Mechanism and Machine Theory 130 (2018) 301–312

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Design of one DOF closed-loop statically balanced planar linkage with link-collinear spring arrangement

Chi-Shiun Jhuang, Young-Yu Kao, Dar-Zen Chen*

Department of Mechanical Engineering, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei 10617, Taiwan

ARTICLE INFO

Article history:
Received 28 December 2017
Revised 15 May 2018
Accepted 30 July 2018

Keyword:
Closed-loop
Balanced
Spring
Link-collinear

ABSTRACT

Several methods have been proposed for reducing the space required for a spanning-spring arrangement in a statically balanced mechanism (SBM) with auxiliary elements. This paper proposed a designing method to overcome the problem traditionally caused by the interference of the spanning-spring arrangement and the use of auxiliary elements. A prismatic joint is required for designing the proposed one DOF, closed-loop, planar linkage with a link-collinear spring arrangement. In our design, two prismatic joints are required for a closed-loop, planar four-link SBM to achieve static balance by deriving the formulation of elastic energy. Moreover, these two prismatic joints should be adjacent and perpendicular to each other. Furthermore, at least one of the prismatic joint must be adjacent to the ground link. On the basis of the design principles of the four-link SBM, a six-link SBM with two independent loops is also discussed. The ground link must be the link connected to the common joints and with the maximum number of ground-adjacent links. And one prismatic joint must be set as the common joint for reducing the number of the prismatic joints from four to three.

1. Introduction

A statically balanced mechanism (SBM) is a well-known mechanism for achieving the sum of the gravitational energy and elastic energy is constant. This mechanism is used to keep the position of payload, such as angle-poise lamp and surgical light, in its workspace by eliminating the influence of gravity [1–3]. The influence of gravity can be eliminated using an SBM, which can improve the efficiency of actuators through system control [4,5]. In addition, SBMs can be used in several applications (such as training or assistance devices) to resist or assist the movement of limbs by acting as a resistance force or resist gravity [6–9].

Static balance is typically achieved through friction, counterweight, and spring methods. However, the counterweight method increases mass and inertia in the system. The friction method would fail because of friction decays due to joint worn out. The spring method is based on the methodology of conservation of potential energy, in which static balance is achieved when the total gravitational energy and elastic energy of the system are perfectly balanced [10–13]. Consequently, the system has a smaller inertia than which is balanced by counterweight method.

Most design principles proposed in the literature focused on the open-loop mechanism along with the spanning-spring arrangement to overcome the influence of gravity. In the open-loop mechanism, the degree of freedom (DOF) can be increased easily; for example, the DOF of a mechanism increases by one when a link is added in the open-loop linkage [14].

* Corresponding author.
E-mail address: dzchen@ntu.edu.tw (D.-Z. Chen).

https://doi.org/10.1016/j.mechmachtheory.2018.07.021

The spanning-spring arrangement is characterized by the connectivity of the spanning structure among the links. Hence, this arrangement is convenient for achieving static balance by installing the spring between two arbitrary links. Lin et al. [15] proposed an admissible spring configuration matrix and reported the admissible configurations of the spanning-spring arrangement of open-loop mechanism for static balance.

However, the SBM has some limitations in practical applications: (1) the rigidity of SBM for open-loop structures is lower than that for closed-loop structures; (2) an SBM with a spanning-spring arrangement requires more space; (3) the spanning spring may interfere with other moving links during the movement of the linkage.

To overcome the aforementioned problems, some studies have proposed SBMs wherein the space required by the spanning spring is reduced using auxiliary elements to rearrange the spring configuration. Most of them have used cables and appropriate cam profiles to reconfigure the spanning springs [16–18]. Nakayama et al. [19] used cables and cams to investigate an SBM implemented in a gear system. The system comprised gears attached to a joint shaft with pulleys and crossing wires. Cho et al. [20] proposed a methodology for designing a bevel gravity compensator with a zero-free-length (ZFL) spring. In an auxiliary gear system with a joint, the spanning spring can be installed within the links; and the spanning spring works with the moving link. However, the use of auxiliary elements has some drawbacks. For example, (1) auxiliary elements impose additional loadings on the system; (2) motion interferences between the auxiliary elements and links may affect the workspace of the SBM; (3) the use of cables may result in a complicated spring configuration, and the cables have low reliability for long-term use.

Therefore, in this paper, a novel and promising SBM using compression springs is proposed the influence of gravity. The compression springs are parallel and coaxial with the moving link; thus, using the compression springs to achieve static balance avoids the aforementioned problems associated with the spanning-spring arrangement. Kim and Song investigated a two-link module with a compression spring for overcoming the influence of gravity [21]. The spring was compressed by a block pulled by the cable. Van Dorsser et al. [22] proposed a gravity equilibrator for adjusting the stiffness of the compression spring; here, a cable was attached to an inverted pendulum and placed on a pulley. Koser [23] reported a cam-type SBM with a compression spring in which the elastic energy is stored based on the designed cam profile. Takesue et al. [24] proposed a compensation mechanism for different gravities by using two 90°-phase-difference compression springs with a rotation follower. However, these studies have focused on the open-loop mechanism and the use of auxiliary elements in the design, and still suffer from the aforementioned problems associated with the open-loop mechanism and auxiliary elements.

In this paper, the method for designing a closed-loop SBM with a link-collinear spring and without auxiliary elements is proposed to enhance the rigidity of SBM and to avoid the use of auxiliary elements. By using the link-collinear spring arrangement, the problems of interference and lack of reliability of cables can be overcome. The remainder of this paper is organized as follows. Section 2 introduces the static balance of the SBM. The link vectors and joints in the system are defined, and the formulation of gravitational energy and elastic energy by using prismatic joint vectors is presented. Hence, the equation of total potential energy for the linkage is obtained. In Section 3, the determination of the link-collinear spring arrangement for the four-link SBM is derived according to the total potential energy equation. The admissible four-link SBM with link-collinear spring arrangement are obtained. In Section 4, on the basis of the design principles of the four-link SBM, the arrangement of link-collinear spring for a six-link SBM with two independent loops is determined. For the six-link linkage, the location of the ground link is determined using additional design rules, and the number of springs can be reduced by setting the prismatic joint as the common joint. In Section 5, the illustrative examples for the four- and six-link SBMs are presented, and the simulated results are presented to verify the design concept.

