Compact Arrangements of Cable-Pulley Type Zero-Free-Length Springs

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A methodology to develop springs with zero-free-length (ZFL) characteristics is presented, and the configurations for placing the springs precisely on the manipulators are introduced. A spring–string arrangement installed between two separate links of a serial-type manipulator is employed and is divided into three regions for mounting, tensioning, and placing the string. The springs can develop ZFL characteristics if adequate length is ensured for mounting the springs. To shorten the length for placing strings, a reference length acquired from the link configurations is utilized. The minimization of the placing length can therefore be described clearly. Because the overextended springs and links occupied the workspace of the other links as a result of the long mounting length, the springs are reorganized using pulleys and wire winding configurations to shorten the mounting length. The springs can then be arranged in alignment on the links. As achieved with this additional arrangement, comprehensive spring configurations on the manipulators can be shown. Two examples are presented after deriving the spring configurations for ZFL characteristics and the configurations with wire winding, respectively. [DOI: 10.1115/1.4036515]

1 Introduction

Springs are used widely in manipulators to facilitate control and energy utilization. For springs, physical length is the length at its initial state without any external force, whereas free length is the length when the spring is unloaded and is shorter than the physical length in preloaded springs. However, the initial force and length in preload springs cannot be eliminated; therefore, the springs in this paper are assumed to be nonpreloaded. The physical length of a nonpreloaded spring is thus equivalent to its free length. The concept underlying zero-free-length (ZFL) springs is that the free length of a spring is eliminated, and therefore, the pull exerted by the spring is always proportional to the spring length. In other words, in a ZFL spring, the force is proportional to its length, and a plot of force versus length extrapolates back to zero force at zero length.

Approaches to arranging springs as ZFL started in the 1930s with the use of guiding elements and pulley–string arrangements [1,2] to provide additional mounting space to the springs. However, systematic arrangement theorems were yet to be developed. Straightforward methods involving linkages, such as a slider or a cam [3,4], were developed; the arrangements derived in Ref. [5] show practicable means to achieve ZFL characteristics, but these arrangements do not consider the spring and string lengths. Also, the elongation equation presented in Ref. [6] does not account for the limitations of springs bent by turning points. Such methods might not be practicable because the springs are arranged through trial and error or by providing excessive spring mounting space. Therefore, a practical method for deriving feasible spring configurations for ZFL characteristics is required.

Theorems to derive configuration arrangements of ZFL springs for manipulators exist, such as the technique for statically balancing any planar revolute-jointed linkage [7] or using stiffness block matrices for arranging springs on manipulators [8,9]. In addition, spring installation configurations can be determined on the basis of the stiffness block matrixes [10]. By deriving and applying spring configurations for ZFL characteristics, the rationality and feasibility of these theorems can be established.

In this paper, a methodology for inducing ZFL characteristics in springs is presented. An elastic system consisting of a spring and a string (or a cable) is employed as a spring–string arrangement and is divided into three regions for mounting, tensioning, and placing the string. ZFL characteristics can be induced if adequate length can be ensured for mounting the spring. However, the overextended springs and links occupy the workspace of the other links because of their long mounting length. To shorten the mounting length, wire winding configurations entailing the use of reels are introduced to shorten the spring elongation. Theoretical springs can then be practically arranged in alignment on the manipulators. Finally, two examples illustrating the derivation of the spring configurations for developing ZFL characteristics as well as the applications of wire winding configurations are discussed.

2 Spring Configurations for Zero-Free-Length Characteristics

2.1 Methodology of Spring Configurations for ZFL Characteristics. A serial-type planar articulated manipulator with rigid multiple links is shown in Fig. 1(*a*). Each succeeding link $i+1$ is connected to its preceding link i by joint J_i. Link k is the distal link and link i is the proximal link closer to the ground link.

Configuration parameters are defined in this section. Each link vector $\mathbf{r}_i, \mathbf{r}_j, \ldots$ and $\mathbf{r}_k$ of the articulated manipulator is placed along the line passing through centers of the joints of links i, j, and k, respectively, and link vectors are represented by 2×1 matrices. Residual link length is the length from the link joints to the

