Design of Statically Balanced Planar Articulated Manipulators With Spring Suspension

Po-Yang Lin, Win-Bin Shieh, and Dar-Zen Chen

***Abstract*—This paper proposes a design methodology for the synthesis of statically balanced planar articulated manipulators through direct spring installation. The proposed method can determine all admissible configurations of spring installation with a given planar articulated manipulator. The fundamental principle for gravity balance of a conservative system of spring-gravity is formulated by a stiffness block matrix, wherein each component matrix implies interacting forces among the links. The distribution of nonzero-component matrices can be related to a type of spring configuration. The minimum number of springs that are required for a statically balanced manipulator can be further determined, based on the number of design parameters and the number of simultaneous equations derived from the zero-component matrices of stiffness block matrix. By the representation of conventional adjacency matrix for connectivity of springs among the links, five characteristics are identified to enumerate all admissible spring configurations with a minimum number of springs. Spring configurations for the statically balanced articulated manipulators with up to four degrees of freedom are obtained and illustrated schematically. An index function for the evaluation of the robustness of such a statically balanced articulated manipulator with respect to each spring configuration is also proposed.**

***Index Terms*—Articulated manipulator, static balance, stiffness block matrix, zero-free-length spring.**

I. Introduction

STATICALLY balanced articulated manipulator is an adaptive mechanical system [1] that is capable of self-balancing all joint torques due to the gravitational forces on configuration. A statically balanced manipulator can gain benefit from the efficient improvement of control and energy in the system [2]–[5], since little actuating torque is required to sustain the weight and payload of manipulator. The balance of gravity in the given system without employing actuating control scheme can be accomplished through many well-known methods [2], for instance, the counterweight method [9]–[14], the spring-balancing method [5], [14], [16]–[20], the cam linkage method [20], [21], the friction method, etc. Among them, the method of spring-balancing is advantageous in the introduction of only small amount of additional inertia to the original system. Therefore, the recent novel and promising applications of statically balanced mechanisms with ideal zero-free-length springs [15], [17], [19] were proposed, for instance, passive rehabilitation devices [6]–[9], flight simulator [10], parallel robots [2]–[4], etc. These spring-gravity systems are all referred to as spring-balancing mechanisms [16].

The energy approach for the synthesis of spring-balancing mechanisms had been investigated and well justified in many studies [1], [16], [17], [19], [20], [23], where the equilibrium of a conservative spring-gravity system in any configuration can be obtained, if the gravitational potential energy and elastic potential energy of system is equally and perfectly exchanged to each other.

The state-of-the-art approaches of spring-balancing mechanisms are mainly achieved by embedding springs along with auxiliary links to form parallelogram linkages in the mechanism that is desired to be balanced statically [17]–[19]. However, there are several disadvantages when adding the auxiliary links to a manipulator. For example, 1) the motion interferences between primary links and auxiliary links may hinder the workspace of the manipulator; 2) mechanical tolerance must be carefully controlled to maintain the parallelism of auxiliary links; 3) auxiliary links produce additional and comparatively large inertia to the system; and 4) the excess number may reduce the robustness of manipulator [23]. Few spring-balancing mechanisms without parallel auxiliary links had been proposed except for some of specific designs [24]. The purpose of this investigation is to propose a general methodology for configuration synthesis of spring-balancing articulated manipulators (SBAMs). The proposed method is based on the approach of direct spring installation in order to avoid the problems of the auxiliary link method mentioned previously.

This paper is laid out as follows. In Section II, the fundamental principle of static balance in spring-gravity system is first reviewed with a quantitative measure; namely, the stiffness block matrix is proposed for the analysis of equilibrium through the energy method. Within such a stiffness block matrix, distributions of nonzero-component matrices can be identified and related to various types of spring configuration. In Section III, based on the number of design parameters and simultaneous equations derived from the zero-component matrices of stiffness block matrix, the minimum number of the springs required for the balance of the system is determined. Section IV describes the spring configuration through conventional adjacency matrix, and it is identified with the enumeration of all possible

Manuscript received February 15, 2011; revised July 30, 2011; accepted September 17, 2011. Date of publication October 14, 2011; date of current version February 9, 2012. This paper was recommended for publication by Associate Editor M. Minor and Editor B. J. Nelson upon evaluation of the reviewers' comments.

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Digital Object Identifier 10.1109/TRO.2011.2169633

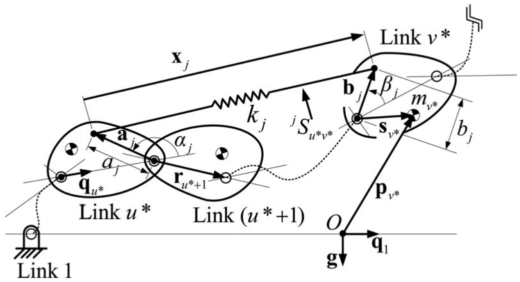

Fig. 1. Spring fitted between links u^* and v^* of a planar articulated manipulator.

spring configurations by five characteristics of an admissible spring configuration matrix. The result of spring configurations of planar SBAM with up to four degrees of freedom (DOFs) is schematically illustrated. In Section V, an index of indication of the robustness of such a statically balanced articulated manipulator corresponding to each achieved spring configuration is proposed. Section VI demonstrates a conceptual design of a statically balanced 3-DOF robotic arm. The analytical solution of all the design parameters is given in detail. The robotic arm is modeled in the software ADAMS, and the results of simulation are consistent with analysis.

II. Principle of Gravity Balance

Configuration of an articulated manipulator can be defined by any unit vector fixed on the link. Denote $\mathbf{q}_u$ as a unit vector of link u of an n-link planar manipulator, where $u = 1, 2, \ldots, n$, and $\mathbf{q}_u$ is a 2×1 column matrix for 2-D space. The configuration of the n-link manipulator can be defined by n vectors and represented as a $2n \times 1$ column matrix of $\mathscr{Q} = [\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_n]^T$. For a system of spring-gravity with N springs without dynamic effect, the balance of a conservative system can be achieved, only if the sum of gravitational and elastic potential energy always remains with

$$\sum_{j=1}^{N} {}^jU(\mathscr{Q}) + {}^gU(\mathscr{Q}) = U(\mathscr{Q}) = \text{constant} \tag{1}$$

where ${}^jU$, ${}^gU$, and U are the function of elastic potential energy of the jth spring, the function of gravitational potential energy of mechanism, and the total potential energy function, respectively.

In this research, the inertia of springs is considered comparatively small with respect to that of the links of mechanism; hence, it is considered negligible. Note that the accuracy of gravity balance considering the effects of inertia of springs has also been studied [23].

A. Stiffness Block Matrix

For simplicity and without loss of generality, each unit vector $\mathbf{q}_u$ is placed along the line passing through centers of the revolute joints on link u, as shown in Fig. 1. The elastic potential energy of the jth spring, which is fitted between links u^* and v^* of the n-link planar articulated manipulator, can be obtained from the deformation length $\mathbf{x}_j$ of zero-free-length spring as

$${}^jU = \frac{1}{2} k_j \mathbf{x}_j^{\mathrm{T}} \mathbf{x}_j \tag{2}$$

where k_j is the spring constant of the jth spring; $\mathbf{x}_j = \mathbf{b}_j - \mathbf{a}_j + \sum_{u=u^*+1}^{v^*-1} \mathbf{r}_u$, where $\mathbf{a}_j$ and $\mathbf{b}_j$ are, respectively, the vectors of the two attachment points of the jth spring on links u^* and v^*; and $\mathbf{r}_u$ is the vector fixed on link u.