2. Potential energy of SBM with link-collinear springs

2.1. Position vector of mass center of revolute joint and prismatic joint

The gravitational energy of the system can be expressed as the scalar product of the vector of gravitational acceleration, the mass and position vector of the mass center, and applied to each of the system's rigid bodies. The coordinate system can be defined using complex number notations for vectors, the complicated algebraic operations of trigonometric functions can be avoided, and the derivations for both gravitational and elastic energy of the system can be simplified. Consider an arbitrary linkage for a planar n-link linkage as shown in Fig. 1; link u connecting the revolute joints or prismatic joint and the link vector $\boldsymbol{r_u}$ can be represented as

$$\boldsymbol{r_u} = |\boldsymbol{r_u}|e^{i\theta_u} \tag{1}$$

where $|\boldsymbol{r_u}|$ denotes the magnitude of $\boldsymbol{r_u}$ and is equal to the length of link u, and θ_u represents the orientation of vector $\boldsymbol{r_u}$ measured with respect to the negative imaginary axis. All orientations measured in this paper are counterclockwise.

Note that $\boldsymbol{r_u}$ is the link vector measured from the proximal joint or proximal end of the link to distal joint, where J_{u-1} is the proximal end of the link u − 1; J_u is the proximal revolute joint of link u or prismatic joint of link u; and J_{u+1} is the distal revolute joint of link u. Hereinafter, all vectors are represented in boldface characters.

Let us assume that the mass of the link is uniformly distributed; the mass center m_u is located at the geometric center of the link u. The coefficient of position vector of mass center p_u is $\frac{1}{2}$ and 1 for revolute joint and prismatic joint, respectively.

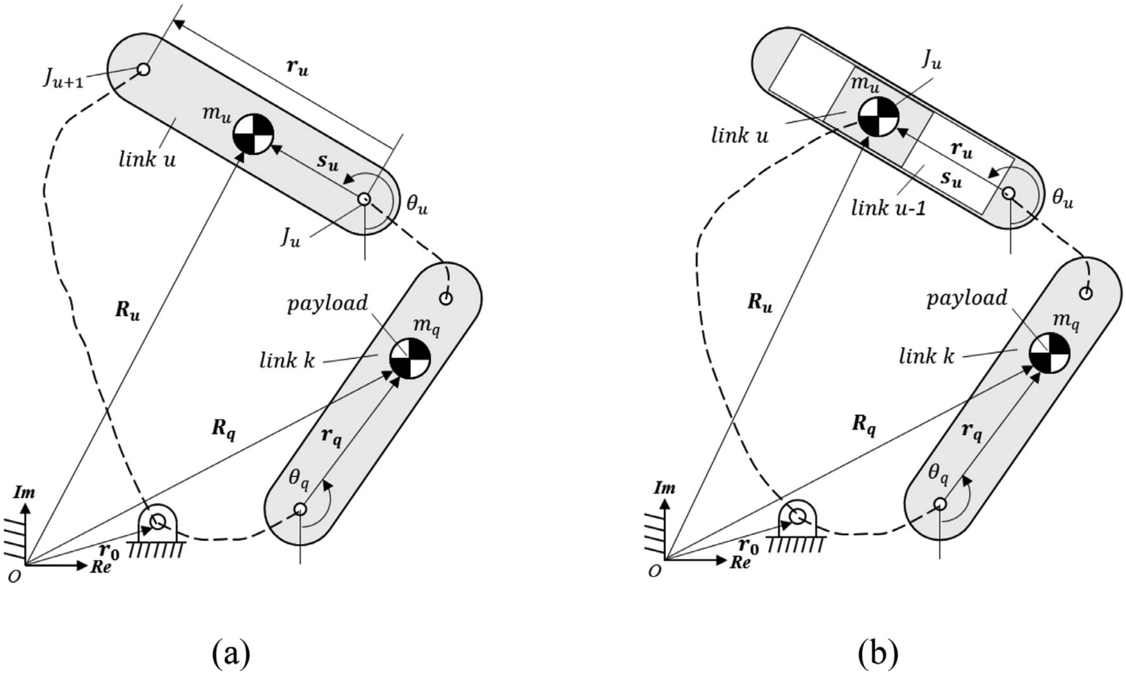


Fig. 1. Position vectors and joints of a planar linkage: (a) Link u connected with revolute joints; (b) Link u and link u − 1 connected with prismatic joint.

The position vector of mass center $\boldsymbol{s_u}$ from J_u or proximal end of link can be expressed as

$$\boldsymbol{s_u} = p_u|\boldsymbol{r_u}|e^{i\theta_u} \;\; p_u = \begin{cases} \frac{1}{2} & for\ R\ joint \\ 1 & for\ P\ joint \end{cases} \tag{2}$$

For n-link linkage, the position vector from the original point O to mass center $\boldsymbol{R_u}$ of each link can be expressed as

$$\boldsymbol{R_u} = \boldsymbol{r_o} + \sum_{j=1}^{u-1} \boldsymbol{r_j} + \boldsymbol{s_u} \tag{3}$$

where $\boldsymbol{r_o}$ is the vector from the original point O to the first joint. Substituting Eqs. (1) and (2) into Eq. (3), the position vector from the original point O to mass center can be rewritten as

$$\boldsymbol{R_u} = |\boldsymbol{r_o}|e^{i\theta_o} + \sum_{j=1}^{u-1} \left|\boldsymbol{r_j}\right|e^{i\theta_j} + p_u|\boldsymbol{r_u}|e^{i\theta_u} \tag{4}$$

2.2. Representation of gravitational energy

Considering a payload with mass m_q or a force F that equals the weight of payload acting on link k, using Eq. (4), the position vector from the original point O to the mass center of payload or force, $\boldsymbol{R_q}$, can be represented as

$$\boldsymbol{R_q} = |\boldsymbol{r_o}|e^{i\theta_o} + \sum_{j=1}^{k-1} \left|\boldsymbol{r_j}\right|e^{i\theta_j} + |\boldsymbol{r_q}|e^{i\theta_q} \tag{5}$$

The gravitational energy of n-link linkage and payload or force is the sum of the mass multiplied by the gravitational acceleration and the position vector from the original point O to the mass center of each link can be represented as

$$e_g = -\sum_{u=1}^{n} m_u\boldsymbol{g}\cdot\boldsymbol{R_u} - m_q\boldsymbol{g}\cdot\boldsymbol{R_q} \tag{6}$$

where gravitational acceleration $\boldsymbol{g}$ is a vector; and equals to $|\boldsymbol{g}|e^{i0}$.

