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# On the Operation Space and Motion Compatibility of Variable Topology Mechanisms[1]

*With the implementation of just one mechanism, variable topology mechanisms can serve the functions of many mechanisms by changing their topology. These types of mechanisms have raised interest and attracted numerous studies in recent years, yet few of these studies have focused discussing of these mechanisms in light of their operation space. As the change of a variable topology mechanism is induced by either intrinsic constraints or constraints due to the change of joint geometry profile, the operation space of kinematic joints and kinematic chains in various working stages is changed in accordance. A theoretic framework based on the concept of the operation space of variable topology mechanisms is presented here. A number of characteristics with regard to the motion compatibility among joints and loops in different working stages are derived, laying a foundation for systematical synthesis of variable topology mechanisms. Design of a novel latch mechanism for the standardized mechanical interface system is given as an illustrative example for the synthesis of a variable topology mechanism.*
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## 1 Introduction

Traditionally, mechanisms have only one fixed topology and therefore possess a fixed set of motions and a fixed operation space, providing a limited number of functions. However, studies on mechanisms with variable topologies and variable operation spaces have arisen in recent years. Because the mechanisms have altered topologies, their motions and thus functions have changed accordingly, enabling them to serve various functions in different working stages.

One of the very first groups to tread into this new realm of variable topology mechanisms is Dai and Rees Jones [1]. They conceived the concept of metamorphic mechanisms from the study of folding and unfolding artifacts and boxes made of flat cards with creases. Later, Dai and Rees Jones [2] used an adjacency matrix representation for the change of distinct topology of configurations possessed by metamorphic mechanisms. Based on the matrix representation, Wang and Dai [3] developed mathematic modeling of metamorphic mechanisms in various working stages. The source mechanism could be synthesized by solving the metamorphic equations. Zhang et al. [4] proposed an approach for the synthesis of metamorphic mechanisms with multiple working stages, where the source mechanism contains the complete topological elements and highest level of mobility of all configuration phases in a full working cycle, while only part of the joints within the source mechanism is active in each working stage. Kuo et al. [5] indicated that the variability of mechanisms can be due to the kinematic geometry arrangements, geometric constraints, designated profiles of links and joints, etc. Figure 1 shows that one joint type of the source mechanism in stage I can be induced equivalently into another type in stage II, while geometric constraints of links are applied.

Yan and Kuo [6,7] investigated a type of joints that changes topology with their geometry [8], as shown in Fig. 2. They indicated that the change of a variable kinematic joint is reversible and continuous, leading to a change in degrees of freedom (DOFs) of such a joint. Joint codes were developed to represent the type and orientation of joints, and joint sequences were used to trace the topology states of a variable kinematic joint. Kuo and Yan [9] further investigated the mobility and configuration singularity of variable topology mechanisms, linking variable mobility of joints with the variable topology of the mechanism. As a result, they were able to derive variable topology mechanisms from an existing one by changing the joints of the mechanism and choosing a few desirable ones from the large pool of mechanisms enumerated [10].

Another branch of mechanisms with changing topology, which emerged a little earlier than the metamorphic mechanisms, are the kinematotropic mechanisms. Named by Wohlhart [11], the kinematotropic mechanisms change their mobility as the relative positions of links vary. Galletti and Fanghella [12] synthesized some single loop kinematotropic mechanisms that contained only lower pairs. These kinematotropic mechanisms change their mobility due to the locking of some particular joints that occurs when they are arranged in certain positions and subsequent unlocking that occurs as the position changes. Such a mechanism is given as an example in Fig. 3.

Aside from the aforementioned mechanisms, a series of existing standard mechanical interfaces (SMIFs), referred to also as latch mechanisms [13–21], are observed to have a variable topology. These SMIF latch mechanisms are deployed in semiconductor wafer plants. They are usually designed intuitively for the purpose of latching and sealing a wafer container box, creating a clean mini-environment to keep the wafers from being contaminated. The latching and sealing movements of the latching-sealing link have two very different movements that are unlikely to be achieved through one topology. To serve these two distinct functional requirements, some latch mechanisms are designed to have two working stages. With each stage having a different operation space and characteristic motion to serve different functional requirements, these mechanisms are said to have a variable topology. A typical example of variable topology mechanisms among these latch mechanisms is the U.S. 5915562 latch mechanism [17], as shown in Fig. 4. It is a planar mechanism in the first working stage; in the second working stage, it becomes a spatial mechanism, changing the space in which it operates.