Since any vector fixed on link u can be obtained by a specific rotation and elongation of the unit vector $\mathbf{q}_u$, any vector on a link can be obtained by premultiplying the unit vector of the link with a constant 2×2 transformation matrix $\mathbf{T}$, where $\mathbf{T} = \mathbf{SR}$. $\mathbf{S}$ is the conventional scaling matrix and $\mathbf{R}$ is the rotation matrix:

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{3}$$

The scaling matrix $\mathbf{S}$ is diagonal. The rotation matrix $\mathbf{R}$ and the transformation matrix $\mathbf{T}$ are in a typical matrix form: sum of a skew-symmetric matrix and a diagonal matrix. With vectors $\mathbf{r}_u$, $-\mathbf{a}_j$, and $\mathbf{b}_j$ represented as $\mathbf{T}_u\mathbf{q}_u$, $\mathbf{T}_{u^*}\mathbf{q}_{u^*}$, and $\mathbf{T}_{v^*}\mathbf{q}_{v^*}$, respectively. Equation (2) can be explicitly expressed as

$${}^jU = \frac{1}{2} \sum_{u,v=1}^{n} \left(\mathbf{q}_u^{\mathrm{T}}\, {}^j\mathbf{K}_{uv} \mathbf{q}_v\right) \tag{4}$$

where

$${}^j\mathbf{K}_{uv} = k_j \mathbf{T}_u^{\mathrm{T}} \mathbf{T}_v. \tag{5}$$

Rearranging (4) in a matrix form yields

$${}^jU(\mathscr{Q}) = \frac{1}{2} \mathscr{Q}^{\mathbf{T}}\, {}^j\mathscr{K}\, \mathscr{Q} \tag{6}$$

where the stiffness block matrix ${}^j\mathscr{K}$ is constituted of $n \times n$ component matrices, and each component matrix ${}^j\mathbf{K}_{uv}$ is a 2×2 matrix, containing the spring parameters of the jth spring as (5) indicates.

According to (5), since ${}^j\mathbf{K}_{uv}$ and its transpose ${}^j\mathbf{K}_{vu}$ have opposite off-diagonal elements, and hence matrix ${}^j\mathscr{K} = [{}^j\mathbf{K}_{uv}]$ is in a matrix form: sum of a symmetric matrix and a skew-symmetric matrix. Since the jth spring is fitted to links u^* and v^*, the elastic potential energy because of the jth spring is only affected by the configurations of links u^*, $(u^* + 1), \ldots, v^*$, and the component matrices ${}^j\mathbf{K}_{uv}$ are zero matrices for $u < u^*$ and $v > v^*$.

Referring to the manipulator of n links in Fig. 1, the gravitational potential energy of the manipulator is

$${}^gU = \sum_{i=2}^{n} m_i \mathbf{g}^{\mathrm{T}} \mathbf{p}_i \tag{7}$$

where m_i and $\mathbf{p}_i$ represent the mass and the position vector of mass center of link i, respectively. The position vector $\mathbf{p}_i$ can be expressed as

$$\mathbf{p}_i = \sum_{v=1}^{i} \tilde{\mathbf{T}}_v \mathbf{q}_v \tag{8}$$

where $\tilde{\mathbf{T}}_v$ is a 2×2 transformation matrix containing the dimensional parameters of link ν of manipulator.

Since the coordinate system fixed on link 1 is the inertia frame of manipulator system, the gravitational acceleration vector $\mathbf{g}$ in (7) can be represented as $\mathbf{g} = \mathbf{T}_g \mathbf{q}_1$, where $\mathbf{T}_g$ is a constant of 2×2 transformation matrix. Substituting $\mathbf{T}_g \mathbf{q}_1$ for $\mathbf{g}$, and (8) for $\mathbf{p}_i$ in (7) yields

$$^{g}U = \sum_{v=1}^{n} (\mathbf{q}_1^{\mathrm{T}}\, {}^{g}\mathbf{K}_{1v} \mathbf{q}_v). \tag{9a}$$

Each term of summation in (9a) is a scalar function for $v = 1, 2, \ldots, n$

$$\mathbf{q}_1^{\mathrm{T}}\ {}^{g}\mathbf{K}_{1v} \mathbf{q}_v = (\mathbf{q}_1^{\mathrm{T}}\, {}^{g}\mathbf{K}_{1v} \mathbf{q}_v)^{\mathrm{T}} = \mathbf{q}_v^{\mathrm{T}}\, {}^{g}\mathbf{K}_{1v}^{\mathrm{T}} \mathbf{q}_1 = \mathbf{q}_v^{\mathrm{T}}\, {}^{g}\mathbf{K}_{v1} \mathbf{q}_1. \tag{9b}$$

Equation (9a) can be rewritten as

$$^{g}U = \frac{1}{2} \sum_{v=1}^{n} (\mathbf{q}_1^{\mathrm{T}}\, {}^{g}\mathbf{K}_{1v} \mathbf{q}_v + \mathbf{q}_v^{\mathrm{T}}\, {}^{g}\mathbf{K}_{v1} \mathbf{q}_1). \tag{9c}$$

Hence, the stiffness block matrix representation for the gravitational potential energy can be obtained as

$$^{g}U(\mathscr{Q}) = \frac{1}{2} \mathscr{Q}^{T}\, {}^{g}\mathscr{K}\, \mathscr{Q} \tag{10}$$

where, similar to ${}^{j}\mathscr{K}$, matrix ${}^{g}\mathscr{K} = [\,{}^{g}\mathbf{K}_{uv}]$ is also sum of a symmetric matrix and a skew-symmetric matrix. ${}^{g}\mathscr{K}$ has $n \times n$ component matrices, and each component matrix is a 2×2 matrix.

Since ${}^{g}\mathbf{K}_{uv} = \mathbf{0}$ for u and v not equal to one, the block matrix, ${}^{g}\mathscr{K}$ has nonzero-component matrices only in its first row and first column. Summing (6) and (10) yields the overall stiffness block matrix of the spring-gravity system as

$$\mathscr{K} = {}^{g}\mathscr{K} + \sum_{j=1}^{N} {}^{j}\mathscr{K} \tag{11}$$

where $\mathscr{K} = [\mathbf{K}_{uv}]$.

Since a nonzero-component matrix $\mathbf{K}_{uv}$ can be regarded as a pseudostiffness component embedded between links u and v, the change of relative angular displacement $\theta_{uv} = \cos^{-1}(\mathbf{q}_u^{\mathrm{T}}\, \mathbf{q}_v)$ between links u and v causes a variation of potential energy of $\mathbf{q}_u^{\mathrm{T}}\, \mathbf{K}_{uv} \mathbf{q}_v$ to the system. Therefore, for a statically balanced system, all stiffness components between any two of distinct links should be zeros, i.e., any off-diagonal component matrix $\mathbf{K}_{uv}$ for $u \neq v$ in $\mathscr{K}$ has to be a zero matrix. Thus, the overall stiffness block matrix has been turned into a diagonal block matrix as

$$\mathscr{K} = \mathrm{diag}(\mathbf{K}_{11}, \mathbf{K}_{22}, \ldots, \mathbf{K}_{nn}). \tag{12}$$

If (12) can be obtained by appropriate arrangement of fitted springs into system, the total potential energy U in (1) is constant for any configuration $\mathscr{Q}$ as

$$U = \frac{1}{2} \sum_{u=1}^{n} \mathbf{q}_u^{\mathrm{T}} \mathbf{K}_{uu} \mathbf{q}_u = \frac{1}{4} \mathrm{tr}(\mathscr{K}). \tag{13}$$

Hence, the condition that satisfies (12) yields design equations to solve for the design parameters of a feasible configuration of spring installation.

