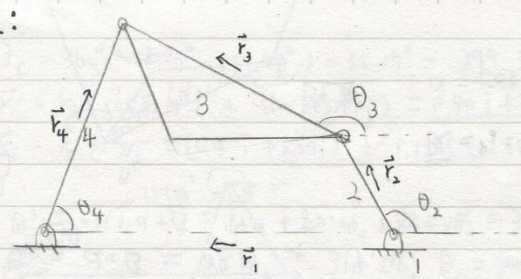
機構設計 HW1 HW2 參考

(參考代表作法與完整度符合預期作業完成的程度，並非正確解答)

HW1：

Hw 1:



$|\vec{r}_1| = 3.8$ $\quad\theta_2 = 125°$

$|\vec{r}_2| = 1.2$ $\quad\theta_1 = 180°$

$|\vec{r}_3| = 3$

$|\vec{r}_4| = 2.3$

(1) $\vec{r}_1 + \vec{r}_4 = \vec{r}_2 + \vec{r}_3$

$$\Rightarrow r_1(\cos\theta_1 i + \sin\theta_1 j) + r_4(\cos\theta_4 i + \sin\theta_4 j) = r_2(\cos\theta_2 i + \sin\theta_2 j) + r_3(\cos\theta_3 i + \sin\theta_3 j)$$

$$\Rightarrow \begin{cases} r_1\cos\theta_1 + r_4\cos\theta_4 = r_2\cos\theta_2 + r_3\cos\theta_3 \\ r_1\sin\theta_1 + r_4\sin\theta_4 = r_2\sin\theta_2 + r_3\sin\theta_3 \end{cases}$$

$$\Rightarrow \begin{cases} r_3\cos\theta_3 = r_1\cos\theta_1 + r_4\cos\theta_4 - r_2\cos\theta_2 & \text{— ①} \\ r_3\sin\theta_3 = r_1\sin\theta_1 + r_4\sin\theta_4 - r_2\sin\theta_2 & \text{— ②} \end{cases}$$

$$①^2 + ②^2 \Rightarrow r_3^2 = r_1^2 + r_4^2\cos^2\theta_4 + r_2^2\cos^2\theta_2 + 2r_1r_2\cos\theta_2 - 2r_1r_4\cos\theta_4 - 2r_4r_2\cos\theta_4\cos\theta_2$$

$(\theta_1 = 180°)$

$$+ r_4^2\sin^2\theta_4 + r_2^2\sin^2\theta_2 - 2r_2r_4\sin\theta_2\sin\theta_4$$

$$\Rightarrow 0 = r_1^2 + r_2^2 + r_4^2 - r_3^2 + 2r_1r_2\cos\theta_2 + (-2r_2r_4\sin\theta_2\sin\theta_4) + (-2r_1r_4 - 2r_2r_4\cos\theta_2)\cos\theta_4$$

令
$$\begin{cases} A = -2r_1r_4 - 2r_2r_4\cos\theta_2 \\ B = -2r_2r_4\sin\theta_2 \\ C = r_1^2 + r_2^2 + r_4^2 - r_3^2 + 2r_1r_2\cos\theta_2 \end{cases} \Rightarrow A\cos\theta_4 + B\sin\theta_4 + C = 0 \quad \text{— ③}$$

$$\because \sin\theta_4 = \frac{2\tan(\theta_4/2)}{1+\tan^2(\theta_4/2)}, \quad \cos\theta_4 = \frac{1-\tan^2(\theta_4/2)}{1+\tan^2(\theta_4/2)}, \quad 令\ t = \tan\frac{\theta_4}{2}$$

$$\therefore ③ \Rightarrow A\frac{1-t^2}{1+t^2} + B\frac{2t}{1+t^2} + C = 0$$

$$\Rightarrow A - At^2 + 2Bt + C + Ct^2 = 0 \Rightarrow (C-A)t^2 + 2Bt + (A+C) = 0$$

$$t = \frac{-2B \pm \sqrt{4B^2 - 4C^2 + 4A^2}}{2(C-A)} = \frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{C-A}$$

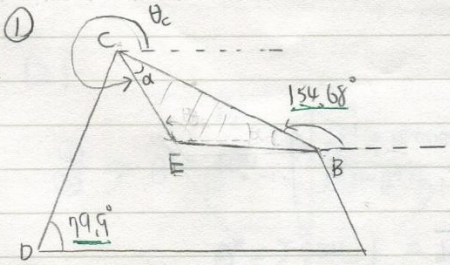
代入已知數值：
$$A = -2\times3.8\times2.3 - 2\times1.2\times2.3\times\cos125° = -14.314$$
$$B = -2\times1.2\times2.3\times\sin125° = -4.522$$
$$C = 3.8^2 + 1.2^2 + 2.3^2 - 3^2 + 2\times3.8\times1.2\times\cos125° = 6.939$$
$$t = \frac{4.522 \pm \sqrt{4.522^2 - 6.939^2 + 14.314^2}}{2(6.939 + 14.314)} = 0.839,\ -0.414$$

$$\theta_4 = 2\tan^{-1}t = \underline{79.9°},\ \underline{-44.9°}$$

$$\frac{②}{①} \Rightarrow \theta_3 = \tan^{-1}\left[\frac{r_4\sin\theta_4 - r_2\sin\theta_2}{-r_1 + r_4\cos\theta_4 - r_2\cos\theta_2}\right] = -25.319°,\ 60.369° \leftarrow \text{Wrong}$$

$$\underline{154.68°},\ \underline{240.369°} \leftarrow \text{Correct}$$

①



$\overline{BC} = r_3 = 3$, $\overline{BE} = 2.05$, $\overline{CE} = 1.35$

$\overline{BE}^2 = \overline{CE}^2 + \overline{BC}^2 - 2 \times \overline{CE} \times \overline{BC} \times \cos\alpha$