Investigation of configuration representation [2,6,7,22,23], con-

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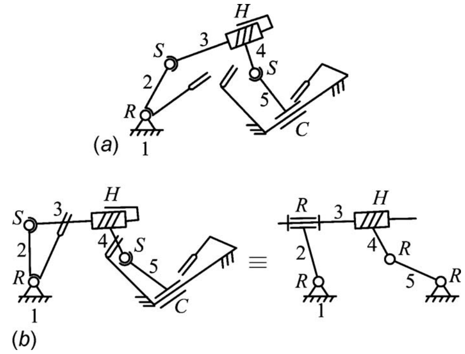

**Fig. 1 A metamorphic mechanism [5]: (*a*) stage I—source mechanism and (*b*) stage II—a subphase mechanism**

straint characteristics [5,11,24], mobility analysis [1,9], and synthesis methodologies [3,4,10,12] of reconfigurable mechanisms, including metamorphic mechanisms and kinematotropic linkages, had been noteworthy [25]. Reconfigurable mechanism based applications [10,24,26] had also been proposed. However, most studies of geometric constraints and synthesis of the mechanisms with variable topology are based on the screw theory [9,24,26–28]. Although screw and reciprocal screw system is an effective tool for solving geometric problems, complicated math-

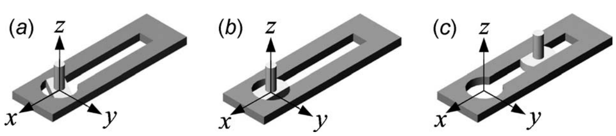

**Fig. 2 Joint type changed by geometry [8]: (*a*) stage I—about *z*-axis, (*b*) stage II—switching stage, and (*c*) stage III—along *x*-axis**

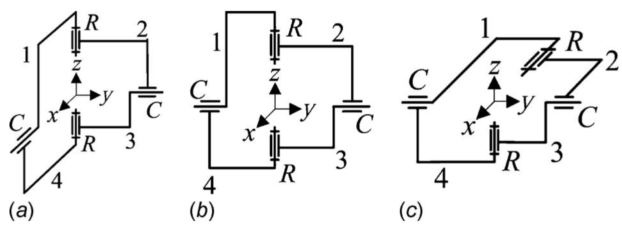

**Fig. 3 A kinematotropic chain [12]: (*a*) stage I, (*b*) stage II—switching stage, and (*c*) stage III**

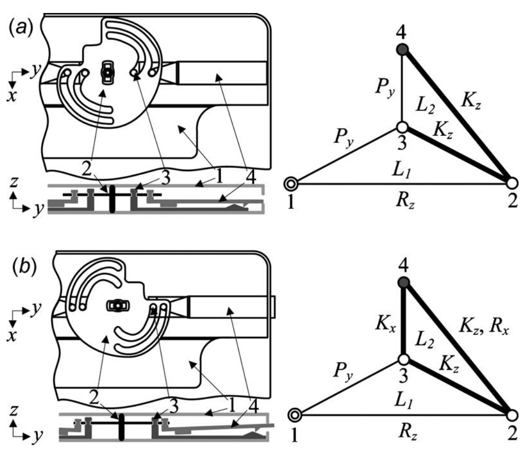

**Fig. 4 Schematic and graph representation of the U.S. 5915562 two-stage latch mechanism [17]: (*a*) working stage I and (*b*) working stage II**

ematic formulation is required. Hence, a theoretic framework based on the concept of the operation space, which is suitable for engineering perception and topological synthesis of variable topology mechanisms, is proposed.

In this paper, variable kinematic joints, variable topology mechanisms, and working stages are first defined, and constraints inducing the mechanisms to have variable topology are classified. The operation space of the kinematic joints and chains with topological variability is then investigated, where a SMIF latch mechanism is used for the illustration of the proposed methodology. Based on the operation space, motion compatibility among joints, loops, and working stages of variable topology mechanisms is derived, and an example for the topological synthesis of a new SMIF latch mechanism is presented. Note that, as the previous studies focused on mechanisms with closed-loop chains only, this paper will follow suit and focus only on mechanisms with close loops.

## 2 Variable Topology Mechanisms

**2.1 Definitions.** DEFINITION 1. *Variable kinematic joint. A variable kinematic joint is capable of transforming its characterized type of kinematic pairs into another and/or varying its representative motion* [6].

DEFINITION 2. *Variable topology mechanism. A variable topology mechanism is a mechanism with one or many variable kinematic joints, providing various characteristic functions of motions.*

DEFINITION 3. *Working stage of variable topology mechanism. A working stage is a state that a variable topology mechanism exhibits one of their many characteristic motions as the characterized type or orientation of one or many variable kinematic joints changes.*

**2.2 Types of Constraints of Variable Topology Mechanisms.** Mechanisms can possess variable topology in two major ways.

1. Topology changes by intrinsic constraints. In a kinematic chain, a default joint type will not be changed if the geometry of the joint remains unchanged. However, due to special arrangements of relative orientations and positions of links and joints, the effective joint type may change due to the change of the configuration of the mechanism. These kinds of constraints are called intrinsic constraints.

Figure 1 is an example of intrinsic constraints. Referring to the source mechanism in stage I in Fig. 1(*a*), the two spherical joints between links 2 and 3 and links 4 and 5 work properly. As the source mechanism moves to a specific configuration, where the range of motion of links 3 and 4 is partially confined, both spherical joints have changed equivalently into revolute joints in stage II, as shown in Fig. 1(*b*). Figure 3 is another example of the intrinsic constraints. In stage I, the two coaxial revolute joints allow each other to work, whereas the two unparalleled cylindrical joints provide constraints and lock each other up; in stage II, where the revolute joints are coaxial and the cylindrical joints are parallel, all joints are workable; in stage III, the revolute joints are not coaxial and lock themselves up, while the cylindrical joints are parallel and still workable. The constraints inducing the joints to be partially or completely locked pertain to the intrinsic constraints.