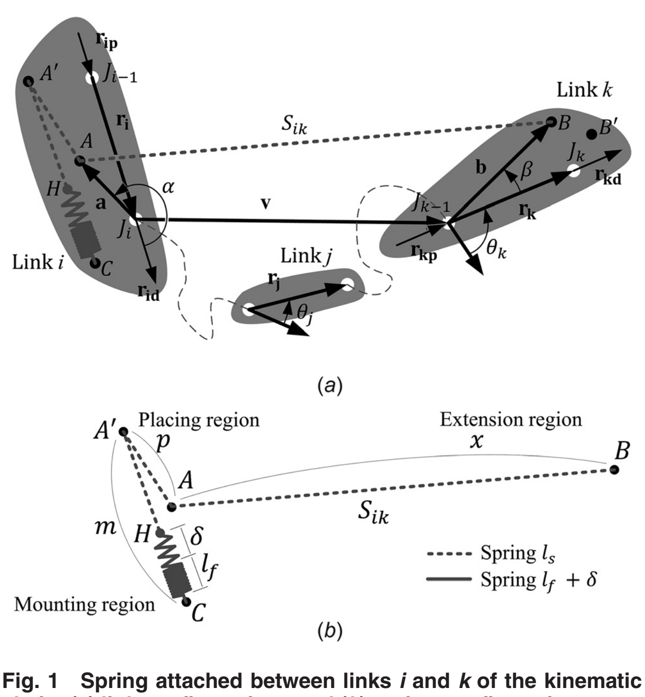

Fig. 1 Spring attached between links *i* and *k* of the kinematic chain: (*a*) link configurations and (*b*) spring configurations

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proximal and distal ends of the link and is represented by r_{ip} and r_{id}, respectively. The succeeding link $i+1$ with respect to its preceding link i can be represented using a 2×2 rotation matrix with joint angle θ_{i+1}, which is measured counterclockwise from $\mathbf{r}_i$ to $\mathbf{r}_{i+1}$. One mean is used to describe the links between link i and k as a virtual link v, which is variable in length, with link vector $\boldsymbol{v}$ from J_i to J_{k-1}, that is, $\mathbf{v} = \mathbf{r}_{i+1} + \cdots + \mathbf{r}_j + \cdots + \mathbf{r}_{k-1}$.

In Fig. 1(*b*), a spring is labeled S_{ik}, free length as l_f, string length as l_s, and spring elongation as δ. The two ends are hinged to links i and k, and the spring coefficient is k_{ik}. If links i and k are contiguous, the spring is a mono-articular spring; otherwise, it is a multi-articular spring. Point H is fixed on the spring for arranging strings; points A, A', B, B', and C are fixed on the links. The point A is a turning point, and A' is a pseudoturning point dividing the spring into three regions, as shown in Fig. 1(*b*). The first region is called the "mounting region" with mounting length m, which is the distance between A' and C; it contains the spring and a part of the string. The second is the "extension region" between separate links in a serial-type manipulator with extension x during motion, which includes the remainder of the string. The turning points are not immutable. The mounting region can be arranged on the distal side such as link k to obtain the turning point and the pseudoturning as B and B', respectively. Furthermore, the mounting region can be arranged elastically on the other links. The third region is the "placing region," represented by p, which is the distance between the two turning points or the two aforementioned regions. The placing length of springs can be theoretically expressed using the following equations when springs are arranged on the proximal and distal side of the manipulator, respectively,

$$p = \overline{A'A} \tag{1a}$$

$$p = \overline{B'B} \tag{1b}$$

Under tension, the length of a spring equals the sum of its free length and elongation. Therefore, the length of the spring–string arrangement can be calculated, i.e., $\delta + l_f + l_s$. By the sum of regions' lengths and the placing length, that is, $x + m + p$, the elongation equation for a springs is derived and expressed in Eq. (2)

$$\delta = x + m + p - l_f - l_s \tag{2}$$

$$m + p - l_f - l_s = 0 \tag{3}$$

A spring is called a ZFL spring if δ equals extension x. The configurations of non-ZFL springs for ZFL characteristics can then be derived by eliminating the nonzero term in Eq. (2). The necessary condition for ZFL characteristics (Eq. (3)) is a function of the mounting length, free length, string length, and placing length.

With the spring configurations for ZFL characteristics built, the elongation of springs in Eq. (2) can be represented as Eq. (4*a*). The extension x is determined by link kinematics and described as a function of link vectors $\mathbf{v}$ and attachment vectors $\mathbf{a}$, $\mathbf{b}$. Attachment vectors are used to describe the positions of points A and B with respect to the link joints J_i and J_{k-1}. α and β are the rotation angles of the attachment vectors measured counterclockwise from $\mathbf{r}_i, \mathbf{r}_k$ to $\mathbf{a}$, $\mathbf{b}$, respectively. $\mathbf{a}$ and $\mathbf{b}$ can be represented by rotation matrices in Eq. (4*b*) and unit vectors of links, that is, $\mathbf{a} = a\mathbf{R}(\alpha)\hat{\mathbf{r}}_i$ and $\mathbf{b} = b\mathbf{R}(\beta)\hat{\mathbf{r}}_k$. ZFL characteristics can be achieved even if a spring is installed at a rotation angle with respect to the line passing through joint centers