Substituting Eqs. (4) and (5) into Eq. (6), the gravitational energy of n-link linkage and payload or force can be rewritten as

$$e_g = -\sum_{u=1}^{n}\left(m_u|\boldsymbol{g}||\boldsymbol{r_o}|\cos\theta_o + \sum_{j=1}^{u-1} m_u|\boldsymbol{g}|\left|\boldsymbol{r_j}\right|\cos\theta_j + p_u m_u|\boldsymbol{g}||\boldsymbol{r_u}|\cos\theta_u\right)$$

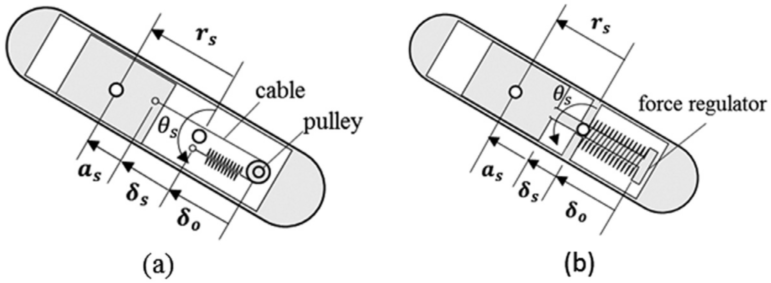


Fig. 2. Arrangement of ZFL, link-collinear spring: (a) Tension spring [25]; (b) Compression spring [26].

$$-m_q|\boldsymbol{g}|\left(|\boldsymbol{r_o}|\cos\theta_o+\sum_{j=1}^{\mathrm{k}-1}\left|\boldsymbol{r_j}\right|\cos\theta_j+|\boldsymbol{r_q}|\cos\theta_q\right) \tag{7}$$

where the gravitational energy of payload or force is q times the link vector of the link k, which is affected by the payload or force, and can be represented as

$$m_q|\boldsymbol{g}||\boldsymbol{r_q}|\cos\theta_q=q|\boldsymbol{r_k}|\cos\theta_k \tag{8}$$

where θ_q is an angle of link vector of payload, θ_k is an angle of link vector of the link acted by payload, and q is a coefficient equaling to $\frac{m_q|\boldsymbol{g}||\boldsymbol{r_q}|\cos\theta_q}{|\boldsymbol{r_k}|\cos\theta_k}$.

Substituting Eq. (8) into (7), the gravitational energy of n-link linkage and payload or force can be rewritten as

$$e_g=-c_0|\boldsymbol{r}_0|\cos\theta_o-\sum_{u=1}^{n}c_u|\boldsymbol{r_u}|\cos\theta_u-\sum_{j=1}^{k-1}c_q\left|\boldsymbol{r_j}\right|\cos\theta_j-\mathrm{q}|\boldsymbol{r_k}|\cos\theta_k \tag{9}$$

where $c_0=\left(m_q+\sum\limits_{u=1}^{n}m_u\right)|\boldsymbol{g}|,\ c_u=\left(\sum\limits_{j=u+1}^{n}m_j+p_u m_u\right)|\boldsymbol{g}|,\ c_q=m_q|\boldsymbol{g}|$

The constant coefficients c_0, c_u, and c_q represent the weight of each link or payload.

2.3. Elastic energy of link-collinear springs

The elastic energy of the system depends on the springs added in the mechanism. In this study, the ZFL springs were installed collinearly with the prismatic joint vectors; that is, the direction of extension or compression of spring is the same as the direction of motion of prismatic joint. The characteristics of the collinear ZFL spring can be determined with a specific arrangement with the nonzero-free-length spring. The sum of elastic energy of each ZFL, link-collinear spring installed in the prismatic joint can be expressed as

$$e_s=\sum_{s=1}^{\mathrm{n_p}}\frac{1}{2}K_s\left|\boldsymbol{\delta_s}\right|^2 \tag{10}$$

where K_s and $|\boldsymbol{\delta_s}|$ represent the stiffness and elongation of the spring, respectively; $\mathrm{n_p}$ represents the number of prismatic joints. By fitting a ZFL, tensional [25] and compression [26], linear spring in the prismatic joint, as shown in Fig. 2, the vector of elongation of the link-collinear tension spring can be written as

$$\boldsymbol{\delta_s}=\boldsymbol{r_s}-\boldsymbol{a_s} \tag{11}$$

where $\boldsymbol{r_s}$ is the vector of spring equaling to the vector of prismatic joint; $\boldsymbol{a_s}$ is the vector from attachment ends of the spring to prismatic joint.