B. Balance of the Off-Diagonal Stiffness Component Matrices

Since the stiffness block matrix is the sum of a symmetric block matrix and a skew-symmetric block matrix, only the off-diagonal component matrices on the upper triangular stiffness block matrix, $\mathbf{K}_{uv}$'s for $v > u$ are considered. Consider the stiffness block matrix, because of gravity, ${}^{g}\mathscr{K}$, the gravitational force on link v and its reciprocal force on ground (link 1) yield the stiffness components ${}^{g}\mathbf{K}_{1v}$ for $v = 2, 3, \ldots, n$. On the other hand, all ${}^{g}\mathbf{K}_{uv}$'s are zero matrices for $u \neq 1$ on the upper triangular part of ${}^{g}\mathscr{K}$. Hence, the stiffness block matrix ${}^{g}\mathscr{K}$ is shown as follows:

$$^{g}\mathscr{K} = \begin{bmatrix} {}^{g}\mathbf{K}_{11} & {}^{g}\mathbf{K}_{12} & \cdots & \cdots & \cdots & {}^{g}\mathbf{K}_{1n} \\ & \mathbf{0} & \cdots & \cdots & \cdots & \mathbf{0} \\ & & \ddots & & & \vdots \\ & & & \ddots & & \vdots \\ & & & & \ddots & \vdots \\ & & & & & \mathbf{0} \end{bmatrix}. \tag{14}$$

Equation (14) shows the stiffness component matrix between any two of moving links is zero, while a stiffness component matrix between ground and any moving link is nonzero. Balancing the nonzero-component matrix, ${}^{g}\mathbf{K}_{1v}$ yields

$$\mathbf{K}_{1v} = {}^{g}\mathbf{K}_{1v} + \sum_{j=1}^{N} \left({}^{j}\mathbf{K}_{1v}\right) = \mathbf{0}, \qquad v = 2, 3, \ldots, n. \tag{15}$$

The stiffness block matrix ${}^{j}\mathscr{K}$, because of the installation of the jth spring between links u^* and v^*, can be demonstrated as

$$^{j}\mathscr{K} = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \cdots & \cdots & \mathbf{0} \\ & {}^{j}\mathbf{K}_{u^*u^*} & \cdots & \cdots & {}^{j}\mathbf{K}_{u^*v^*} & \vdots \\ & & \ddots & & \vdots & \vdots \\ & & & \ddots & \vdots & \vdots \\ & & & & {}^{j}\mathbf{K}_{v^*v^*} & \vdots \\ & & & & & \mathbf{0} \end{bmatrix} \tag{16}$$

where the nonzero-component matrix ${}^{j}\mathbf{K}_{uv}$ exists for $u \geqq u^*$ and $v \leqq v^*$. For any component matrix $\mathbf{K}_{uv}$ with $u \neq 1$, as a zero matrix, the relation has to be satisfied as follows:

$$\mathbf{K}_{uv} = \sum_{j=1}^{N} \left({}^{j}\mathbf{K}_{uv}\right) = \mathbf{0}, \qquad u, v \neq 1. \tag{17}$$

The total number of off-diagonal component matrices in the upper triangular stiffness block matrix is $n(n-1)/2$, and each of them is with the sum of a skew-symmetric matrix and a diagonal matrix. Therefore, (15) and (17) yield a total number of $n(n-1)$ simultaneous design equations.

III. Minimum Number of Springs

Referring to spring j in Fig. 1, each spring possesses one design parameter as the spring constant k_j and four design parameters as the positioning constants a_j, α_j, b_j, and β_j for the attachment points of the spring. These design parameters are unknowns to be solved through the design equations given in (15) and (17). Take spring constants, k_j's for $j = 1, 2, \ldots, N$ to be the free variables and make it arbitrarily predetermined. For a manipulator with N fitted springs, a total number of $4N$ design parameters in the design equations are to be solved. Generally, the number of unknowns must be greater than or equal to the number of simultaneous equations

$$4N \geq n(n-1). \tag{18}$$

Based on (18), the minimum number of spring N^* can be obtained as

$$N^* = \left\lceil \frac{n(n-1)}{4} \right\rceil \tag{19}$$

where $\lceil * \rceil$ denotes the ceiling function and returns the smallest integer which is no less than the real number within bracket. Note that the proposed minimum number of springs for an SBAM is on the premise without auxiliary links. The minimum number of springs for a method of auxiliary link proposed by Streit and Shin [17] is equal to $(n-1)$ for an n-link system, while $2(n-2)$ auxiliary links are added into the system. Given in (19), N^* is equal to $(n-1)$ for $n \leqq 4$, i.e., the minimum number of springs with direct spring installation is equal to that of the auxiliary link method for the system with four or less number of links. However, since N^* is a second-order function of n, the minimum number of springs may tremendously increase for higher DOF SBAMs if the direct spring installation method is used.

IV. Determination of the Spring Configuration

A. Spring Configuration Matrix

The distribution of nonzero-component matrices in a stiffness block matrix is determined by configuration of a spring installation, as shown in (16). The spring configuration of an SBAM, which is similar to the topological configuration of the links of mechanism, can be used to characterize and classify the distinct types of designs. Hence, a matrix $\boldsymbol{\Lambda}$ of a direct spring installation system is defined to represent the configuration of fitted springs. Matrix $\boldsymbol{\Lambda} = [\lambda_{uv}]$ follows the conventional usage of adjacency matrix, and it is an $n \times n$ square matrix. For an n-link SBAM, element $\lambda_{u^*v^*}$ is an integer that is equal to the number of springs between links u^* and v^*. For simplicity, the spring installation that is proposed in this paper allows only single installation of a spring between a pair of distinct links, and hence, element λ_{uv} equals either 1 or 0. If the solution of spring design parameters of an SBAM can be obtained from the simultaneous design equations given in (15) and (17), the spring configuration matrix $\boldsymbol{\Lambda}$ and the corresponding spring configuration of spring system are admissible.

B. Characteristics of the Admissible Spring Configuration Matrix

In this section, characteristics of an admissible spring configuration matrix for the design of an SBAM are investigated. First, consider the off-diagonal stiffness component matrix $\mathbf{K}_{1n}$ of (15). An associated spring component matrix ${}^j\mathbf{K}_{1n}$ is required to balance the gravity component matrix of ${}^g\mathbf{K}_{1n}$, in order to make the component matrix $\mathbf{K}_{1n}$ to be zero, according to (14) and (16). It implies that a spring has to be fitted between link 1 (ground) and link n. Denote the first spring as ${}^1S_{1n}$ and thus

Characteristic **C1**: *For an n-link planar SBAM, a spring is required to be fitted between ground and the end link and element λ_{1n} of an $n \times n$ spring configuration matrix $\boldsymbol{\Lambda}$ is equal to* 1.