$$\delta = x = \sqrt{(\mathbf{v} + \mathbf{b} - \mathbf{a}) \cdot (\mathbf{v} + \mathbf{b} - \mathbf{a})} \tag{4a}$$

$$\mathbf{R}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}, \quad \mathbf{R}(\beta) = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \tag{4b}$$

and

$$\delta = \sqrt{v^2 + b^2 + a^2 + 2vb(\hat{\mathbf{v}}^T\mathbf{R}(\beta)\hat{\mathbf{r}}_k) - 2va(\hat{\mathbf{v}}^T\mathbf{R}(\alpha)\hat{\mathbf{r}}_i) + 2ab(\hat{\mathbf{r}}_i\mathbf{R}(\alpha - \beta)\hat{\mathbf{r}}_k)} \tag{4c}$$

However, the necessary condition (Eq. (3)) is unconstrained, which may yield infeasible solutions, such as the problem of springs bent at turning points, if the mounting length is shorter than the sum of the elongation and free length of the springs, that is, $m < \delta + l_f$.

To ensure that the springs do not bend, the constraint is that the mounting length of a spring should be longer than the sum of its elongation and free length at any time, that is, $m \geq \delta + l_f$, so that an adequate workspace for springs is available. Because the elongation varies, one conservative design is set with the constraint of maximum elongation $\delta_{\max}$ rather than tracing the variation of elongations, as expressed by Eq. (5*a*). The string length l_s is then given by Eq. (5*b*), obtained by substituting Eq. (5*a*) into Eq. (3). Two available choices for designers to arrange non-ZFL springs for ZFL characteristics with the spring motion unlimited are shown. Subsequently, the minimum string length or mounting length can be determined

$$m \geq \delta_{\max} + l_f \tag{5a}$$

$$l_s \geq \delta_{\max} + q \tag{5b}$$

In Eq. (5*b*), the string length is proportional to the placing length. Intentionally minimized the placing length helps reduce the string length. The spring configurations for the minimized placing length are determined by three parameters, namely, the length of the spring attachment vectors a and b and the pseudomounting length m'. The pseudomounting length is the maximum length available for the springs to be arranged with a minimal placing length and is theoretically limited by the link configurations. The magnitude of the pseudomounting length is based on residual link length defined away from link joints to link's proximal or distal ends (e.g., r_{ip} and r_{id} in Fig. 1(*a*)). An appropriate link configuration has a long pseudomounting length, and candidates for the pseudomounting length can be described using Eqs. (6*a*) and (6*b*) for scenarios where the springs are arranged on adjacent links, as shown in Fig. 2

$$m'_{i-1} = \max(r_{(i-1)p} + r_{i-1}, r_{(i-1)d}) \tag{6a}$$

$$m'_{i+1} = \max(r_{(i+1)p}, r_{i+1} + r_{(i+1)d}) \tag{6b}$$

The pseudomounting length for the springs on proximal (distal) links can be realized by giving a vertical distance from spring to the link that shortens the placing length such that p equals $a\sin\alpha$ ($b\sin\beta$), that is to say, the vertical distance is orthogonal to the link i. The pseudomounting length is therefore solved as the longest length measured from the point of vertical intersection. However, this leads to various placing length scenarios, making the

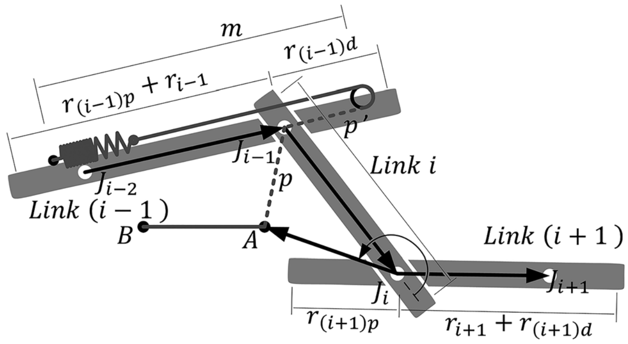

Fig. 2 Mounting length and pseudomounting length of springs on the adjacent links

equations more complex because the rotation angle can be any value, as shown in Fig. 3.