Substituting Eq. (12) into Eq. (11), because the number of prismatic joints is equal to the number of springs, the elastic energy of n_p link-collinear springs can be expressed as

$$e_s=\sum_{s=1}^{\mathrm{n_p}}\frac{1}{2}K_s|\boldsymbol{r_s}-\boldsymbol{a_s}|^2=\sum_{s=1}^{\mathrm{n_p}}\frac{1}{2}K_s\left(|\boldsymbol{r_s}|^2-2|\boldsymbol{r_s}||\boldsymbol{a_s}|+|\boldsymbol{a_s}|^2\right) \tag{12}$$

2.4. Total potential energy of four-link SBM with link-collinear springs

Based on the principle of conservation of energy, full compensation of the gravity can be achieved by installing the springs. Static balance is achieved when the sum of the gravitational energy and elastic energy is constant. Therefore, by summing gravitational energy and elastic energy, the total potential energy of the system can be obtained, using Eqs. (9)–(12), as

$$e_{total}=-c_o|\boldsymbol{r_o}|\cos\theta_o-\sum_{u=1}^{n}c_u|\boldsymbol{r_u}|\cos\theta_u-\sum_{j=1}^{k-1}c_q\left|\boldsymbol{r_j}\right|\cos\theta_j-q|\boldsymbol{r_k}|\cos\theta_k+\sum_{s=1}^{n_p}\frac{1}{2}K_s\left(|\boldsymbol{r_s}|^2-2|\boldsymbol{r_s}||\boldsymbol{a}_s|+|\boldsymbol{a}_s|^2\right)=\text{const.} \tag{13}$$

The potential energy of link O and length of attachment ends of the link-collinear spring are constant; Eq. (13) can be rewritten as

$$e_{total}=-\sum_{u=1}^{n}c_u|\boldsymbol{r_u}|\cos\theta_u-\sum_{j=1}^{k-1}c_q\left|\boldsymbol{r_j}\right|\cos\theta_j-q|\boldsymbol{r_k}|\cos\theta_k+\sum_{s=1}^{n_p}\frac{1}{2}K_s\left(|\boldsymbol{r_s}|^2-2|\boldsymbol{r_s}||\boldsymbol{a}_s|\right)=\text{const.} \tag{14}$$

To achieve static balance, the total potential energy, which is the summation of the gravitational energy and elastic energy of the system, must be equal to a constant value for any configuration. The angle of all joints and displacement of prismatic joint are variable terms should be eliminated or kept a constant to let the total potential energy is constant. The summation of prismatic joint vectors in elastic energy has quadratic terms and linear terms. The summation of quadratic terms in elastic energy must be constant to ensure that the total potential energy is constant. Thus, this is a necessary condition for the summation of quadratic prismatic joint vectors. By deriving the necessary conditions for the summation of quadratic prismatic joint vectors, the link-collinear spring arrangement can be determined. The design equation of the stiffness and installed attachment ends for each spring can be derived by summing the linear prismatic joint vectors.

3. Arrangements of prismatic joints of the four-link SBM

3.1. Number of prismatic joints

To ensure that the summation of quadratic terms in Eq. (14) is constant, the number of prismatic joints for a four-link linkage, represented by n_p, must be determined. The condition of $n_p = 1$ is invalid for making the summation of quadratic term equal to a constant because the term of the prismatic joint vector represents the position of the prismatic joint and cannot equal to a constant or zero value for any configuration for the four-link linkage; that is, if there is only one prismatic joint, the summation of the quadratic term of total potential energy must be a variable value. Therefore, n_p cannot equal 1 for achieving a static balance.

A feasible condition is $n_p = 2$; the summation of two prismatic joint vectors can remain constant by maintaining a specific arrangement of the prismatic joints. The arrangement of link-collinear springs for the four-link linkage has the following rule.

R1: For a link-collinear spring arrangement in a four-link planar linkage, two prismatic joints are required to achieve static balance.

3.2. Configuration of the four-link SBM

For the four-link linkage, not considering the position of ground link in the graph representation, there are only two types of linkage: RRPP and RPRP. Assume that there are two revolute joints and two prismatic joints in the four-link SBM, R_1, R_2, P_1, and P_2 represent the revolute joints and two prismatic joints, and the payload is attached to link 2, which connects revolute joint, R_2, and prismatic joint, P_1. The total potential energy of the linkage and payload must be constant for the static balance to be expressed as

$$\begin{aligned}e_{total}=&-((c_1+m_q)|\boldsymbol{r}_{R_1}|\cos\theta_{R_1}+(q+c_2)|\boldsymbol{r}_{R_2}|\cos\theta_{R_2}+c_3|\boldsymbol{r}_{P_1}|\cos\theta_{P_1}+c_4|\boldsymbol{r}_{P_2}|\cos\theta_{P_2})\\&+\frac{1}{2}K_1\left(|\boldsymbol{r}_{P_1}|^2-2|\boldsymbol{r}_{P_1}||\boldsymbol{a}_{S_2}|\right)+\frac{1}{2}K_2\left(|\boldsymbol{r}_{P_2}|^2-2|\boldsymbol{r}_{P_2}||\boldsymbol{a}_{S_2}|\right)=\text{const.}\end{aligned} \tag{15}$$

Regarding the total potential energy, the angle of the revolute joints and prismatic joints, and displacement of prismatic joints are variable, and the total potential energy can be rewritten as

$$\begin{aligned}e_{total}=&-((c_1+m_q)|\boldsymbol{r}_{R_1}|\cos\theta_{R_1}+(q+c_2)|\boldsymbol{r}_{R_2}|\cos\theta_{R_2})+\frac{1}{2}\left(K_1|\boldsymbol{r}_{P_1}|^2+K_2|\boldsymbol{r}_{P_2}|^2\right)-|\boldsymbol{r}_{P_1}|(c_3\cos\theta_{P_1}+|\boldsymbol{a}_{S_1}|)\\&-|\boldsymbol{r}_{P_2}|(c_4\cos\theta_{P_2}+|\boldsymbol{a}_{S_2}|)=\text{const.}\end{aligned} \tag{16}$$

To balance the four-link SBM, first, the sum of quadratic terms of the prismatic joints must be constant, and other linear terms must also be constant, that is, the angles of the revolute joints and prismatic joints must be constant during the movement of the linkages. To ensure that the sum of quadratic terms is constant, the following expression can be used:

$$K_1|\boldsymbol{r}_{P_1}|^2+K_2|\boldsymbol{r}_{P_2}|^2=const. \tag{17}$$

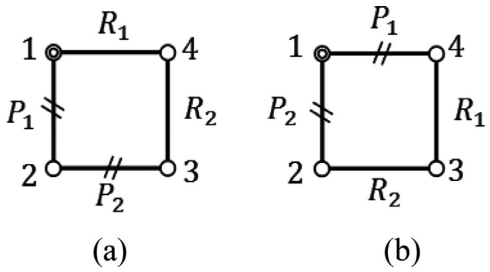


Fig. 3. Admissible arrangement of four-link SBM: (a) RRPP; (b) PRRP.