2. Topology changes by joint geometry. Joints provide their constraints through their geometry. As the geometric profile of the joint changes, orientation or type of the joint may change in accordance.

An example of topology change by the joint geometry is shown in Fig. 2, where the joint is changed from a revolute joint in stage I to a prismatic joint in stage III via a switching stage in stage II.

Another example of topology change due to the joint geometry can also be observed in Fig. 4, where the joint between links 3 and 4 is changed from a prismatic joint in stage I to a cam pair in stage II. A classic example of topology change by joint geometry change is the Geneva wheel.

Changes of the configuration of a variable topology mechanism may change the effective numbers, types, and orientation of links and joints, thereby changing the mechanism's topology. The metamorphic mechanisms, the kinematotropic mechanisms, and the mechanisms with variable kinematic joints are variable topology mechanisms with various working stages.

## 3 Operation Space of Variable Topology Mechanisms

As the characterized motion of the variable topology mechanisms changes, the operation space of the mechanisms is changed in accordance. To correctly depict the behavior of a mechanism, the operation space of a mechanism in which it operates is investigated.

**3.1 Operation Space of a Joint.** DEFINITION 4. *Joint operation space. Operation space of a joint is defined as the space of the relative motion permitted by that joint.*

*Axiom 1.* Representation of joint operation space. Denoting $S(J_{ij})$ as the operation space of the joint between link $i$ and link $j$, and $r$ as the number of the DOFs possessed by the joint, $S(J_{ij})$ can be represented as

$$S(J_{ij}) = S(J_{ij}^1) \cup S(J_{ij}^2) \cdots \cup S(J_{ij}^r) = \bigcup_{k=1,r} J_{ij}^k \tag{1}$$

where $S(J_{ij}^k)$, $k=1,\ldots,r$, is the operation space of the $k$th DOF of $J_{ij}$.

Denote $R$, $P$, $K$, $C$, $U$, and $S$ for revolute, prismatic, cam, cylindrical, universal, and spherical joints, respectively. Referring to the latch mechanism in Fig. 4($a$), joint $J_{12}$ is an $R$ joint about the $z$-axis; the operation space of the relative motion permitted by $J_{12}$ is the space swept by one rotation about the $z$-axis on planes parallel to the $xy$-plane and thus is represented as $S(J_{12})=xy$-1R. Also, in Fig. 4($a$), joint $J_{13}$ is a $P$ joint along the $y$-axis; the operation space of $J_{13}$ is the space traced by one translation along a line parallel to the $y$-axis and hence is represented as $S(J_{13})=y$-1T. Although the operation space of a prismatic joint is a line, the prismatic joint can operate on any plane that includes it. Both joints $J_{12}$ and $J_{13}$ are 1DOF joints. Referring to joint $J_{24}$ in working stage I, as shown in Fig. 4($a$), since it is a 2DOF $K$ joint about the $z$-axis, the operation space of joint $J_{24}$ in stage I is constituted by one rotation and one translation on planes parallel to the $xy$-plane and therefore is represented as $S(J_{24})=xy$-1R1T according to Axiom 1.

**3.2 Operation Space of a Loop.** DEFINITION 5. *Operation space of a close kinematic chain. The operation space of a closed-loop kinematic chain (or loop operation space) is defined as the space of the loop in which the relative motions of all links with respect to any link of the loop are permitted to operate.*

Note that the space in which all links within a loop are permitted to operate is referred to as the loop motion space. In general, the loop motion space of a loop is the union of the loop operation space and the space of any link within the loop. Loop operation space and the loop motion space should be carefully distinguished from each other.

Since the topology change of a variable kinematic joint can be induced by intrinsic constraints or due to the geometric change of joint profile, a specific DOF of a joint may be active or inactive in various working stages.

THEOREM 1. *Status of a specific DOF of a joint within a loop. A specific DOF of a joint, $J_{ij}^k$, within a loop is said to be active if*

$$S(J_{ij}^k) \subset \bigcup_{a,b,c} S(J_{ab}^c) \tag{2}$$

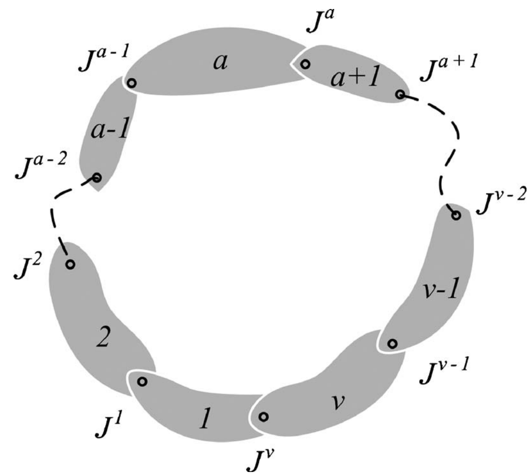


**Fig. 5 A generalized closed-loop kinematic chain of *v* links and *v* 1DOF joints**

*and $J_{ij}^k$ is said to be inactive if*

$$S(J_{ij}^k) \not\subset \bigcup_{a,b,c} S(J_{ab}^c) \tag{3}$$

*where $\subset$ and $\not\subset$ represent for inclusion and noninclusion, respectively, and $J_{ab}^c$ is referred to any DOF of joints within the loop, except for $J_{ij}^k$.*