Two scenarios of pseudomounting lengths for arranging springs on proximal and distal links are as expressed in Eqs. (7*a*) and (7*b*), respectively. If the point of vertical intersection is located on the links, the pseudomounting length can be determined as the longest length of the two sections separated by the point. For scenarios where the point of vertical intersection of the spring is not located on the links, the total link length can be utilized fully as the pseudomounting length. Depending on the rotation angle of the springs, the pseudomounting length with the aligned or unaligned springs can also be classified. The pseudomounting length of the springs is presented in Table 1

$$m'_i = \max(r_{ip} + r_i + a\cos\alpha, r_{id} - a\cos\alpha) \quad \text{or} \quad r_{ip} + r_i + r_{id} \tag{7a}$$

$$m'_k = \max(r_{kp} + b\cos\beta, r_k + r_{kd} - b\cos\beta) \quad \text{or} \quad r_{kp} + r_k + r_{kd} \tag{7b}$$

According to the spring configurations on the proximal link, distal link, or adjacent links, the minimization of the placing length on the basis of the pseudomounting length can be described. Moreover, two scenarios—where the spring attachment vectors are either aligned or not aligned on the links—are discussed separately. Table 2 is a reference table for all the cases of placing length equations and classifications. Gray table cells are cases with an overextending mounting length, i.e., $m > m'$, whereas the other cells are cases where $m < m'$. A function is utilized for simplifying the law of cosines such that $\mathrm{Loc}(a, r, \alpha)$ represents $\sqrt{a^2 + r^2 - 2ar\cos\alpha}$, as shown in Table 2.

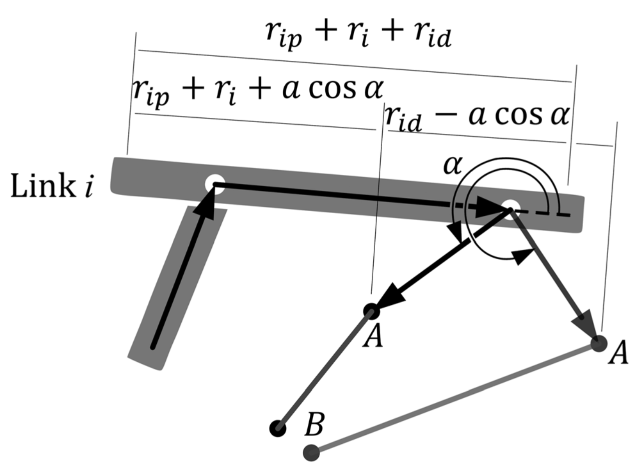

Fig. 3 Mounting length and pseudomounting length of springs on the proximal link

2.2 Example of Spring Configurations for ZFL Characteristics. An example of spring configurations for ZFL characteristics is introduced herein, as shown in Fig. 4. The parameters of the example from Ref. [10] are shown and summarized in Table 3, by using the definitions presented in this paper. Springs are idealized such that the free length l_f is 0 when not using strings, that is, l_s is 0. r_1, r_2, and r_3 of the links are 0.18, 0.18, and 0.07 m, respectively. For the practical configuration with the use of springs, the configurations should be modified such that spring–string arrangements are utilized or $l_s > 0$, and the springs have nonzero free length.

Using the aforementioned methods, the minimum required mounting length according to Eq. (5*a*) is assumed as expressed by Eq. (8*a*). For full rotation, the maximum elongation can be determined easily as the sum of links' lengths, as given by Eq. (4*c*). Because the free length is undetermined and the maximum elongation of the springs is proportional to their free length, maximum elongation is therefore considered as n times the free length, as expressed in Eq. (8*b*). In product lists of spring manufacturers and suppliers (Vanel,[2] GutekunstFedern,[3] and Lesjoforsab[4]), n equals 2–4 is the elongation of springs with respect to the free length without yielding, so n is assumed to equal 3 in this example especially to determine the mounting length according to Eq. (8*c*), which is thereby a function of the maximum elongation only

$$m = \delta_{\max} + l_f \tag{8a}$$

$$l_f = \frac{\delta_{\max}}{n} \tag{8b}$$

and

$$m = \frac{4\delta_{\max}}{3} \tag{8c}$$

With the solved mounting length, data in Table 2 can be used for designing the placing length p, which is based on the pseudomounting lengths acquired from the link configurations in Table 1. The difference in this example is that the springs are arranged on the same link for sharing the mounting region, for example, arranging S_{12} and S_{24} on link 2 and arranging S_{34} and S_{14} on link 3 (Table 4). This sharing configuration elongates the pseudomounting length for each spring configuration. The minimum placing length based on these conditions can then be redetermined using the new pseudomounting length. The placing length equations are chosen on the basis of the aligned case and are described as below

$$\begin{aligned} p_{12} &= m_{12} - m'_{12} \\ p_{24} &= m_{24} - m'_{24} \\ p_{34} &= m_{34} - m'_{34} \\ p_{14} &= m_{14} - m'_{14} + b_{14} \end{aligned} \tag{8d}$$