Dividing into K_1K_2, Eq. (17) can be rewritten as

$$\frac{|\boldsymbol{r}_{P_1}|^2}{\left(\sqrt{K_2}\right)^2}+\frac{|\boldsymbol{r}_{P_2}|^2}{\left(\sqrt{K_1}\right)^2}=const \tag{18}$$

Eq. (18) is an ellipse equation, which indicates that the orientation of these two prismatic joint vectors should be perpendicular to each other during the motion of the four-link linkage.

$$\theta_{P_1}=\theta_{P_2}\pm\frac{1}{2}\pi \tag{19}$$

Because there is always a constant phase difference between θ_{P_1} and θ_{P_2}, a simple approach to arrange these two prismatic joints is to make these two prismatic joints adjacent to each other in the graph representation. The relation of two prismatic joints for a four-link linkage has the following rule:

R2: For a link-collinear spring arrangement in a four-link planar linkage, the orientation of two prismatic joints should be perpendicular; therefore, these two prismatic joints are adjacent to each other.

By design rule **R2**, RPRP-type mechanism cannot achieve static balance with the link-collinear spring arrangement. Two methods can be adopted to ensure that the angles of the revolute joints and prismatic joints are constant: setting the joints adjacent to the ground link, and using a closure equation to change the position vector of one joint into the combinations of the position vector of other joints. Because the angle of the prismatic joints can be varied if they are not adjacent to the ground link, the relation of the ground link and two prismatic joints for a four-link linkage has the following rule:

R3: For link-collinear spring arrangement in a four-link planar linkage, at least one prismatic joint must be attached to the ground link to make the orientation equal to a constant.

Applying design rules **R1–R3**, the configuration of the four-link SBM are only RRPP and PRRP, as shown in Fig. 3.

Fig. 3. shows that revolute joint, R1, is adjacent to the ground link or between the ground-adjacent prismatic joints; thus, the angle of this joint is constant. Using closure equation to change the position vector of revolute joint, R2, into the combinations of position vector of other joints, can be represented as

$$|\boldsymbol{r}_{R_2}|\cos\theta_{R_2}=-(|\boldsymbol{r}_{R_1}|\cos\theta_{R_1}+|\boldsymbol{r}_{P_1}|\cos\theta_{P_1}+|\boldsymbol{r}_{P_2}|\cos\theta_{P_2}) \tag{20}$$

Substituting Eq. (20) into Eq. (16), the total potential energy can be rewritten as

$$\begin{aligned}e_{total}=&(\mathrm{q}+c_2-c_1-m_q)|\boldsymbol{r}_{R_1}|\cos\theta_{R_1}+\frac{1}{2}\left(K_1|\boldsymbol{r}_{P_1}|^2+K_2|\boldsymbol{r}_{P_2}|^2\right)+|\boldsymbol{r}_{P_1}|((\mathrm{q}+c_2-c_3)\cos\theta_{P_1}-K_1|\boldsymbol{a}_{S_1}|)\\&+|\boldsymbol{r}_{P_2}|((\mathrm{q}+c_2-c_4)\cos\theta_{P_2}-K_2|\boldsymbol{a}_{S_2}|)=\mathrm{const.}\end{aligned} \tag{21}$$

The angles of the revolute joint, prismatic joints, and sum of the quadratic terms are constant according to the aforementioned derived processes. To achieve static balance, the difference in constant coefficients and spring terms must be equal to zero because the displacement of prismatic joints is variable and can be represented as

$$(\mathrm{q}+c_2-c_3)\cos\theta_{P_1}-K_1|\boldsymbol{a}_{s_1}|=0 \tag{22a}$$

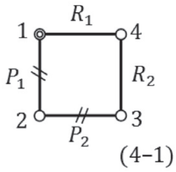
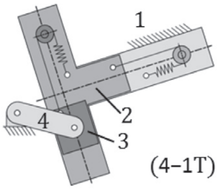
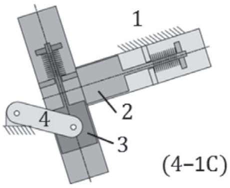
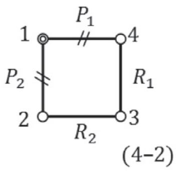
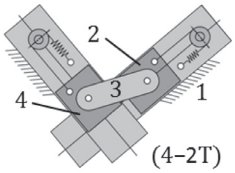
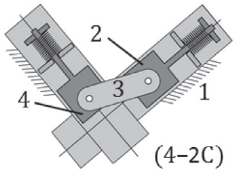
$$(\mathrm{q}+c_2-c_4)\cos\theta_{P_2}-K_2|\boldsymbol{a}_{s_2}|=0 \tag{22b}$$

The length of the attachment ends of the link-collinear spring, $|\boldsymbol{a}_s|$, equals to the sum of length of the attachment ends of the spring, and Eq. (23) can be rewritten as

$$|\boldsymbol{a}_{s_1}|=\frac{(\mathrm{q}+c_2-c_3)\cos\theta_{P_1}}{K_1} \tag{23a}$$

$$|\boldsymbol{a}_{s_2}|=\frac{(\mathrm{q}+c_2-c_4)\cos\theta_{P_2}}{K_2} \tag{23b}$$

Table 1
Admissible arrangement of link-collinear spring for the four-link SBM.

Graph representation	Tension spring	Compression spring
 (4–1)	 (4–1T)	 (4–1C)
 (4–2)	 (4–2T)	 (4–2C)

In case of three prismatic joints for the four-link linkage, two of the prismatic joints are used to make the summation of quadratic terms equal to a constant value, and remains a quadratic term of prismatic joint vector. This remaining prismatic joint causes the summation of quadratic terms of the system to not be equal to a constant similar to when $n_p = 1$. Therefore, the condition of $n_p = 3$ is also invalid to make the summation of quadratic prismatic joint vectors equal to a constant.