*Proof.* For convenience and without loss of generality, a closed-loop kinematic chain can be generalized as a loop of $v$ links with $v$ 1DOF joints, as shown in Fig. 5. Consider the operation space of any link, say, link $v$, of the loop. The motion space of link $v$ can be considered as the union of the space of link 1 and the space of link $v$ with respect to link 1, according to Definition 5. Since link $v$ is constrained by both sides of the open kinematic chain from link 1, the operation space of link $v$ with respect to link 1 can be obtained as the intersection of the space from both chains as

$$S(J^v) \cap [S(J^1) \cup \cdots \cup S(J^{v-1})] = S(J^v) \cap \bigcup_{k=1,v-1} S(J^k) \tag{4}$$

If the intersection in Eq. (4) is an empty space, it means that the motion space of link $v$ is essentially equal to the space of link 1, i.e., there is no relative motion between links $v$ and 1. In turn, this suggests that joint $J^v$ is locked or inactive. In contrast, if the intersection in Eq. (4) is not an empty space, indicating that $S(J^v)$ is a subset of the union of $S(J^k)$, $k=1$, $v-1$. In this case, the relative motion between links $v$ and 1 is permitted, i.e., joint $J^v$ is unlocked or active. The status of any DOF, say, $J^k$, within the loop depends on the relative motion of link $k$ with respect to link $k+1$ and can be determined similarly by that of Eq. (4). Thus, the theorem is proved. ■

THEOREM 2. *Representation of loop operation space. The operation space, $S(L_i)$, of loop $i$ with respect to any link within the loop can be obtained as*

$$S(L_i) = \bigcup_{i,j,k} S(J_{ij}^k) \tag{5}$$

*where $J_{ij}^k$'s represent all active DOFs of joints within loop $i$.*

*Proof.* All inactive DOFs of joints within loop $i$ can be identified and excluded according to Theorem 1. Consider loop $i$ consisting of $v$ links and $v$ active DOFs of joints. Referring to the kinematic chains, as shown in Fig. 5, the operation space of link $a$ with respect to link 1 can be obtained by taking the intersection of the spaces of the end links of the open-loop chains from both sides as

$$S(\text{Link}_a) = [S(J^1) \cup \cdots \cup S(J^{a-1})] \cap [S(J^a) \cup \cdots \cup S(J^v)] \quad (6)$$

Summing the operation spaces of all link $a$ for $a=2,\ldots,v$ yields the operation space of loop $i$ with respect to link 1 as

$$S(L_i) = \bigcup_{a=2,v} S(\text{Link}_a) = \bigcup_{j,k=1,v,j\neq k} [S(J^j) \cap S(J^k)], \quad (7)$$

Rearranging Eq. (7) gives

$$S(L_i) = \{S(J^a) \cap [S(J^1) \cup \cdots \cup S(J^{(a-1)}) \cup S(J^{(a+1)}) \cup \cdots \cup S(J^v)]\} \cup \cdots \quad (8)$$

According to Theorem 1, since $J^a$ is an active 1DOF joint, the operation space of $J^a$ is included in the union of the space formed by all active DOFs of joints within the loop, except for itself. Hence, for all active DOFs within the loop, $S(J^a) \subset S(L_i)$ for $a=1,v$. Therefore,

$$S(J^1) \cup S(J^2) \cup \cdots \cup S(J^v) = \bigcup_{a=1,v} S(J^a) \subset S(L_i) \quad (9)$$

According to Eq. (7),

$$S(L_i) = \bigcup_{j,k=1,v,j\neq k} [S(J^j) \cap S(J^k)] \subset \bigcup_{a=1,v} S(J^a) \quad (10)$$

Equations (9) and (10) yield the operation space of loop $i$ as

$$S(L_i) = \bigcup_{a=1,v} S(J^a) \quad (11)$$

The operation space of loop $i$ with respect to link $k$ for $k=2$, $v$, can also be obtained in a similar procedure as the equation given in Eq. (11). Hence, the theorem is proved. ■

DEFINITION 6. *Motion parameter of a mechanism. Motion parameter of a mechanism is defined as the minimum number of the DOFs of the space in which all links within the mechanism are intended to function [29].*

For variable topology mechanisms with multiple loops, since the motion parameter of each loop can be different in various working stages, the motion parameter of a loop is derived as follows.

COROLLARY 1. *Motion parameter of a loop. Given $F(L_i)$ as the loop mobility of loop i and $v(L_i)$ as the number of active DOFs within loop i, the motion parameter $\lambda(L_i)$ of the loop can be obtained as*

$$\lambda(L_i) = v(L_i) - F(L_i) \quad (12)$$

*Proof.* Consider a loop with a mobility of 1. Since the loop mobility is the number of the independent parameters required to completely specify the configuration of the loop in the space [29], no active DOF can be locked in a loop with a mobility of 1; otherwise, the loop becomes immobilized. According to Theorem 2, the operation space of loop $i$ is obtained by the union of the space of all active DOFs within the loop. Moreover, according to Theorem 1, the operation space of any DOF of the $v(L_i)$ active DOFs within the loop is a subset of the union of the space of the remaining $v(L_i)$-1 active DOFs within a loop with a mobility of 1. For a loop with a mobility of 2, by locking up any active DOF within the loop, the remaining $v(L_i)$-1DOFs are still able to move, and hence the operation space of the loop is obtained by the union of the space of any $v(L_i)$-2 active DOFs within the loop. Therefore, for a loop with mobility of $F(L_i)$, the operation space of loop $i$ can be obtained by the union of the space of any $v(L_i)-F(L_i)$ DOFs of joints within the loop. Hence, the minimum DOFs of the space in which all links within the loop are permitted to operate are obtained as $\lambda(L_i)=v(L_i)-F(L_i)$. The corollary is proved. ■