The pseudomounting lengths for the springs can be found in Table 4 according to the spring configuration. The residual link length of r_{2d} and r_{3p} is extended using springs 24 and 34, respectively. The length of r_{2p} and r_{3d} is zero in this example. Because the maximum elongations of the springs and the pseudomounting lengths are known, the minimum string length of the spring configurations can therefore be determined using Eqs. (8*e*) and (5*b*)

$$l_s = \delta_{\max} + p \tag{8e}$$

With the aforementioned configurations of the springs for ZFL characteristics, the springs can then be arranged practically on the manipulator. The practical configurations of springs for ZFL

[2] http://www.vanel.com/index.php?lang=english
[3] https://www.federnshop.com/en/
[4] http://www.lesjoforsab.com/default-uk.asp

Table 1 The pseudomounting length of springs on the basis of the mounting sides and the attachment links with the springs aligned or not aligned

Mounting side	Attachment link		Pseudomounting length (m') Not aligned ($\alpha, \beta \neq 0, \pi$)	Aligned ($\alpha, \beta = 0, \pi$)
Proximal (a)	Link i	$r_{id} - a\cos\alpha > 0$ and $r_{ip} + r_i + a\cos\alpha > 0$	$\max\left\{\begin{matrix} r_{ip} + r_i + a\cos\alpha \\ r_{id} - a\cos\alpha \end{matrix}\right\}$	$\max\left\{\begin{matrix} r_{ip} + r_i + a\cos\alpha \\ r_{id} - a\cos\alpha \end{matrix}\right\}$
		$r_{id} - a\cos\alpha < 0$ or $r_{ip} + r_i + a\cos\alpha < 0$	$r_{ip} + r_i + r_{id}$	
	Link $i-1$		$\max(r_{(i-1)p} + r_{i-1}, r_{(i-1)d})$	
	Link $i+1$		$\max(r_{(i+1)p}, r_{i+1} + r_{(i+1)d})$	
Distal (b)	Link k	$r_{kp} - b\cos\beta > 0$ and $r_k + r_{kd} + b\cos\beta > 0$	$\max\left\{\begin{matrix} r_{kp} + b\cos\beta \\ r_k + r_{kd} - b\cos\beta \end{matrix}\right\}$	$\max\left\{\begin{matrix} r_{kp} + b\cos\beta \\ r_k + r_{kd} - b\cos\beta \end{matrix}\right\}$
		$r_{kp} - b\cos\beta < 0$ or $r_k + r_{kd} + b\cos\beta < 0$	$r_{kp} + r_k + r_{kd}$	
	Link $k-1$		$\max(r_{(k-1)p} + r_{k-1}, r_{(k-1)d})$	
	Link $k+1$		$\max(r_{(k+1)p}, r_{k+1} + r_{(k+1)d})$	

characteristics, as shown in Fig. 5, calculated using the necessary conditions for the configurations and their constraints, given by Eqs. (8*b*)–(8*e*), are summarized and shown in Table 4. The mounting length, pseudomounting length, placing length, free length, and string length are all determined such that the springs are practical for applications with whose motions are assured. The practical configurations of the individual spring with the mounting and placing lengths are shown in Fig. 6.

3 Spring Configurations With Zero-Free-Length Characteristics

3.1 Reel-Based Wire Winding Configurations of Spring. The aforementioned example illustrates the practical arrangements of springs. However, the spring mounting region is invalid because of the overextended mounting region with respect to the links. The overextended regions hinder the motion of the manipulators because

Table 2 Placing length equations of springs based on the mounting sides and attachment links

Mounting side	Attachment link		Placing length q Not aligned ($\alpha, \beta \neq 0$ or π)	Aligned ($\alpha, \beta = 0$ or π)
Proximal (a)	Link i	$r_{id} - a\cos\alpha > 0$ and $r_{ip} + r_i + a\cos\alpha > 0$	$\lvert a\sin\alpha\rvert$	$m - m'$
			$\mathrm{LoC}\left(a\sin\alpha, (m - m'), \frac{\pi}{2}\right)$	
		$r_{id} - a\cos\alpha < 0$ or $r_{ip} + r_i + a\cos\alpha < 0$	$\min\left\{\begin{matrix} \mathrm{LoC}(a, r_{id}, \alpha) \\ \mathrm{LoC}(a, (r_i + r_{ip}), (\alpha - \pi)) \end{matrix}\right\}$	
			$\min\left\{\begin{matrix} \mathrm{LoC}(a, (m - r_i - r_{ip}), \alpha) \\ \mathrm{LoC}(a, (m - r_{id}), (\alpha - \pi)) \end{matrix}\right\}$	
	Link $i-1$		$\mathrm{LoC}(a, r_i, (\alpha - \pi))$	$\lvert r_i \pm a\rvert$
			$\mathrm{LoC}(a, r_i, (\alpha - \pi)) + (m - m')$	$\lvert r_i \pm a\rvert \pm (m - m')$
	Link $i+1$		a	a
			$a + (m - m')$	$a + (m - m')$
Distal (b)	Link k	$r_{kp} - b\cos\beta > 0$ and $r_k + r_{kd} + b\cos\beta > 0$	$\lvert b\sin\beta\rvert$	$m - m'$
			$\mathrm{LoC}\left(b\sin\beta, (m - m'), \frac{\pi}{2}\right)$	
		$r_{kp} - b\cos\beta < 0$ or $r_k + r_{kd} + b\cos\beta < 0$	$\min\left\{\begin{matrix} \mathrm{LoC}(b, (r_k + r_{kd}), \beta) \\ \mathrm{LoC}(b, r_{kp}, (\beta - \pi)) \end{matrix}\right\}$	
			$\min\left\{\begin{matrix} \mathrm{LoC}(b, (m - r_{kp}), \beta) \\ \mathrm{LoC}(a, (m - r_k - r_{kd}), (\beta - \pi)) \end{matrix}\right\}$	
	Link $k-1$		b	b
			$b + (m - m')$	$b + (m - m')$
	Link $k+1$		$\mathrm{LoC}(b, r_k, \beta)$	$\lvert r_k \pm b\rvert$
			$\mathrm{LoC}(b, r_k, \beta) + (m - m')$	$\lvert r_k \pm b\rvert \pm (m - m')$

