S_3 = g_3(g_2 I_1 + g_1 I_2) = g_2 g_3 I_1 + g_1 g_3 I_2 \tag{20}$$

where g_3 is the local gain of KU_3.

Since KU_2 has two local outputs, the second output, S_5, is generated through the second path of KU_2 and is expressed as

$$S_5 = g_2' I_1 \tag{21}$$

where g_2' is the local gain of the second propagation path of KU_2.

Since signal S_6 is the signal composed of S_4 and S_5 and O_1 is the local output of KU_4, they are obtained as

$$S_6 = S_4 + S_5 = g_2 g_3 I_1 + g_1 g_3 I_2 + g_2' I_1 \tag{22}$$

and

$$O_1 = g_4 S_6 = g_4(g_2 g_3 I_1 + g_1 g_3 I_2 + g_2' I_1) = g_2 g_3 g_4 I_1 + g_1 g_3 g_4 I_2 + g_2' g_4 I_1 = (g_2 g_3 g_4 + g_2' g_4) I_1 + g_1 g_3 g_4 I_2 \tag{23}$$

where g_4 is the local gain of KU_4.

Expressing the local output O_1 in a matrix form, Eq. (23) becomes

$$[O_1] = [g_2 g_3 g_4 + g_2' g_4 \quad g_1 g_3 g_4][I_1 \quad I_2]^T \tag{24}$$

Making use of the derivation procedure presented in this section, all corresponding kinematic gain matrices of the admissible propagation paths are obtained. The results are shown in the fourth column of Table 1. Note that since an entity in a gain matrix shows the influence from a specific input to a prescribed output, in a MIMO system, if certain inputs and outputs are not related to each other, their associated gains would be zero.

Once the global output of a geared mechanism is obtained in terms of the gains of the fractionated KUs, the actual output displacement of the geared mechanism can be readily obtained. Consider the three-KU geared mechanism of graph 4-1-1 shown in Fig. 2(*e*). Since angular displacements $\theta_{1,2}$ and $\theta_{3,2}$ are the local inputs of KU_1 and KU_2, respectively, their local gains can be obtained as

$$g_1 = -e_{1,4} = -\theta_{4,2}/\theta_{1,2} \tag{25}$$

$$g_2 = e_{3,5} = \theta_{5,2}/\theta_{3,2} \tag{26}$$

where $\theta_{4,2}$ and $\theta_{5,2}$ are the local outputs of KU_1 and KU_2, respectively.

Note that $\theta_{i,j}$ represents the relative angular displacement of vertex i with respective to vertex j and can be expressed as $\theta_{i,j} = \theta_{i,k} - \theta_{i,k}$. The local gain of KU_3 is

$$g_3 = e_{5,6} = \theta_{6,4}/\theta_{5,4} \tag{27}$$

where $\theta_{6,4}$ and $\theta_{5,4}$ are the local output and input of KU_3, respectively.

Note that the sign of gear ratio $-e_{1,4}$ in Eq. (25) inherits the sign of the angular output of KU_1. Referring to the configuration of KUs shown in Figs. 2(*c*) and 2(*e*), the angular input displacement of KU_3 can be expressed as $\theta_{5,4} = \theta_{5,2} - \theta_{4,2} = \theta_{5,2} + (-\theta_{4,2})$. Since $-\theta_{4,2}$ and $\theta_{5,2}$ are the local outputs of KU_1 and KU_2, respectively, the gear ratio is, thus, written as $-e_{1,4}$.

Substituting Eqs. (25)–(27) in Eq. (16) yields

$$[O_1] = [\theta_{6,4}] = [g_1 g_3 \quad g_2 g_3][I_1 \quad I_2]^T = [-e_{5,6} e_{1,4} \quad e_{5,6} e_{3,5}] \times [\theta_{1,2} \quad \theta_{3,2}]^T = [(-e_{5,6} e_{1,4} \theta_{1,2}) + e_{5,6} e_{3,5} \theta_{3,2}] \tag{28}$$

6 Kinematic Classification

Since the entities in a gain matrix represent the kinematic relations between input and output links of geared mechanisms, propagation paths of geared mechanisms are classified according to the forms of entities in gain matrices to disclose the kinematic behaviors of geared mechanisms. Three types of propagation path are obtained as follows:

1. Cascade type. For the cascade type, there is no feedback or feed-forward loop existing in the block diagram. Entities in a gain matrix are all products of local gains. All propagation paths other than VII(a) and VII(b) in Table 1 belong to this type.

2. Parallel type. Propagation paths with any downstream KU receiving signals from the same upstream KU via different propagation paths are referred to the parallel type. Referring to the propagation path shown in Fig. 3(*d*), KU_4 receives two signals as its input: one is fed directly from KU_2, and the other comes indirectly from KU_2 via KU_3. For this type of propagation path, there exists at least one entity in the gain matrix constituted by summation of products of local gains due to the composition of a feed-forward loop in the block diagram. The propagation path VII(a) in Table 1 is pertains to this type.