3.3. Admissible arrangement of four-link SBM

The admissible arrangements of link-collinear spring for the four-link linkage are derived as described in Section 3.2. Two feasible link-collinear springs balancing the four-link SBM are obtained and illustrated as the graph representation in Table 1, where the links and joints are represented by vertices and edges, respectively. The ground link is represented by double vertices and the link-collinear springs are represented by double slash lines. The ZFL tension spring and ZFL compression spring are arranged as illustrated in the table.

For the link-collinear spring arrangement, the use of a pulley and cable to obtain the characteristics of ZFL can be avoided using the compression spring. Hence, in this paper, all design examples use the compression spring to verify the results.

4. Arrangement of prismatic joints of the six-link SBM

4.1. Relation of redundant loop and ground link

Six-link linkage is of two types, Watt type and Stephenson type, each with independent loops. The joints connecting the two independent loops are defined as the common joints. For the Watt-type linkage, the two loops are connected by one common joint, whereas for the Stephenson-type linkage, the number of common joints is two. Unlike the four-link linkage, the location of the ground link for the six-link linkage must be identified because the location of the ground link for the six-link linkage is not uniquely determined. To exclude the redundant loop, the ground link must be set as the link that is connected to the common joints. That is, two independent loops of the six-link linkage should be connected by the ground link. To enhance the rigidity of the linkage, the maximum number of ground-adjacent links is required. Given these two reasons, the location of the ground link for the six-link linkage must adhere to the following rule:

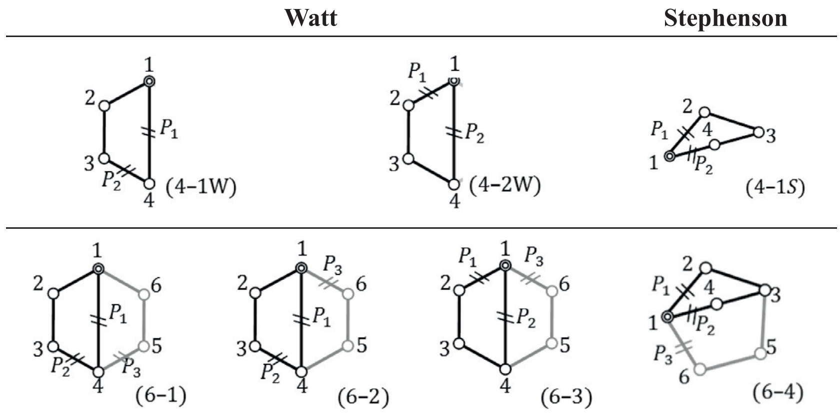
R4: The ground link must be the link connected to the common joints and with the maximum number of ground-adjacent links; that is, the ground link of the six-link linkage must have three ground-adjacent links.

4.2. Reducing the number of prismatic joints

For designing a SBM with link-collinear spring arrangement, prismatic joints are required for storing the elastic energy of the springs. As the number of prismatic joints increases, the arrangement of the link-collinear spring becomes more difficult because the planar linkage suffers from interference during configuration. To avoid this situation, the minimum number of prismatic joints required must be determined.

According to design rule **R1**, the total number of prismatic joints for the six-link SBM is determined to be four because each independent loop has two prismatic joints. By fitting the link-collinear springs between the links that are connected by the common joint, the prismatic joints are set as the common joints for the six-link linkage. These prismatic joint vectors work on two independent loops simultaneously. Although the total number of prismatic joints in the system can be reduced

Table 2
Arrangement of prismatic joints for Watt and Stephenson SBM.



by one, the number of prismatic joints in each independent loop remains two. Therefore, the following rule based on the reduced number of prismatic joints applies.

R5: One prismatic joint must be set as the common joint for reducing the number of the prismatic joints from four to three; that is, the number of prismatic joints must be three for the six-link linkage.

4.3. Admissible arrangement of prismatic joint for the Watt mechanism

The Watt mechanism has two independent four-link loops. Considering that the first four-link loop must follow the design rules **R1–R5**, two types of first four-link loop are feasible in the Watt mechanism. The light gray edges and vertices represent the second loop, which follows the design rules **R1–R5** and connects to the first loop. The results are illustrated in Table 2.

The two types of first four-link loop in the Watt mechanism can be used to derive four arrangements of prismatic joints; however, the two types of Watt mechanism SBM have the same arrangement, and the admissible arrangement of the Watt mechanism SBM has three types.

The results show that type 6-1 has a ground-adjacent prismatic joint; type 6-2 has two ground-adjacent prismatic joints; and all prismatic joints of type 6-3 are ground-adjacent prismatic joints. For rigidity of linkage, type 6-3 is better than type 6-1 and 6-2 because of the number of ground-adjacent prismatic joints. The admissible arrangements of link-collinear spring for the Watt mechanism are presented in Table 3.

4.4. Admissible arrangement of prismatic joint for the Stephenson mechanism

The Stephenson mechanism has one four-link loop and one five-link loop. Similar to the Watt mechanism, considering the first four-link loop that must follow the design rules **R1–R5**, there is only one type of first four-link loop in the Stephenson mechanism. Because Stephenson mechanism has two common joints, the second loop of Stephenson mechanism can achieve static balance by the following rule:

R6: The five-link loop of Stephenson mechanism has no more than two revolute joints in three joints which are not common joints.