According to (16), as spring ${}^1S_{1n}$ is fitted to the system, ${}^1\mathbf{K}_{uv}$'s for $u \geqq 1$ and $v \leqq n$ are nonzero-component matrices. Consider the $(2, n)$th component matrix of the entire stiffness block matrix, i.e., $\mathbf{K}_{2n}$. Since there is no gravitational influence but the elastic influence ${}^1\mathbf{K}_{2n}$ because of spring ${}^1S_{1n}$, a second spring, regarded as ${}^2S_{1n}$, has to be fitted between links 2 and n in order to generate a counterbalancing component matrix ${}^2\mathbf{K}_{2n}$ to compensate the influence of ${}^1\mathbf{K}_{2n}$, because of spring ${}^1S_{1n}$. Hence,

Characteristic **C2**: *For an n-link planar SBAM, a spring is required to be attached between the ground-pivoted link and the end link, and the element λ_{2n} of an $n \times n$ spring configuration matrix $\boldsymbol{\Lambda}$ is equal to* 1 *for any n greater than* 2.

Since the gravity only induces nonzero-component matrices on the first row (and the first column) of stiffness block matrix, the equilibrium for each relative angular displacements θ_{1v}'s for $v = 2, 3, \ldots, n$ are achieved by one gravitational influence and one or more elastic influence, whereas the equilibrium for each relative angular displacements θ_{uv}'s for $u, v \neq 1$ is accomplished by two or more elastic influences.

For a set of simultaneous equations with nontrivial solutions, number of unknowns in any subset of equations has to be greater than or equal to the number of equations in the subset. Consider the subset of equations by the component matrices on the (u^*)th row or the (v^*)th column of stiffness block matrix. There are $(n-u^*)$ component matrices on the (u^*)th row as $\mathbf{K}_{u^*(u^*+1)}$, $\mathbf{K}_{u^*(u^*+2)}, \ldots, \mathbf{K}_{u^*n}$; and (v^*-1) component matrices on the (v^*)th column as $\mathbf{K}_{1v^*}, \mathbf{K}_{2v^*}, \ldots, \mathbf{K}_{(v^*-1)v^*}$. Respectively, there are $2(n-u^*)$ equations and $2(v^*-1)$ equations by the (u^*)th row and the (v^*)th column of stiffness block matrix. Next, consider the number of unknowns. For each fitted spring ${}^jS_{u^*v^*}$, i.e., $\lambda_{u^*v^*} = 1$, four distinct unknowns a_j, α_j, b_j, and β_j are added to the system. According to the derivations from (2) to (5), component matrix ${}^j\mathbf{K}_{u^*v^*}$ possesses all the four unknowns of spring ${}^jS_{u^*v^*}$, each component matrix ${}^j\mathbf{K}_{uv}$ for either $u = u^*$ or $v = v^*$ possesses two unknowns of spring ${}^jS_{u^*v^*}$. Therefore, the number of spring design parameters in the equations by component matrices on row u^* of stiffness block matrix n_{R,u^*} is

$$n_{\mathrm{R},u^*} = \left(\sum_{v=u^*+1}^{n} 4\lambda_{u^*v} \right) + \left(\sum_{v=u^*+1}^{n} \sum_{u=1}^{u^*-1} 2\lambda_{uv} \right) \tag{20}$$

where the first term is resulted from springs that are attached to link u^*, providing four spring design parameters for each spring. And the second term is from springs spanning over link u^*, providing two spring design parameters. Similarly, the number of spring design parameters in the equations by component matrices on column v^* of stiffness block matrix n_{C,v^*} is

$$n_{C,v^*} = \left(\sum_{u=1}^{v^*-1} 4\lambda_{uv^*}\right) + \left(\sum_{v=v^*+1}^{n} \sum_{u=1}^{v^*-1} 2\lambda_{uv}\right) \tag{21}$$

where the first term results from springs attached to link v^*, and the second term is from springs spanning over link v^*.

Hence, a necessary condition based on the number of spring design parameters and the number of equations can be identified as

Characteristic **C3**: *In a planar n-link SBAM, the numbers of spring design parameters, n_{R,u^*} and n_{C,v^*}, in the component matrices on row u^* and on columnv* of stiffness block matrix must, respectively, satisfy*

$$n_{\mathrm{R},u^*} \geq 2(n-u^*) \quad \forall u^* = 1, 2, \ldots, (n-1) \tag{22}$$

and

$$n_{\mathrm{C},v^*} \geq 2(v^*-1) \quad \forall v^* = 2, 3, \ldots, n. \tag{23}$$

The conditions that are given in (20)–(23) can be examined computationally for any given configuration matrix, although a manual inspection method is provided by the following spring configuration matrix as an example in illustration:

$$\Lambda = \begin{bmatrix} 0 & & & 0 & 1 & 1 \\ & 0 & & 0 & 0 & 1 \\ & & \ddots & 0 & 1 & 0 \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & 0 \end{bmatrix}. \tag{24}$$

Equation (24) shows only part of a 6 × 6 spring configuration matrix. To examine the third row in the matrix, since $\lambda_{35} = 1$, it indicates that one spring is attached to link 3, providing four spring design parameters. According to (20), where $u^* = 3$, each of nonzero λ_{uv} for $1 \leqq u \leqq 2$ and $4 \leqq v \leqq 6$ contributes two spring design parameters to the third row. Since λ_{15}, λ_{16}, and λ_{26} are all equal to 1, it indicates that three distinct springs spanning over link 3, each spring provides two design parameters to the third row. Therefore, based on (20), (22) is satisfied as

$$4 \times (1) + 2 \times (3) > 2 \times (6-3). \tag{25}$$

Similarly, for the fourth column of the spring configuration matrix, i.e., $v^* = 4$, since all λ_{uv}'s on the fourth column are all in zero and four λ_{uv}'s in the range of $1 \leqq u \leqq 4$ and $5 \leqq v \leqq 6$. Namely λ_{15}, λ_{16}, λ_{26}, and λ_{35}, all equal to 1, based on (21), (23) is satisfied as

$$4 \times (0) + 2 \times (4) > 2 \times (4-1). \tag{26}$$

Furthermore, consider a smaller subset of equations, i.e., the subset of simultaneous equations that are derived from a single component matrix. Consider the component matrix $\mathbf{K}_{u^*v^*}$ on the (u^*)th row and the (v^*)th column of the stiffness block matrix, two associated design equations are obtained from $\mathbf{K}_{u^*v^*} = \mathbf{0}$, and the number of design parameters is

$$n_{u^*v^*} = 4\lambda_{u^*v^*} + \sum_{u=1}^{u^*-1} 2\lambda_{uv^*} + \sum_{v=v^*+1}^{n} 2\lambda_{u^*v}. \tag{27}$$

Judiciously, the number of design parameters must be equal to or greater than the number of design equations

$$n_{u^*v^*} \geq 2 \quad \forall u^* = 1, 2, \ldots, (n-1); \forall v^* = 2, 3, \ldots, n. \tag{28}$$

Characteristic **C4**: *In an $n \times n$ spring configuration matrix $\mathbf{\Lambda}$ of a planar SBAM, for any entry (u^*, v^*), elements λ_{u^*v}'s for $v \geqq v^*$ and elements λ_{uv^*}'s for $u \leqq u^*$ cannot be all in zero.*

$$\begin{array}{c c} & \text{The } (v^*)^{\text{th}} \text{ column} \downarrow \\ \text{The } (u^*)^{\text{th}} \text{ row} \rightarrow & \begin{bmatrix} 0 & & & 0 & & & \\ & 0 & & \vdots & & & \\ & & \ddots & 0 & \cdots & 0 & 0 \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{bmatrix} \end{array}$$

Fig. 2 Inadmissible spring configuration matrix violating **C4**.