**3.3 Operation Space of a Mechanism.** COROLLARY 2. *Mechanism operation space. The operation space, $S(M)$, of the mechanism with m loops is obtained as*

$$S(M) = \bigcup_{i=1,m} S(L_i) \quad (13)$$

*Proof.* Since the operation space of loop $i$ is the union of the space of its all active DOFs of joints according to Theorem 2 and the mechanism possesses more or equal number of joints than that of loop $i$, the operation space of each loop $i$ in the mechanism is a subset of the space of the mechanism. Hence,

$$\bigcup_{i=1,m} S(L_i) \subset S(M) \quad (14)$$

Contrarily, the operation space of every active DOF of joints in the mechanism is a subset of the space of its associated loop. Hence,

$$\bigcup_{i=1,m} S(L_i) \supset S(M) \quad (15)$$

Based on Eqs. (14) and (15), Eq. (13) is obtained and the corollary is proved. ■

Note that the operation space of a mechanism is equivalent to the motion space of the mechanism because the ground link is fixed. When the operation space of a mechanism changes, it indicates that the topology of the mechanism has varied. In practice, it is desired for the mechanism to achieve different functions; thus, the operation space of a variable topology mechanism is varied to suit this purpose.

## 4 Typical Variable Topology Mechanism

**4.1 A SMIF Latch Mechanism.** The U.S. 5915562 latch mechanism, as shown in Fig. 4, is a variable topology mechanism with two working stages, where link 1 is the ground link, link 2 is the cam disk, link 3 is for displacing the output link, and link 4 is the output link for latching. As the latching link moves to the right to provide the latching function in the first stage, the latching link rotates and moves upward to seal the wafer box air-tight in the second stage.

In the graph representation of the latch mechanism in Fig. 4, thin edges represent for low pairs, thick edges for high pairs, vertices for links, double vertices for the ground, and solid vertex for the output link. Joints $J_{12}$, $J_{13}$, and $J_{23}$ are an $R_z$, a $P_y$, and a $K_z$ joint in both working stages, respectively; joint $J_{34}$ is a $P_y$ joint in stage I and a $K_x$ joint in stage II; joint $J_{24}$ is a $K_z$ joint in stage I, while in stage II, it had a combined cam motion on the $xy$-plane and a rotation about the $x$-axis; therefore, joint $J_{24}$ is a composite joint comprising of a $K_z$ and an $R_x$ joint.

In this design, links 3 and 4 are simultaneously pushed horizontally to the right together by the cam disk in stage I to latch the box. Due to the profile of the cam disk, link 3 moves faster than link 4; hence, the ramp on link 3 forces link 4 to move upward in stage II, sealing the box air-tight. Due to the profile change of joint $J_{34}$, the planar contact of joint $J_{34}$ in stage I becomes a line contact in stage II. As a result, joint $J_{34}$ changes from a $P_y$ to a $K_x$ joint. The topology change of joint $J_{34}$ is due to the joint geometry change, while the change of joint $J_{24}$ is pertaining to intrinsic constraints induced by the change of joint $J_{34}$, resulting in a previously locked up $R_x$ joint to be set free in stage II. Note that the $R_x$ motion of joint $J_{24}$, which is permitted in stage II, is due to a designated allowance between the extruded pin of link 4 and the slot of link 2 at the joint.

**4.2 Operation Space Representation.** To fully describe the variability of the mechanism, a topological graph representation supplemented with a table of operation space of joints, loops, and mechanism is required. The operation spaces of joints, loops, and the mechanism in both stages are shown in Table 1, where the interface joint $J_{23}$ is included in both loops, and the joints in boldface are the variable kinematic joints and the joints marked with footnote a are inactive joints.

*4.2.1 Working Stage I.* Since the operation space of every DOF of joints of loop 1 is a subset of the union of the space of the

**Table 1 Operation space representation of the two-stage SMIF latch mechanism**

| Loop | Joint | $S(J_{jk})$ | $S(L_i)$ | $S(M)$ |
|---|---|---|---|---|
| | | Working stage I | | |
| 1 | $J_{12\ Rz}$ | $xy$-1R | $xy$-plane | $xy$-plane |
| | $J_{13\ Py}$ | $y$-1T | | |
| 2 | $J_{23\ Kz}$ | $xy$-1T1R | $xy$-plane | |
| | $J_{24\ Kz}$ | $xy$-1T1R | | |
| | $\boldsymbol{J}_{24\ Rx}$ [a] | $yz$-1R [a] | | |
| | $\boldsymbol{J}_{34\ Py}$ | $y$-1T | | |
| | | Working stage II | | |
| 1 | $J_{12\ Rz}$ | $xy$-1R | $xy$-plane | $xy$-plane and $yz$-plane |
| | $J_{13\ Py}$ | $y$-1T | | |
| 2 | $J_{23\ Kz}$ | $xy$-1T1R | $xy$-plane and $yz$-plane | |
| | $J_{24\ Kz}$ | $xy$-1T1R | | |
| | $\boldsymbol{J}_{24\ Rx}$ | $yz$-1R | | |
| | $\boldsymbol{J}_{34\ Kx}$ | $yz$-1T1R | | |