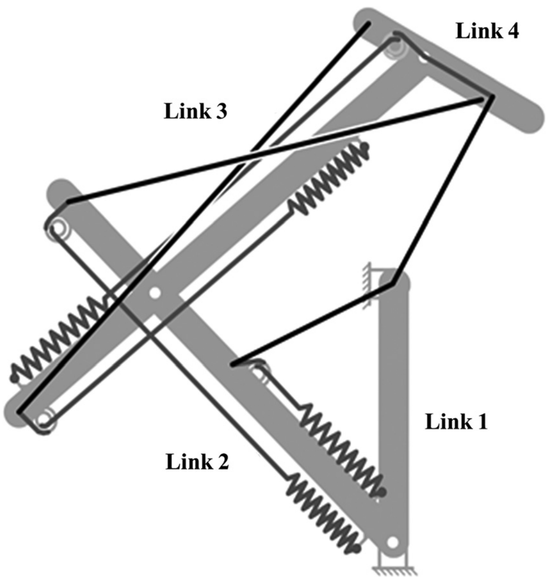

Fig. 4 Theoretical configurations of the springs

Table 3 Parameters for theoretical configurations of springs

S_{ik}	k(N/m)	a (m)	b (m)	α (rad)	β (rad)	$\delta_{\max}$(m)	l_f(m)	l_s(m)
12	93	0.134	0.120	π	0	0.254	0	0
24	320	0.072	0.045	0	0	0.297	0	0
34	400	0.262	0.035	π	π	0.297	0	0
14	128	0.134	0.045	π	0	0.539	0	0

Table 4 Practical configurations of springs for ZFL characteristics

S_{ik}	k(N/m)	m(m)	m'(m)		p(m)	l_f(m)	l_s(m)
12	93	0.34	0.28	$r_2 + r_{2d} - b_{12}$	0.06	0.08	0.31
24	320	0.40	0.25	$r_{2p} + r_2 + a_{24}$	0.15	0.10	0.45
34	400	0.40	0.26	$r_{3d} + a_{34}$	0.14	0.10	0.44
14	128	0.72	0.40	$r_{3p} + r_3 + r_{3d}$	0.37	0.18	0.91

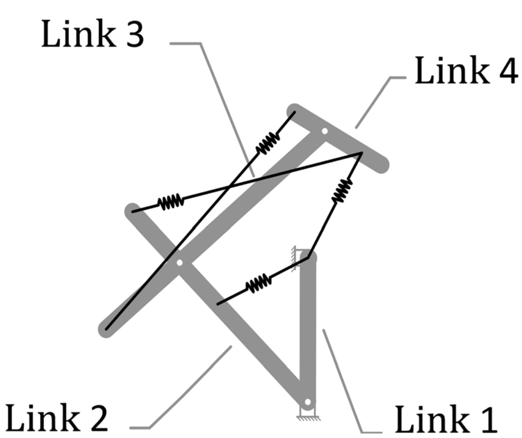

Fig. 5 Practical configurations of ZFL springs

the workspace of the distal link may be occupied. Therefore, a proper mounting length should be determined. The mounting length is a function of the spring elongation. To make the mounting region feasible for the given configurations, a pretension spring can be used to shorten the spring elongation while maintaining the spring force, as shown in Fig. 7(*a*). The pretension force can be achieved using winding springs [11].