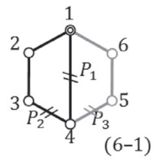
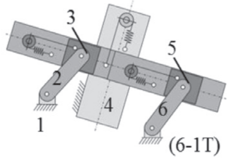
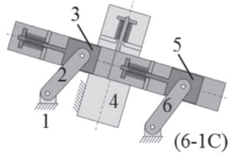
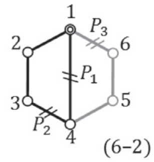
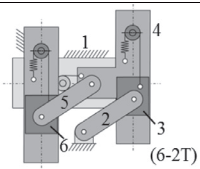
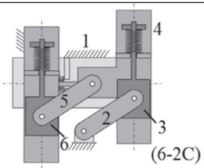
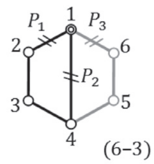
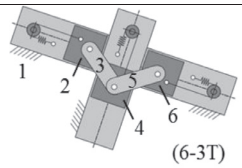
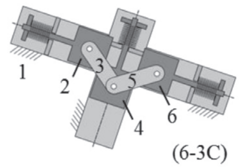
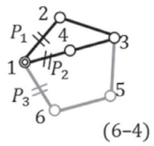
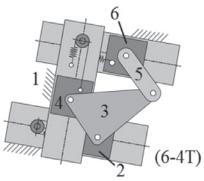
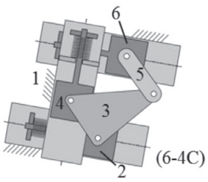
The light gray edges and vertices represent the second loop, which follows the design rules **R1–R6** and is connected to the first loop. The results are shown in Table 2. The results show that type (6-4) has three ground-adjacent prismatic joints; similar to type 6-3, the rigidity of linkage is also better than type 6-1 and 6-2. The admissible arrangements of the link-collinear spring for the Stephenson mechanism are presented in Table 3.

5. Illustrative examples

5.1. Example I: Four-link SBM with link-collinear spring arrangement

In real world, SBM suitable for many cases like TV mounts and console of surgical light, which let the devices operate easily by users because their gravity effect is eliminated. Consider the admissible arrangement of link-collinear spring for the four-link SBM shown in Section 3. The arrangement (4-2C) is considered as an illustrative example (Fig. 4); the dimensions and mass properties of the four-link linkage and payload are shown in Table 4. The results of stiffness and attachment points of the springs, obtained using the equations derived in Section 3, are shown in Table 5. The stiffness and one attachment point of springs can be set arbitrarily.

Table 3
Admissible arrangement of link-collinear spring for Watt and Stephenson SBM.

Graph representation	Tension spring	Compression spring
 (6–1)	 (6-1T)	 (6-1C)
 (6–2)	 (6-2T)	 (6-2C)
 (6–3)	 (6-3T)	 (6-3C)
 (6–4)	 (6-4T)	 (6-4C)

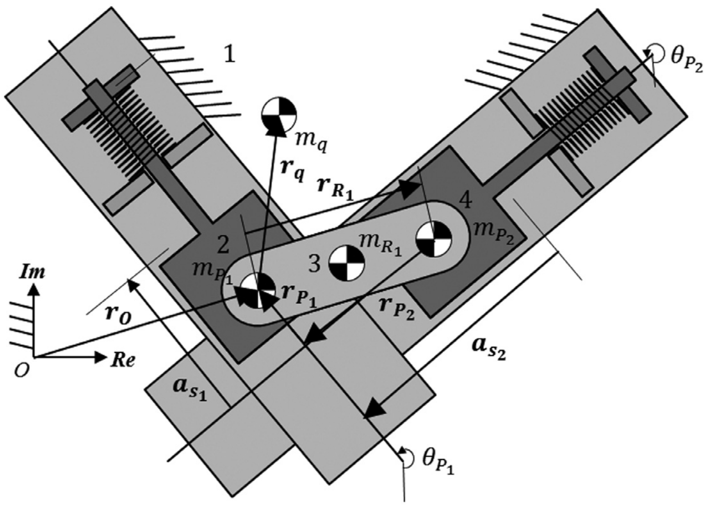


Fig. 4. Four-link SBM with two link-collinear springs.

Table 4
Dimensions and mass properties of the four-link SBM.

Link	Dimension (m)	Mass (kg)
1	$\|r_{R_2}\| = 0$	–
2	$\|r_{P_1}\| =$ variable	2
3	$\|r_{R_1}\| = 0.7$	1
4	$\|r_{P_2}\| =$ variable	−2
Payload	$\|r_q\| = 0.80$	25

Table 5
Parameters of the link-collinear springs for four-link SBM.

Stiffness (N/m)	Angle (degree)	Attachment points (m)
$K_1 = 500$	$\theta_{P_1} = 225$	$\lvert a_{s_1}\rvert = 0.1$
$K_2 = 500$	$\theta_{P_2} = 315$	$\lvert a_{s_2}\rvert = 0.31$

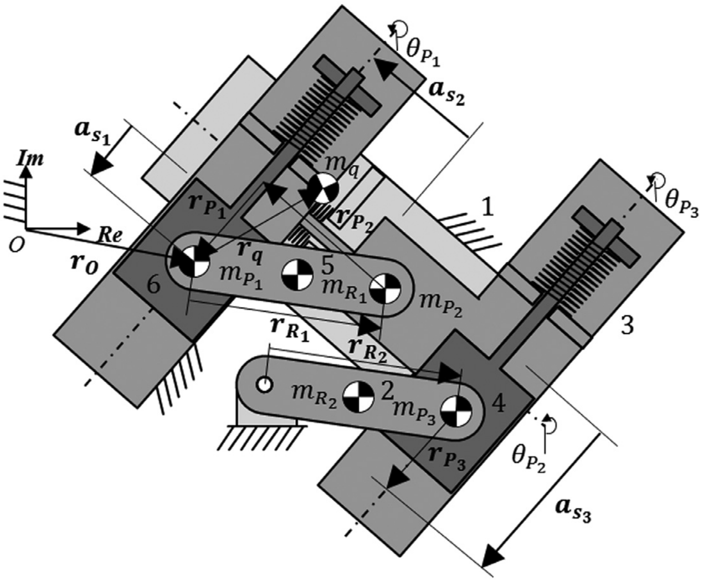


Fig. 5. Stephenson-type six-link SBM with two link-collinear springs.

Table 6
Dimensions and mass properties of the Stephenson-type six-link SBM.