[a]Inactive joints.

rest of the DOFs of joints within the loop, all joints in loop 1 are active according to Theorem 1. According to Theorem 2, since the operation spaces of joints $J_{12\ Rz}$, $J_{13\ Py}$, and $J_{23\ Kz}$ of loop 1 are $xy$-1R, $y$-1T, and $xy$-1T1R, respectively, $\upsilon(L_1)=4$, and the operation space of loop 1 is in the $xy$-plane with $\lambda(L_1)=3$. According to Corollary 1 or Eq. (12), $F(L_1)=1$. While the operation spaces of joints $J_{23\ Kz}$, $J_{24\ Kz}$, and $J_{34\ Py}$ within loop 2 are $xy$-1T1R, $xy$-1T1R, and $y$-1T, respectively, $\upsilon(L_2)=5$, and $S(L_2)$ is also in the $xy$-plane with $\lambda(L_2)=3$. In this stage, $F(L_2)=2$. Note that since the operation space of $J_{24\ Rx}$ is $yz$-1R and is not included in the $xy$-plane, joint $J_{24\ Rx}$ is inactive in this stage.

*4.2.2 Working Stage II.* In this stage, all joints in loop 1 are active and $S(L_1)$ is in the $xy$-plane, where $\upsilon(L_1)=4$, $\lambda(L_1)=3$, and $F(L_1)=1$. While in loop 2, joints $J_{23\ Kz}$, $J_{24\ Kz}$, $J_{24\ Rx}$, and $J_{34\ Kx}$ are active according to Theorem 1 and their operation spaces are $xy$-1T1R, $xy$-1T1R, $yz$-1R, and $yz$-1T1R, respectively. Hence, $\upsilon(L_2)=7$. In loop 2, a full $xy$ planar motion can be contributed by $J_{23\ Kz}$ and $J_{24\ Kz}$, while a full $yz$ planar motion can be provided by $J_{24\ Rx}$ and $J_{34\ Kx}$. Therefore, the operation space of loop 2 is in both the $xy$- and $yz$-planes. However, since the operation space of $xz$-1R is not included, the motion parameter $\lambda(L_2)=5$ and thus $F(L_2)=2$. Noteworthily, for relatively less complex variable topology mechanisms, the proposed expression can assist one in developing an in-depth perception on the determination of the joint status and the operation space, mobility, and motion parameter of a loop without going through a detailed mathematic modeling for the design. However, for mechanisms with complex topology, a mathematic modeling is required and helpful.

## 5 Motion Compatibility of Variable Topology Mechanisms

Intrinsic and/or joint geometry constraints alter the operation space of joints and thereafter change the topology of a loop as well as a mechanism. As a variable topology mechanism transforms from one working stage to another, the joints and loops in various working stages should be compatible to one another. In each working stage, the joints should be compatible with their loop, and the loops should be compatible with one another.

**5.1 Joint Compatibility.** For a variable kinematic joint to change from working stage $i$ to stage $j$, (i) the motion that is allowed in stage $i$ but not allowed in stage $j$ must be locked up and (ii) the constraint restricting the motion that is allowed in stage $j$ but not in stage $i$ must be lifted.

DEFINITION 7. *Generic and degenerated working stages. A generic working stage is defined as the stage in which all DOFs of joints in a variable topology mechanism are active; otherwise, it is a degenerated working stage.*

*Characteristic 1.* Compatibility of a joint. Denoting the operation space of joint $J_{ij}$ in the generic stage as $S_G(J_{ij})$ and that in the $k$th working stage as $S_k(J_{ij})$, $S_G(J_{ij})$ and $S_k(J_{ij})$ are related as

$$S_k(J_{ij}) \subset S_G(J_{ij}) \tag{16}$$

In the example of U.S. 5915562 latch mechanism, the operation space of joint $J_{24}$ in stage I is a subset of the operation space of joint $J_{24}$ in stage II. For this mechanism, stage I and stage II are, respectively, the degenerated and the generic stage. Based on Characteristic 1, a joint, a loop, or a mechanism in the generic stage can be degenerated into various working stages by locking up some of the joints. As a result, a designer can enumerate admissible working stages from either a generic or a degenerated stage.