Feedback type. For the feedback type, there exists at least one feedback loop in the propagation path and the entity of the gain matrix is in fractional form. Referring to the propagation path VII(b) in Table 1, since the feedback loop contains KU_2 and KU_3, both entities of the gain matrix are in fractional form, in which $(1-g_3g_2')$ is the denominator and $[g_1g_3g_2\ g_4g_2]$ is the numerator.

According to the classification of gain types, unique kinematic behaviors of geared mechanisms are expected. For those one- and two-DOF geared mechanisms tabulated in Table 2, one is able to look up the table, choose the fit type and pick up the desired graph number to proceed further in the design process. Hence, based on the functional requirements and making use of Table 2, design of one- and two-DOF geared mechanisms of up to six and seven links, respectively, in the atlas can be easily accomplished.

7 Conclusion

A new approach based on the concept of kinematic fractionation is successfully applied in the identification of the topological characteristics of geared mechanisms and in the determination of the kinematic relations between input and output links. Using KUs as basic blocks, admissible configurations of up to four KUs in one- and two-DOF geared mechanisms are unveiled. By adequately designating input(s) and output(s), all possible propagation paths of motion of geared mechanisms are constructed, and their corresponding gains are obtained by block diagram algebra. As a result, the kinematic behaviors of geared mechanisms can be classified. This, in turn, provides the design process with kinematic insight into the synthesis of geared mechanisms with up to four KUs.

References

[1] Freudenstein, F., 1971, "An Application of Boolean Algebra to the Motion of Epicyclic Drives," ASME J. Eng. Ind., **93**, pp. 176–182.

[2] Freudenstein, F., and Yang, A. T., 1972, "Kinematics and Statics of a Coupled Epicyclic Spur-Gear Train," Mech. Mach. Theory, **7**, pp. 263–275.

[3] Tsai, L. W., 1988, "The Kinematics of Spatial Robotic Bevel-Gear Trains," IEEE J. Rob. Autom., **4**, pp. 150–155.

[4] Chatterjee, G., and Tsai, L. W., 1996, "Computer-Aided Sketching of Epicyclic-Type Automatic Transmission of Gear Trains," ASME J. Mech. Des., **118**, pp. 405–411.

[5] Chen, D. Z., and Shiue, S. C., 1998, "Topological Synthesis of Geared Robotic Mechanism," ASME J. Mech. Des., **120**, pp. 230–239.

[6] Chen, D. Z., 1998, "Kinematic Analysis of Geared Robot Manipulators by the Concept of Structural Decomposition," Mech. Mach. Theory, **33**, pp. 975–986.

[7] Liu, C. P., and Chen, D. Z., 2001, "On the Application of Kinematic Units to the Topological Analysis of Geared Mechanism," ASME J. Mech. Des., **123**, pp. 240–246.

[8] Liu, C. P., Chen, D. Z., and Chang, Y. T., 2004, "Kinematic Analysis of Geared Mechanisms Using the Concept of Kinematic Units," Mech. Mach. Theory, **39**, pp. 1207–1221.

[9] Rao, A. C., 2003, "A Genetic Algorithm for Epicyclic Gear Trains," Mech. Mach. Theory, **38**, pp. 135–147.

[10] Kahraman, A., Ligata, H., Kienzle, K., and Zini, D. M., 2004, "A Kinematics and Power Flow Analysis Methodology for Automatic Transmission Planetary Gear Trains," ASME J. Mech. Des., **126**, pp. 1071–1081.

[11] Salgado, D. R., and Del Castillo, J. M., 2005, "Selection and Design of Planetary Gear Trains Based on Power Flow Maps," ASME J. Mech. Des., **127**, pp. 120–134.

[12] Talpasanu, I., Yih, T. C., and Simionescu, P. A., 2006, "Application of Matroid Method in Kinematic Analysis of Parallel Axes Epicyclic Gear Trains," ASME J. Mech. Des., **128**, pp. 1307–1314.

[13] Liu, C. P., and Chen, D. Z., 2000, "On the Embedded Kinematic Fractionation of Epicyclic Gear Trains," ASME J. Mech. Des., **122**, pp. 479–483.

[14] Tsai, L. W., 1987, "An Application of the Linkage Characteristic Polynomial to the Topological Synthesis of Epicyclic Gear Trains," ASME J. Mech., Transm., Autom. Des., **109**, pp. 329–336.

[15] Tsai, L. W., and Lin, C. C., 1989, "The Creation of Non-fractionated Two-Degree-of-Freedom Epicyclic Gear Trains," ASME J. Mech., Transm., Autom. Des., **111**, pp. 524–529.