Link	Dimension (m)	Mass (kg)
1	$\lvert \boldsymbol{r}_{R_3}\rvert = 0$	–
2	$\lvert \boldsymbol{r}_{R_2}\rvert = 0.\mathbf{7}$	−1
3	$\lvert \boldsymbol{r}_{P_2}\rvert =$ variable	−2
4	$\lvert \boldsymbol{r}_{P_3}\rvert =$ variable	−5
5	$\lvert \boldsymbol{r}_{R_1}\rvert = 0.7$	1
6	$\lvert \boldsymbol{r}_{P_1}\rvert =$ variable	−2
Payload	$\lvert \boldsymbol{r}_{q}\rvert = 0.80$	25

Table 7
Installed parameters of the link-collinear springs for Stephenson-type six-link SBM.

Stiffness (N/m)	Angle (degree)	Attachment points (m)
$K_1 = 250$	$\theta_{P_1} = 315$	$\lvert a_{s_1}\rvert = 0.20$
$K_2 = 500$	$\theta_{P_2} = 225$	$\lvert a_{s_2}\rvert = 0.35$
$K_3 = 250$	$\theta_{P_3} = 315$	$\lvert a_{s_3}\rvert = 0.12$

5.2. Example II: Stephenson-type six-link SBM with link-collinear spring arrangement

Consider arrangement (6-2C) as an illustrative example. The arrangement (6-2C) with link vectors is shown in Fig. 5, and the dimensions and mass properties of the four-link linkage are listed in Table 6. The results of stiffness and attachment points of springs, obtained using the equations derived in Section 3, are listed in Table 7. The stiffness and one attachment point of the springs can be set arbitrarily.

The model was simulated using Matlab® software; the gravitational energy function and elastic energy function for four-link and six-link SBMs are plotted in Fig. 6(a) and (b), respectively. Fig. 6 shows that the gravitational energy and elastic energy are transferred completely, and the total potential energy remains constant. For the four-link SBM with link-collinear

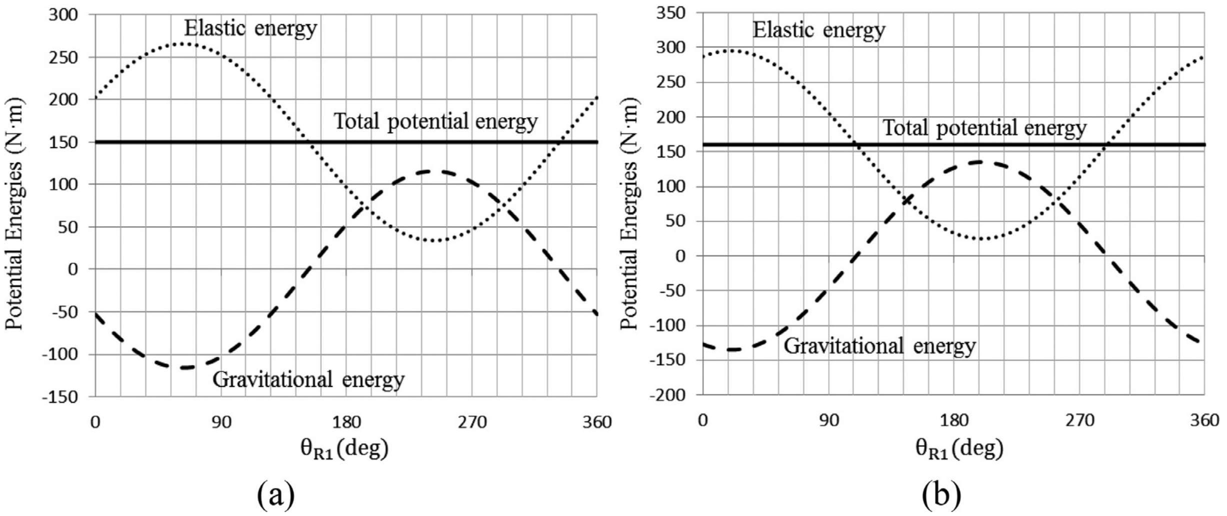


Fig. 6. Simulated potential energies of the illustrated examples: (a) four-link SBM with link-collinear springs; (b) six-link SBM with link-collinear springs.

springs, the total potential energy is equal to 149.863 J, and the total potential energy of the six-link SBM with link-collinear springs is equal to 160.137 J.

In general, the Watt-type six-link linkage with only three link-collinear springs cannot achieve static balance. However, because of setting the prismatic joints as the common joints, the link-collinear spring connected between two independent loops works simultaneously during the motion of the linkage. In practice, SBMs with link-collinear springs can be obtained as long as the design principles are applied.

6. Conclusion

This paper presents a design methodology for designing a one DOF closed-loop, planar linkage with link-collinear spring arrangement. The link-collinear spring arrangement avoids the problems of interference by using the spanning-spring arrangement. The formulations of gravitational and elastic potential energy for the n-link SBM with prismatic joints are introduced. Moreover, the static balance formulations of the four-link SBM are derived. The admissible arrangement of link-collinear spring for the four-link SBM is determined with the necessary conditions. We conclude that two prismatic joints that are perpendicular to each other are required for a four-link SBM. Furthermore, at least one of the prismatic joint must be adjacent to the ground link. On the basis of the design principles of the four-link SBM, we further discuss the link-collinear spring arrangement of the six-link SBM. To avoid the redundant loop, the location of ground link is determined to be the link with three ground-adjacent links. Moreover, reducing the number of prismatic joints is derived by setting the prismatic joint as the common joint; that is, the number of prismatic joints of the six-link SBM is three. Five-link loop of Stephenson mechanism has no more than two revolute joints in three joints which are not common joints. On the basis of the aforementioned design principles, we can determine the arrangement of link-collinear spring in the six-link linkage. The Watt mechanism has three types of SBM for one, two, and three ground-adjacent prismatic joints, respectively. The Stephenson mechanism has only one type of SBM for three ground-adjacent prismatic joints. Finally, a four-link and six-link SBM are illustrated with examples; the computer-simulated results verified the static balance of the illustrated examples, and the potential energies were estimated accordingly.

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