Fig. 2 illustrates a configuration matrix violating (28) as an example to identify inadmissible configuration matrices.

According to (16), it is found that a spring between links u^* and v^* affects the stiffness component matrices between links u^*, $(u^*+1), \ldots,$ and v^*. If a spring is fitted between two adjacent links, i.e., ${}^jS_{u^*(u^*+1)}$ for $u^* = 1, 2, \ldots, (n-1)$, a spring is referred to as a monoarticular spring since the specific spring only spans over one articulated joint. A monoarticular spring affects the stiffness between two adjacent links, and all four design parameters of the monoarticular spring ${}^jS_{u^*(u^*+1)}$ exist only in component matrix $\mathbf{K}_{u^*(u^*+1)}$ of an overall stiffness block matrix. Since $\mathbf{K}_{u^*(u^*+1)} = 0$ yields only two design equations, two out of the four design parameters are dependent. Therefore, a monoarticular spring contributes only two unknown design parameters to the overall simultaneous equations. Since the number of design parameters with the minimum number of spring is $4N^*$, according to (18) and (19), the number of design parameters exceeds the number of equations by

$$4N^* - n(n-1) = \begin{cases} 0 & n(n-1)/4 \in \mathbf{Z} \\ 2 & n(n-1)/4 \notin \mathbf{Z} \end{cases} \tag{29}$$

where $\mathbf{Z}$ denotes the set of integers. Equation (29) suggests that the number of design parameters is equal to the number of equations when $n(n-1)/4$ is an integer. This implies that all

TABLE I
ADMISSIBLE SPRING CONFIGURATION MATRICES

1-DOF ($n = 2$)	2-DOF ($n = 3$)	3-DOF ($n = 4$)
$\begin{bmatrix} 0 & 1 \\ & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ & 0 & 1 \\ & & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 & 1 \\ & 0 & 0 & 1 \\ & & 0 & 0 \\ & & & 0 \end{bmatrix}$
4-DOF ($n = 5$)		
$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ & 0 & 0 & 1 & 1 \\ & & 0 & 0 & 0 \\ & & & 0 & 0 \\ & & & & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ & 0 & 0 & 0 & 1 \\ & & 0 & 0 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{bmatrix}$		
$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ & 0 & 0 & 1 & 1 \\ & & 0 & 0 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ & 0 & 0 & 1 & 1 \\ & & 0 & 0 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{bmatrix}$		

design parameters are unknowns to the design equations, and there is no monoarticular spring to be used. Contrarily, when $n(n-1)/4$ is not an integer, two design parameters must be given in advance; therefore, only one monoarticular spring can be used.

Characteristic **C5**: *For an n-link SBAM, where $n(n-1)/4$ is an integer, no monoarticular spring can be used if the minimum number of spring is desired, and each of elements $\lambda_{u(u+1)}$ for $u = 1, 2, \ldots, (n-1)$ of the $n \times n$ spring configuration matrix must be zero. For an n-link SBAM, where $n(n-1)/4$ is not an integer, only one monoarticular spring can be used and only one of the elements $\lambda_{u(u+1)}$ for $u = 1, 2, \ldots, (n-1)$ is one with the others being all zeros. Therefore, the sum of all the elements on superdiagonal line of a configuration matrix must satisfy*

$$\sum_{u=1}^{n-1} \lambda_{u(u+1)} = \begin{cases} 0 & n(n-1)/4 \in \mathbf{Z} \\ 1 & n(n-1)/4 \notin \mathbf{Z}. \end{cases} \tag{30}$$

Among **C1–C5**, characteristics **C1** and **C2** are necessary and sufficient conditions for an admissible spring configuration matrix, while **C3–C5** are merely necessary.

C. Admissible Spring Configurations for an Articulated Manipulator With Up to Four Degrees of Freedom

Based on **C1–C5**, admissible spring configuration matrices for 1-, 2-, 3- and 4-DOF SBAMs are tabulated in Table I. According to (19), the minimum numbers of required springs for $n = 2$, 3, 4, and 5, respectively, are $N^* = 1$, 2, 3, and 5. From **C1** and **C2**, in each admissible configuration matrix, the first and the second elements of the last column on the upper triangular matrix are equal to 1. Thus, configuration matrices of $n = 2$ and 3 can be determined, as shown in Table I. For the configuration matrices of $n = 4$ and 5, the third element with a figure of nonzero cannot be placed on the superdiagonal line according to **C5**. Hence, one and four feasible configuration matrices, respectively, for $n = 4$ and 5, are obtained as shown in Table I. Since the four feasible configuration matrices for $n = 5$ satisfy **C3** and **C4**, all of them are admissible configuration matrices. Corresponding spring configurations are schematically illustrated in Fig. 3, where the circles, the heavy edges, and thin edges, respectively, represent as joints, links, and springs of a SBAM.

V. MINIMUM PRETENSILE ENERGY

Upon an admissible spring configuration, the total potential energy as shown in (13) is constant, regardless of the configuration change in manipulator. Substituting (13) with (11), the constant potential energy can be decomposed by two parts as $\mathrm{tr}({}^g\mathscr{K})/4$ and $\Sigma_{j=1}^{N}\mathrm{tr}({}^j\mathscr{K})/4$, where the latter is the constant potential energy induced by all the spring forces of an SBAM, in which the jth term in the summation can be rewritten as

$$\frac{1}{4}\mathrm{tr}({}^j\mathscr{K}) = \frac{1}{4}\mathrm{tr}\left(\sum_{u=u^*}^{v*} {}^j\mathbf{K}_{uu}\right) = \frac{1}{4}\mathrm{tr}\left(\sum_{u=u^*}^{v*} k_j \mathbf{T}_u^{\mathrm{T}}\mathbf{T}_u\right)$$
$$= \frac{1}{2}k_j\left(a_j^2 + b_j^2 + \sum_{i=u^*+1}^{v^*-1} r_i^2\right). \tag{31}$$

Equation (31) relates to the elastic potential energy of the jth spring of a "stretched configuration," as illustrated in Fig. 4. It is referred to as the pretensile energy of the jth spring as well. In Fig. 4, all the centers of joints spanned over by the spring and the two spring attachment points are collinear. On the stretched configuration, the spring force of spring ${}^jS_{u^*v^*}$ is a maximum in comparison with other possible configurations of manipulator, and it results in the maximum compression forces to all revolute joints between link u^* and link v^*. In practice, compression forces may increase friction in the revolute joints and weaken the structural strength of the system. Hence, it should be minimized.