However, the springs in this paper are not preloaded. With the configurations providing sufficient mounting length for the springs, a real extension-type spring can show ZFL characteristics (Figs. 7(*b*) and 7(*c*)). The length of linkage is proportional to the spring elongation. For the mounting regions, the spring curve can be realized as shown in Fig. 7(*c*), while that for the extension regions can be realized as shown in Fig. 7(*b*).

In this paper, the approach of scaling the elongation is called wire winding; this is different from special tension members such as tensegrity structures [12] and from pulley systems [13] used to ensure a constant wrapping angle during motion. Wire winding is employed in this paper to shorten the elongation of springs and to elongate the extension. Although this result can be achieved using sets of movable pulley systems, reels are employed in this paper. The benefit of using the reels is that only one reel is required for one spring; furthermore, the reels are attached to the links, leading to higher stability of the pulley systems. By using a reel which has two different sizes (e.g., R_w and R_s for the wire and spring, respectively; Fig. 8), the elongation of the springs can be reduced through shrink performance, s_r, which is a positive number, as described in Eq. (9*a*). To ensure consistent performance of the pull, the torque on the reels should be kept as usual. Therefore, spring coefficient k is enlarged as the square of s_r; that is, k becomes $s_r^2 k$, which can be achieved using parallel sets of springs.

The wire winding configuration decreases the size of the spring mounting region, as expressed by Eq. (9*b*), where the spring's mounting length is shorter than its pseudomounting length m' such that $m' \geq m$ always holds. To quantify the minimum performance of s_r, the maximum elongation, pseudomounting length, and spring free length are used as described in Eq. (9*c*). The string length considering the remaining space of the mounting region is derived as Eq. (9*d*)

$$s_r = \frac{R_w}{R_s} \tag{9a}$$

$$m' \geq m = \frac{\delta_{\max}}{s_r} + l_f \tag{9b}$$

and

$$s_r \geq \frac{\delta_{\max}}{m' - l_f} \tag{9c}$$

$$l_s = \delta_{\max} + p + (m' - m) \tag{9d}$$

3.2 Example of Spring Configurations With Wire Winding Configurations.

The configurations given in aforementioned example are infeasible because the mounting length of spring S_{14} is overextended, as shown in Fig. 6. By the wire winding configurations for the springs, the spring elongations can then be shortened and feasible for the springs to be arranged on the linkages or manipulators.

The shrink performance required for the elongation is realized according to Eq. (9*c*). Because the free length is assumed to be related to the elongation, the free length is also scaled by wire winding configurations, that is, $l_f \rightarrow l_f/ns$, and the shrink performance is then given by Eq. (10). Increasing n and m' and decreasing $\delta_{\max}$ are the options to minimize the boundary of the shrink performances. Since n is limited by the springs, and $\delta_{\max}$ is limited by the spring arrangements, without changing the arrangements of springs, the minimum shrink performances are determined and calculated with m' and $\delta_{\max}$, which are known from link kinematics and are listed in Table 4. With the performance, the spring coefficients, maximum elongation, and free length are all determined simultaneously. Furthermore, the mounting length can be calculated according to Eq. (9*b*); the string

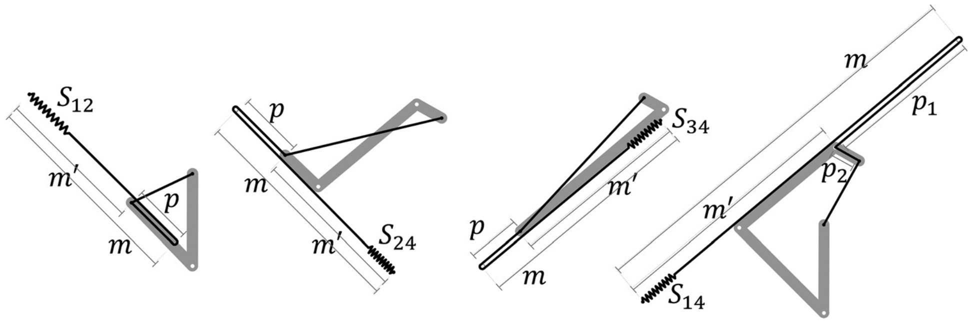

Fig. 6 Practical configurations of the individual spring for ZFL characteristics: (*a*) spring S_{12}, (*b*) spring S_{24}, (*c*) spring S_{34}, and (*d*) spring S_{14}

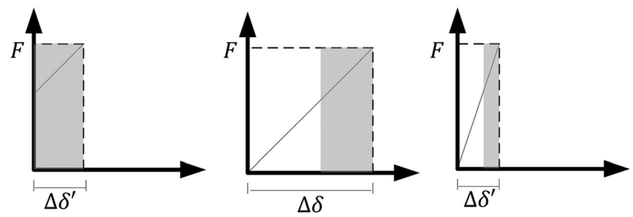

Fig. 7 Spring elongation and its workspace (in gray blocks) in three cases: (*a*) curve of pretension springs, (*b*) curve of original springs, and (*c*) curve of stronger springs