*Characteristic 2.* Compatibility of a joint and its associated loop in various working stages. The operation space of joint $J_{ij}$ is a subset of the space of its associated loop $q$ in any working stage $k$ as

$$S_k(J_{ij}) \subset S_k(L_q) \tag{17}$$

*Characteristic 3.* Constraints inducing locked joints. Joint $J_{ij}$ is said to be partially or completely locked up if and only if the operation space of $J_{ij}$ in the generic stage is not included in the space of its associated loop $q$ in the $k$th working stage as

$$S_G(J_{ij}) \not\subset S_k(L_q) \tag{18}$$

In the example of the U.S. 5915562 latch mechanism of Fig. 4, the operation space of loop 2 in stage I is the $xy$-plane, and in stage II, it is the $xy$- and $yz$-planes. Hence, $S_{\mathrm{I}}(J_{34\ Py})$ is a subset of $S_{\mathrm{I}}(L_2)$ and $S_{\mathrm{II}}(J_{34\ Kx})$ is a subset of $S_{\mathrm{II}}(L_2)$, satisfying Characteristic 2. However, because the operation space of $J_{34\ Kx}$ is not a subset of $S_I(L_2)$, the rotation motion of $J_{34\ Kx}$ in stage I is locked up. Also, since $S(J_{24\ Rx})$ is not a subset of $S_I(L_2)$, the $R_x$ motion of $J_{24}$ of loop 2 is inactive, which is complied with Characteristic 3.

A joint can be designated to be compatible or incompatible within its loop, thereby serving the designer's purpose. Locking up joints decreases the number of effective DOFs of joints and that of links within the loop. Locking up a joint may lock up the loop as a whole.

**5.2 Loop Compatibility.** Two loops not locking each other up are defined as being compatible with each other. Loops are joined by the loop interface; the joints on the interface convey the constraints of the two loops to each other. The combined constraints from both loops may lock up the joints on the interface and decrease their mobility. If the loop interface is locked or incompatible with one of the joints, motion cannot convey through these locked interface pairs. Therefore, if loops with locked interface pairs or their interface pairs are not compatible with one of them, these two are incompatible with each other.

*Characteristic 4.* Compatible loop interface joints. Denoting $I_{ij}$ as the interface joints between two adjacent loops $L_i$ and $L_j$, the operation space of $I_{ij}$, $L_i$, and $L_j$ in any working stage $k$ is related as

$$S_k(I_{ij}) \subset (S_k(L_i) \cap S_k(L_j)) \tag{19}$$

Since the interface joints pertain to both loop $i$ and loop $j$, $S_k(I_{ij})$ must be a subset of $S_k(L_i)$ and $S_k(L_j)$. In the example of U.S. 5915562 latch mechanism, the interface of $L_1$ and $L_2$ in both stages is $J_{23\ Kz}$. The joint operation space of $J_{23\ Kz}$ is a subset of the operation space of either loop, satisfying Characteristic 4. If a loop in a working stage is capable of transforming into another working stage, these two working stages are compatible with each other.

*Characteristic 5.* Compatible successive working stages of a loop. If two successive working stages $j$ and $k$ of loop $i$ are capable of transforming into each other, the operation space of the loop in the two working stages must be related as

$$S_j(L_i) \subset S_k(L_i) \tag{20}$$

or

$$S_j(L_i) \supset S_k(L_i) \tag{21}$$

When a variable kinematic joint transforms, the joint locks up unwanted motion or releases the motion that was previously constrained. Therefore, for two successive working stages, the operation space of a loop in one working stage may include or be included by that of the other stage. In the example of U.S. 5915562 latch mechanism, it can be seen that $S_{\mathrm{I}}(L_2)$ is a subset of $S_{\mathrm{II}}(L_2)$, which is complied with Characteristic 5.

In Fig. 3, there is a transient switching stage between the first and last stages. As the first and last working stages are not subsets to each other, a switching stage with the motion of both stages appears to bridge the transformation. For such mechanisms, the transient switching stage can be the generic stage.

The creation or design of a variable kinematic joint may be attempted by designing the generic stage of many separate stages, obtaining the form of the joint through the joint operation space of the generic stage. Yet, if a joint in the generic stage possesses too many DOFs, it is deemed impractical and thus inadmissible since a practical joint cannot harbor large number of DOFs.

**5.3 Mobility Compatibility.** Since the motion parameter of the space of a loop in a variable topology mechanism may be different from one loop to another, the mobility of a loop has to be evaluated separately.

*Characteristic 6.* Compatible loop mobility. For any working stage $k$, the DOFs of a mechanism [30] can be obtained as

$$F_k(M) = \sum_i F_k(L_i) - \sum_{i,j} F_k(I_{ij}) \tag{22}$$

where $F_k(M)$ is the DOF of a mechanism, $F_k(L_i)$ is the mobility of loop $i$, and $F_k(I_{ij})$ is the mobility of the interface joints between loops $i$ and $j$ in working stage $k$.

In the example of U.S. 5915562 latch mechanism, $F(L_1)=1$, $F(L_2)=2$, and $F(I_{12})=2$ in both working stages. Hence, $F(M)=1$. The loop mobility can be changed as the status of the interface joints is changed.

## 6 Topological Synthesis of a Novel SMIF Latch Mechanism

Design of a new two-stage SMIF latch mechanism with the same function of U.S. 5915562 mechanism is used to illustrate the synthesis of a variable topology mechanism. The latch mechanism on the SMIF box has to serve two major functions: (i) to latch the wafer box shut and (ii) to seal the wafer box air-tight. The functional requirements are sorted as follows.