To obtain a robust SBAM, it intends to minimize the pretensile energy. According to (31), the pretensile energy of spring ${}^jS_{u^*v^*}$ is proportional to the trace of the sum of main diagonal component matrices ${}^j\mathbf{K}_{uu}$'s for $u = u^*, (u^*+1), \ldots, v^*$. Thus, minimizing the pretensile energy is to obtain a spring configuration, where its associated stiffness block matrix has minimum number of nonzero diagonal component matrices. From (16), one can infer that the number of nonzero diagonal component matrices by spring ${}^jS_{u^*v^*}$ is

$${}^jn_{\mathrm{D}} = v^* - u^* + 1. \tag{32}$$

Note that $({}^jn_{\mathrm{D}} - 1)$ is equal to the number of articulated joints spanned over by spring ${}^jS$. Thus, an index function $R(\mathbf{\Lambda})$, for an admissible configuration matrix $\mathbf{\Lambda}$, can be used to evaluate the robustness of spring configuration by the total number of nonzero diagonal component matrices for springs

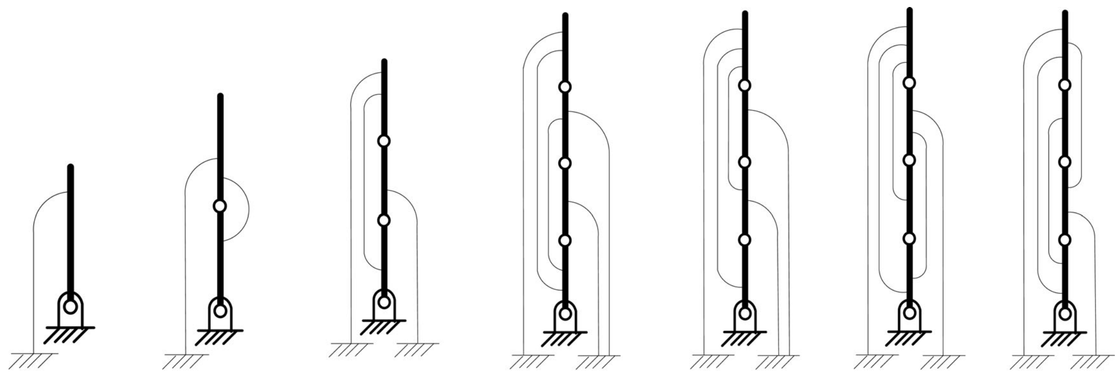

Fig. 3. Functional schematics of admissible spring configurations.

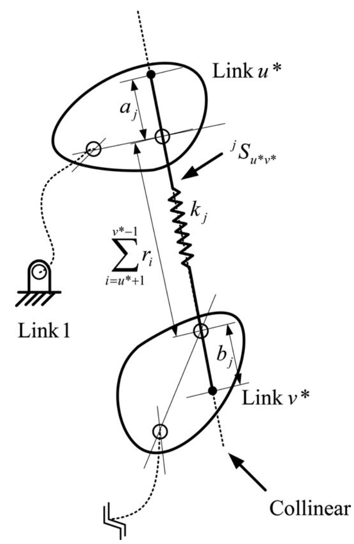

Fig. 4. Stretched configuration for spring ${}^jS_{u^*v^*}$.

${}^1S, {}^2S, \ldots, {}^NS$, as

$$R(\mathbf{\Lambda}) = \sum_{j=1}^{N} {}^jn_{\mathrm{D}}. \tag{33}$$

From Table I, the robustness indexes of each admissible spring configuration matrix in a 4-DOF SBAM, respectively, are 19, 19, 19, and 18. Conclusively, the spring configuration of Fig. 3(g) is said to be more robust than those in Fig. 3(d)–(f).

VI. Statically Balanced 3-DOF Robotic Arm

A. Solving the Design Equations

The inertia and dimension properties of a 3-DOF robotic arm intended to be statically balanced are listed in Table II, and the associated parameters are illustrated in Fig. 5. The component matrices in ${}^g\mathrm{K}$ of interest can be obtained as follows:

$$ {}^g\mathbf{K}_{12} = -\left[m_2 g s_2 + (m_3 + m_4) g r_2\right]\mathbf{I} \tag{34a}$$

$$ {}^g\mathbf{K}_{13} = -(m_3 g s_3 + m_4 g r_3)\mathbf{I} \tag{34b}$$

$$ {}^g\mathbf{K}_{14} = -m_4 g s_4 \mathbf{I} \tag{34c}$$

TABLE II
Dimensional and Inertia Parameters of the 3-DOF Robotic Arm
(Data Given in Kilograms and Meters)

Link i	m_i	s_i	r_i
2	4.55	.127	.127
3	11.42	.378	.757
4	42.60	.140	

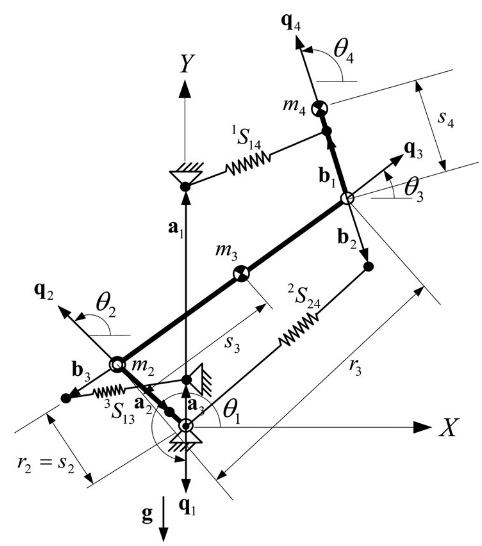

Fig. 5. Schematic of the statically balanced 3-DOF robotic arm.

$$ {}^g\mathbf{K}_{23} = {}^g\mathbf{K}_{24} = {}^g\mathbf{K}_{34} = \mathbf{0} \tag{34d}$$

where $\mathbf{I}$ is a 2×2 identity matrix, and the component matrices are diagonal, since $\mathbf{q}_i$ are parallel to $\mathbf{r}_i$ and $\mathbf{s}_i$ in Fig. 1.

Based on the result of Fig. 3(c), springs ${}^1S_{14}$ of k_1, ${}^2S_{24}$ of k_2, and ${}^3S_{13}$ of k_3 are installed in the system as shown in Fig. 5, where the nonzero elastic component matrices can be derived accordingly. In ${}^1\mathscr{K}$

$$ {}^1\mathbf{K}_{12} = -k_1 a_1 r_2 \begin{bmatrix} C(-\alpha_1) & -S(-\alpha_1) \\ S(-\alpha_1) & C(-\alpha_1) \end{bmatrix} = -k_1 a_1 r_2 \mathbf{R}(-\alpha_1) \tag{35a}$$

$$ {}^1\mathbf{K}_{13} = -k_1 a_1 r_3 \mathbf{R}(-\alpha_1) \tag{35b}$$

$$^1\mathbf{K}_{14} = -k_1 a_1 b_1 \mathbf{R}(\beta_1 - \alpha_1) \tag{35c}$$

$$^1\mathbf{K}_{23} = k_1 r_2 r_3 \mathbf{I} \tag{35d}$$

$$^1\mathbf{K}_{24} = k_1 r_2 b_1 \mathbf{R}(\beta_1) \tag{35e}$$

$$^1\mathbf{K}_{34} = k_1 r_3 b_1 \mathbf{R}(\beta_1) \tag{35f}$$

where α_i and β_i are the constant angles of vectors, C and S represent as cosine and sine functions, respectively. $\mathbf{R}$ is a 2×2 rotation matrix as defined in (3), and $\mathbf{a}_i$ and $\mathbf{b}_i$ are the position vectors of the spring attachment points as shown in Fig. 1. In $^2\mathscr{K}$