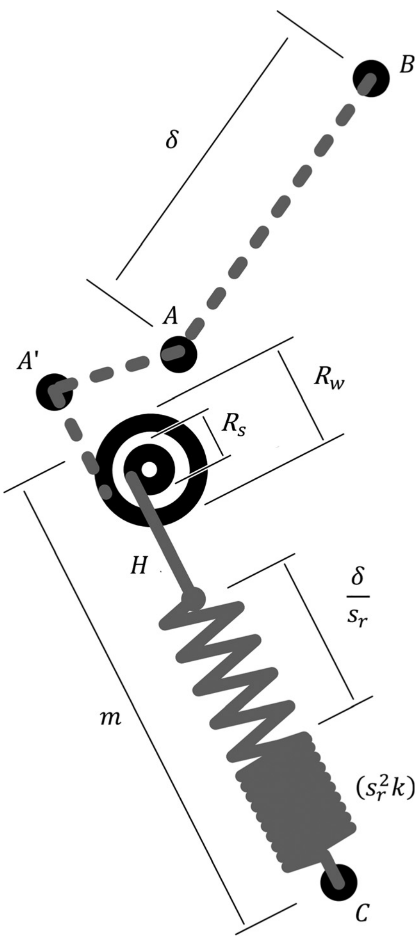

Fig. 8 Reel-based wire winding configurations

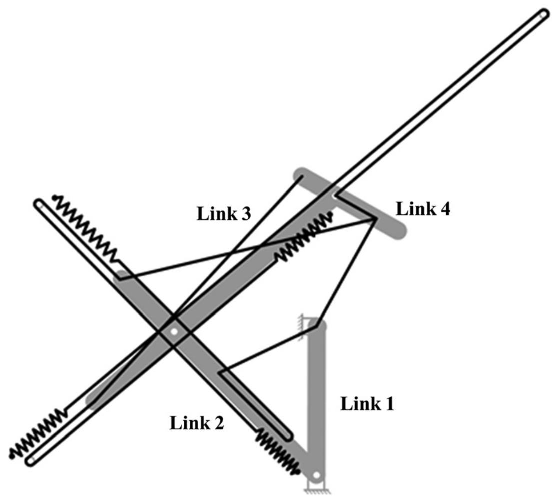

Fig. 9 Practical configurations of springs with the use of wire winding

length under this condition is given by Eq. (9*d*). The results for wire winding configurations are shown in Fig. 9, and the parameters are provided in Table 5

$$s_r \geq \frac{(n+1)\delta_{\max}}{nm'} = \frac{4\delta_{\max}}{3m'} \tag{10}$$

A comprehensive spring configuration for ZFL characteristics with unlimited motion is presented. The spring elongation is controlled. The overextended mounting length is also reduced. The pins for spring attachments points are utilized; hence, imperfections due to changes in wrapping angle throughout the motion of the manipulator can be omitted. With the wire winding methods, it is more practical and feasible for arranging springs on the linkages or manipulators. In practice, a wire is guided about a pulley, a finite radius causes a balancing error because the cable is not originating from a point but wraps from a circle, and the resulting balancers are not perfect in terms of balancing quality.

Table 5 Parameters of practical configurations of springs with the use of wire winding

S_{ik}	Shrink performances	k(N/m)	m(m)	m'(m)	p(m)	l_f(m)	l_s(m)
12	sr $\geq$ 2.57 (2.6)	629	0.13	0.13	0	0.03	0.25
24	sr $\geq$ 1.57 (1.6)	819	0.25	0.25	0	0.06	0.30
34	sr $\geq$ 1.51 (1.6)	1024	0.24	0.26	0	0.06	0.30
14	sr $\geq$ 2.74 (2.8)	1004	0.26	0.26	0.05	0.06	0.58

4 Conclusions

In this paper, comprehensive spring configurations for ZFL characteristics with unrestricted motion are presented. According to the spring configurations, with the necessary condition satisfied, the springs can possess ZFL characteristics. Furthermore, by mounting springs on purpose, a minimal placing length is left and the string is shortened, which further minimizes the string length of the springs. In wire winding configurations of the springs, the springs are organized such that the mounting length is reduced. The elongation is curtailed by the reels, enabling the arrangement of the springs on the links in an aligned manner such that the over-extended links and springs are eliminated. To summarize, these methods comprehensively establish spring configurations on manipulators, including mounting springs, placing springs, and tensioning springs with ZFL characteristics.

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