F1: The standard SMIF input implements a 1DOF rotation about a vertical axis.
F2: A horizontal displacement for the latching link is required to latch shut the wafer box.
F3: To seal the wafer box air-tight, a vertical displacement of the latching link is required.

For simplicity, the new design has the same topological characteristics as that of U.S. 5915562 latch mechanism in the first working stage, as shown in the graph representation of Fig. 6(*a*), where the operation space of each loop is in the *xy*-plane and the mobilities of loops 1 and 2 are 1 and 2, respectively. According to F1–F3, the motion of the input link is determined to be invariant and that of the output link is variable. Thus, the input loop $L_1$ is invariant, while the output loop $L_2$ is of variable topology. Since, for the U.S. 5915562 mechanism, the motion parameter of the

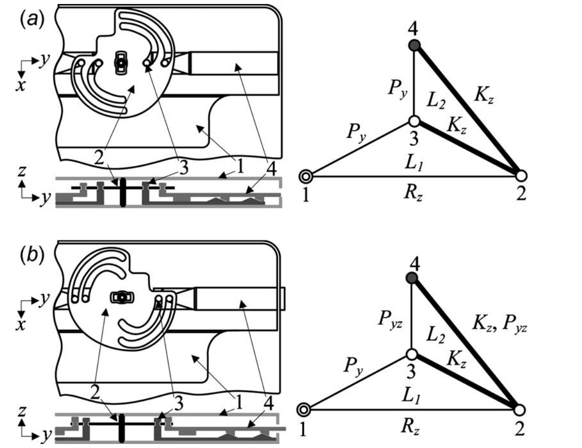


**Fig. 6 Schematic and graph representation of a new two-stage latch mechanism: (*a*) working stage I and (*b*) working stage II**

space of loop 2 in the second working stage is 5, an effort to design a mechanism with $\lambda(L_2)=4$ in working stage II is attempted as follows.

For the new design, since the operation space in stage I is a subset of that in stage II, working stage II is a generic stage according to Characteristic 5. For the new design, because loop 1 is invariant, the interface joint in both stages is also invariant and thus $F_{\mathrm{I}}(I_{12})=F_{\mathrm{II}}(I_{12})=2$. The design is a single-input system and the mobility of loop 1 is equal to 1 for both stages, i.e., $F_{\mathrm{I}}(M)=F_{\mathrm{II}}(M)=1$ and $F_{\mathrm{I}}(L_1)=F_{\mathrm{II}}(L_1)=1$. Based on Characteristic 6 or Eq. (22), the mobilities of loops 1 and 2 are evaluated as $F_{\mathrm{I}}(L_2)=2$ and $F_{\mathrm{II}}(L_2)=2$, respectively. Since $\lambda_{\mathrm{II}}(L_2)=4$ is desired, the number of the active DOFs of joints within loop 2 can be evaluated as $\upsilon_{\mathrm{II}}(L_2)=6$ according to Corollary 1 or Eq. (12), i.e., the total number of the active DOFs of joints $J_{23}$, $J_{24}$, and $J_{34}$ in stage II is 6. Among these joints, the interface joint $J_{23}$ is invariant and the active DOF of joint $J_{23}$ is 2 and the active DOFs of joint $J_{24}$ in stage II (generic stage) must be greater than or equal to 2 according to Characteristic 1. Hence, admissible numbers of DOFs for joints $J_{24}$ and $J_{34}$ should be, respectively, either 3 and 1 or 2 and 2. If the DOFs of $J_{24}$ and $J_{34}$ are both 2, $J_{24}$ has to be remained as a $K_z$ joint. However, if both joints $J_{23}$ and $J_{24}$ are $K_z$ joints, $S(J_{23})$ and $S(J_{24})$ are on the *xy*-plane; any DOF of joint $J_{34}$ required to lift the output link in the $z$-axis will be locked up according to Characteristic 3. Therefore, the numbers of DOFs possessed by joints $J_{24}$ and $J_{34}$ should be 3 and 1, respectively. There are two feasible solutions for such a design: One is an $R_x$ joint for $J_{34}$ and a composite joint of an $R_x$ and a $K_Z$ for $J_{24}$; the other is a $P_{yz}$ joint for joint $J_{34}$ and a composite joint of a $P_{yz}$ and a $K_Z$ for $J_{24}$, where $P_{yz}$ joint is a prismatic joint along a line in the *yz*-plane. Figure 6 shows the schematic and graph representation of the novel latch mechanism of the latter case.

## 7 Conclusion

The operation space and motion compatibility of variable topology mechanisms are presented. With the topological expression of a variable topology mechanism including a graph with labeled joint type and the operation space of joints, loops, and the mechanism, the topology of variable kinematic joints and variable topology mechanisms in various working stages can be depicted precisely. Based on the derivation of the operation space for joints, loops, and a mechanism, it is proved that the changed status of a joint is determined by the operation space within the loop and hence changes the topology of the mechanism as well. As a variable topology mechanism transforms from one working stage to another, the motion compatibility of the joints and loops among various working stages and the compatibility of a joint and its associated loop in any working stage are investigated, providing an invaluable aid in the systematic design of a variable topology mechanism. A novel SMIF latch mechanism is presented for the illustration of the topological synthesis of a variable topology mechanism based on the compatibility characteristics revealed in this work.

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