$$^2\mathbf{K}_{23} = -k_2 a_2 r_3 \mathbf{R}(-\alpha_2) \tag{36a}$$

$$^2\mathbf{K}_{24} = -k_2 a_2 b_2 \mathbf{R}(\beta_2 - \alpha_2) \tag{36b}$$

$$^2\mathbf{K}_{34} = k_2 r_3 b_2 \mathbf{R}(\beta_2 - \alpha_2). \tag{36c}$$

In $^3\mathscr{K}$

$$^3\mathbf{K}_{12} = -k_3 a_3 r_2 \mathbf{R}(-\alpha_3) \tag{37a}$$

$$^3\mathbf{K}_{13} = -k_3 a_3 b_3 \mathbf{R}(\beta_3 - \alpha_3) \tag{37b}$$

$$^3\mathbf{K}_{23} = k_3 r_2 b_3 \mathbf{R}(\beta_3). \tag{37c}$$

According to (15) and (17), 12 design equations can be obtained as

$$k_1 a_1 r_2 \mathrm{C}(-\alpha_1) + k_3 a_3 r_2 \mathrm{C}(-\alpha_3) + m_2 g s_2 + (m_3 + m_4) g r_2 = 0 \tag{38a}$$

$$k_1 a_1 r_2 S(-\alpha_1) + k_3 a_3 r_2 S(-\alpha_3) = 0 \tag{38b}$$

$$k_1 a_1 r_3 C(-\alpha_1) + k_3 a_3 b_3 C(\beta_3 - \alpha_3) + m_3 g s_3 + m_4 g r_3 = 0 \tag{39a}$$

$$k_1 a_1 r_3 \mathrm{S}(-\alpha_1) + k_3 a_3 b_3 \mathrm{S}(\beta_3 - \alpha_3) = 0 \tag{39b}$$

$$k_1 a_1 b_1 \mathrm{C}(\beta_1 - \alpha_1) + m_4 g s_4 = 0 \tag{40a}$$

$$k_1 a_1 b_1 \mathrm{S}(\beta_1 - \alpha_1) = 0 \tag{40b}$$

$$-k_1 r_2 r_3 + k_2 a_2 r_3 \mathrm{C}(-\alpha_2) - k_3 r_2 b_3 \mathrm{C}(\beta_3) = 0 \tag{41a}$$

$$k_2 a_2 r_3 \mathrm{S}(-\alpha_2) - k_3 r_2 b_3 \mathrm{S}(\beta_3) = 0 \tag{41b}$$

$$-k_1 r_2 b_1 \mathrm{C}(\beta_1) + k_2 a_2 b_2 \mathrm{C}(\beta_2 - \alpha_2) = 0 \tag{42a}$$

$$-k_1 r_2 b_1 \mathrm{S}(\beta_1) + k_2 a_2 b_2 \mathrm{S}(\beta_2 - \alpha_2) = 0 \tag{42b}$$

$$k_1 r_3 b_1 \mathrm{C}(\beta_1) + k_2 r_3 b_2 \mathrm{C}(\beta_2) = 0 \tag{43a}$$

$$k_1 r_3 b_1 \mathrm{S}(\beta_1) + k_2 r_3 b_2 \mathrm{S}(\beta_2) = 0. \tag{43b}$$

The numerical solution can be solved by the "NSolve" function in the software of Mathematica through assigning 12 auxiliary variables, $a_i\mathrm{C}(\alpha_i)$, $a_i\mathrm{S}(\alpha_i)$, $b_i\mathrm{C}(\beta_i)$, and $b_i\mathrm{S}(\beta_i)$ for $i = 1, 2, 3$ [see (38)–(43)].

To obtain the closed-form solution, summing the squared (42a) and (42b) yields

$$k_1 r_2 b_1 = k_2 a_2 b_2 \tag{44a}$$

$$\beta_1 = \beta_2 - \alpha_2. \tag{44b}$$

TABLE III
SPRING DESIGN PARAMETERS OF THE 3-DOF ROBOTIC ARM (DATA GIVEN IN NEWTON PER METER, METER, AND DEGREE)

Spring i	k_i	a_i	α_i	b_i	β_i
1	1000	.491	180	.119	0
2	600	.127	180	.198	180
3	8000	.010	180	.151	180

Summing the squared (43a) and (43b) yields

$$k_1 r_3 b_1 = k_2 r_3 b_2 \tag{45a}$$

$$\beta_1 = \beta_2 \pm \pi. \tag{45b}$$

Dividing (44a) by (45a) yields

$$a_2 = r_2. \tag{46}$$

Subtracting (44b) from (45b) yields

$$\alpha_2 = \pm\pi. \tag{47}$$

Substituting (46) and (47) into (41a) and (41b) yields

$$-k_1 r_2 r_3 + k_2 r_2 r_3 - k_3 r_2 b_3 \mathrm{C}(\beta_3) = 0 \tag{48a}$$

$$-k_3 r_2 b_3 \mathrm{S}(\beta_3) = 0 \tag{48b}$$

where parameters b_3 and β_3 can be determined as

$$b_3 = \frac{(k_1 + k_2) r_3}{k_3} \tag{49a}$$

$$\beta_3 = \pm\pi. \tag{49b}$$

Assign four auxiliary variables Y_i as $Y_1 = a_1\mathrm{C}(\alpha_1)$, $Y_2 = a_1\mathrm{S}(\alpha_1)$, $Y_3 = a_3\mathrm{C}(\alpha_3)$, and $Y_4 = a_3\mathrm{S}(\alpha_3)$. Applying the sum and the difference of formulas for the functions of sine and cosine in (38a), (38b), (39a), and (39b), they yield a linear system as

$$\sum_{i=1}^{4} A_{ij} Y_i = B_j, \quad j = 1, 2, 3, 4 \tag{50}$$

where A_{ij} and B_j are constituted of determined parameters. With Cramer's rule, auxiliary variables of Y_1, Y_2, Y_3, and Y_4 can be solved. Parameters of a_1, α_1, a_3, and α_3 can also be determined accordingly. In the end, parameters of b_1 and β_1 with (40a) and (40b) can be solved as

$$b_1 = \frac{m_4 g s_4}{k_1 a_1} \tag{51a}$$

$$\beta_1 = \alpha_1. \tag{51b}$$

Substituting (51a) and (51b) into (45a) and (45b), respectively, parameters b_2 and β_2 can be obtained. The spring constants of k_1, k_2, and k_3 are free variables that are assigned through trial-error for the reasonable spring constants and spring attachments. The results of design parameters are tabulated in Table III.

B. Results and Discussions

The CAD model of 3-DOF robotic arm is shown in Fig. 6. There is an adjustable length of r_3 through a lead screw device of the robotic arm in link 3. In the comparison with the method

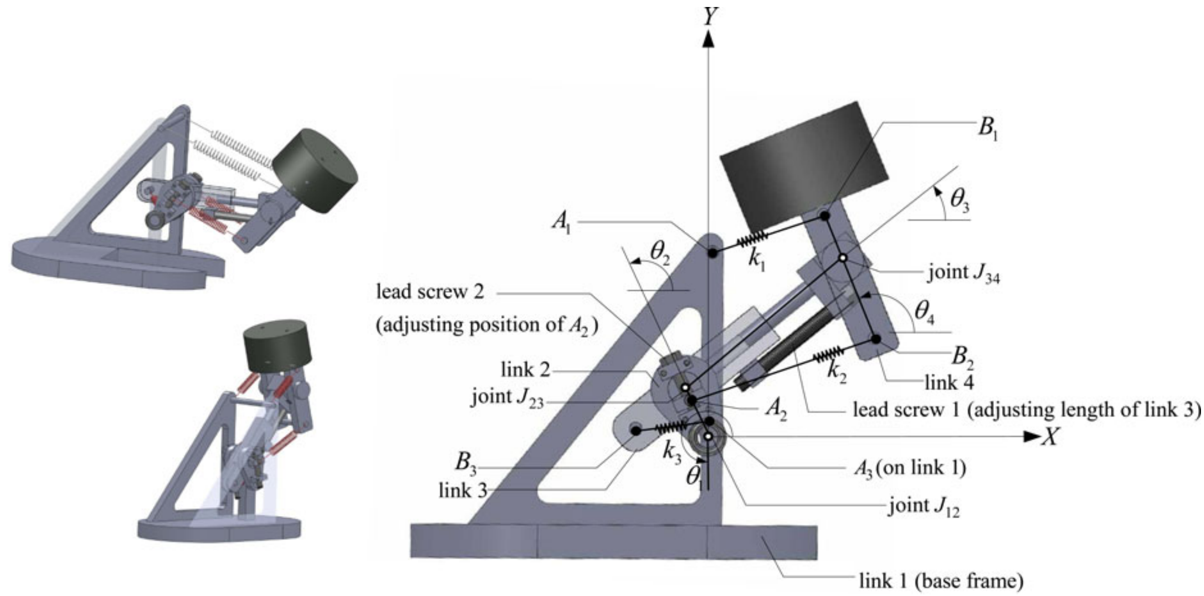

Fig. 6. Statically balanced 3-DOF robotic arm.

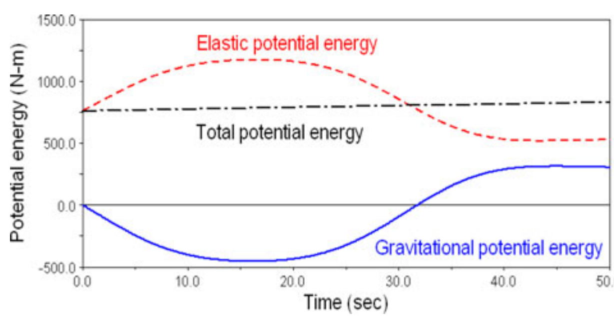

Fig. 7. Time history of the simulated potential energy.

of auxiliary link, a linkage of telescopic parallelogram demands a simultaneous and equal length adjustment in the opposite side of parallelogram, which might present in a more complicated perform in the structure. Note that the change of length r_3 requires different positions of attachment points of corresponding spring for static balance. Generally, the direct spring installation requires less number in mechanical parts. However, the direct spring installation sometimes requires a spring to transversely crosscut a potentially large workspace, and a capable working spring in such a large stroke may be difficult to be obtained and manufactured. Thus, the selection of spring constants and points of spring attachment are the most time-consuming phase in the design.

This model is simulated in the software ADAMS, in which the potential energy functions of springs and gravity are plotted in Fig. 7, with an arbitrary task of the robotic arm. The robotic arm of link 3 is initially horizontal ($\theta_1 = 270°$, $\theta_2 = 282°$, $\theta_3 = 8°$, and $\theta_4 = 81°$). Next, the large mass of the final part is lowered ($\theta_1 = 270°$, $\theta_2 = 247°$, $\theta_3 = 293°$, and $\theta_4 = 343°$) and raised ($\theta_1 = 270°$, $\theta_2 = 314°$, $\theta_3 = 78°$, and $\theta_4 = 140°$) to complete the motion. Fig. 8 illustrates a series of snapshots of the mechanism at time instances $t = 0$, 10, 20, 30, 40, 50 s. The six corresponding configuration matrices $\mathscr{Q}$'s are, respectively, $[0, -1, 0.21, -0.98, 0.99, 0.14, 0.16, 0.99]^T$, $[0, -1, -0.29, -0.96, 0.57, -0.82, 0.99, -0.14]^T$, $[0, -1, -0.32, -0.95, 0.52, -0.86, 0.91, -0.41]^T$, $[0, -1, 0.10, -0.99, 0.99, -0.05, 0.86, 0.52]^T$, $[0, -1, 0.60, -0.80, 0.45, 0.89, -0.29, 0.96]^T$, and $[0, -1, 0.69, -0.72, 0.21, 0.98, -0.77, 0.64]^T$, where the component vectors of $\mathbf{q}_j$ ($i = 1, 2, 3, 4$) in each $\mathscr{Q}$ are expressed as coordinates of the frame of XY in Fig. 5. The 8×8 diagonal stiffness matrix of statically balanced robotic arm is

$$\mathscr{K} = \mathrm{diag}(241.88,\ 241.88,\ 155.93,\ 155.93,\ 1098.41,\ 1098.41,\ 37.68,\ 37.68). \quad (52)$$

The total potential energy can be calculated by (13) and (52) as $U = \mathrm{tr}(\mathscr{K})/4 = 766$ (Nm). The constant value is consistent with the result of simulation as shown in Fig. 7, where the total potential energy remains constant in any configuration. The elastic potential energy and the gravitational potential energy can also be easily justified by (52) with the corresponding configuration matrices $\mathscr{Q}$'s at various mechanism configurations. The simulated static torques of the revolute joints J_{12}, J_{23}, and J_{34} are all zero during entire motion.

VII. Conclusion

This paper presents a design methodology on the synthesis of spring configuration of SBAMs. Based on the structural analysis of stiffness block matrix, five criteria are identified as fundamental characteristics for admissible spring configuration. The spring configuration of an SBAM is represented by a proposed configuration matrix. Admissible spring configurations are enumerated for 1-, 2-, 3-, and 4-DOF planar articulated manipulators, and therefore, candidate designs for various spring arrangements can be provided. Furthermore, an optima selection of admissible spring configurations can be followed by the robustness index. The design of a 3-DOF SBAM is proposed as

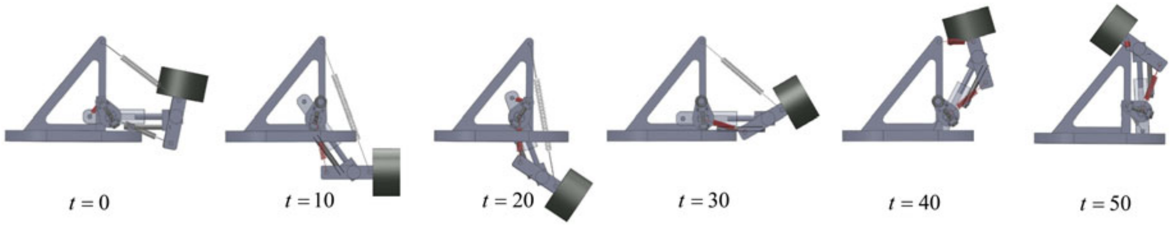

Fig. 8. Snapshots of the 3-DOF robotic arm.

an illustrative design example. Without the auxiliary links, the 3-DOF SBAM has less of geometric constraints. The statically balanced robotic arm can manipulate a payload which is much heavier than its weight of link with little effort. The results from computer simulation justify the static balance of the proposed design.

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