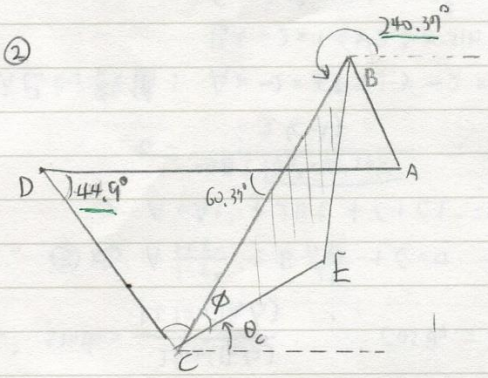
$\Rightarrow \alpha = \cos^{-1}\left(\dfrac{2.05^2 - 1.35^2 - 3^2}{-2 \times 1.35 \times 3}\right) = 35.19°$

$\theta_c = 360° - (180° - 154.68°) - 35.19° = 299.49°$

$\vec{r}_c = r_4(\cos\theta_4 i + \sin\theta_4 j) = 2.3\cos 79.9° i + 2.3 \sin 79.9° j = \underline{0.4i + 2.26j}$ C

$\vec{r}_E = \vec{r}_c + \vec{r}_{E/c} = (0.4i + 2.26j) + (1.35\cos 299.49° i + 1.35 \sin 299.49° j) = \underline{1.06 i + 1.09 j}$ E

②



$\phi = \alpha = 35.19°$

$\therefore \theta_c = 60.39° - \phi = 25.18°$

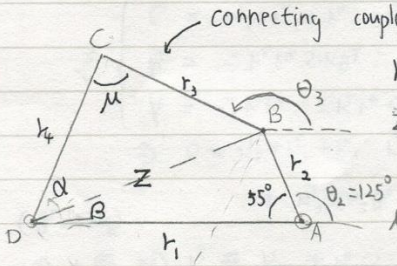
$\vec{r}_c = r_4(\cos\theta_4 i + \sin\theta_4 j)$

$= 2.3\cos(-44.9°) i + 2.3 \sin(-44.9°) j = \underline{1.63 i + (-1.62) j}$ C

$\vec{r}_E = \vec{r}_c + \vec{r}_{E/c} = 1.63 i + (-1.62) j + 1.35(\cos 25.18° i + \sin 25.18° j)$

$= \underline{2.85 i - 1.05 j}$ E

(2)



$r_1 = 3.8$, $r_2 = 1.2$, $r_3 = 3$, $r_4 = 2.3$

$Z^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos 55°$

$= r_3^2 + r_4^2 - 2 r_3 r_4 \cos\mu$

$\mu = \cos^{-1}\left(\dfrac{r_1^2 + r_2^2 - r_3^2 - r_4^2 - 2 r_1 r_2 \cos 55°}{-2 r_3 r_4}\right)$

$= 74.7°$ or $285.3°$ (crossed)

$Z^2 = 3.8^2 + 1.2^2 - 2 \times 3.8 \times 1.2 \times \cos 55° \Rightarrow Z = 3.26$

$r_2^2 = Z^2 + r_1^2 - 2 Z r_1 \cos\beta$

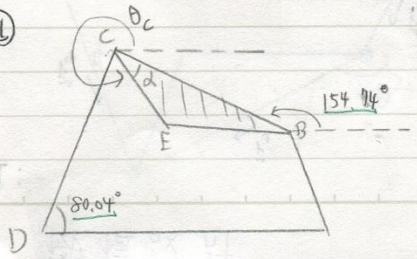
$\Rightarrow \beta = \cos^{-1}\left(\dfrac{1.2^2 - 3.26^2 + 3.8^2}{-2 \times 3.26 \times 3.8}\right) = 17.51°$, $342.49°$ (這是 invalid) $\because \theta_2 < 180°$

$\alpha = \cos^{-1}\left(\dfrac{3^2 - 2.3^2 - 3.26^2}{-2 \times 2.3 \times 3.26}\right) = 62.53°$, $297.47°$

$\theta_4 = \alpha + \beta = 80.04°$ or $314.98°$

$\theta_3 = \mu + \alpha + \beta = 74.7 + 80.04° = 154.74°$, $\theta_3' = \theta_3 + (180 - \alpha - \mu) + (180 - 74.7 - 62.53) = 240.28°$

①



$\overline{BC} = r_3 = 3$, $\overline{BE} = 2.05$, $\overline{CE} = 1.35$

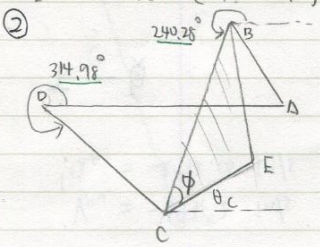
$\alpha = \cos^{-1}\left(\dfrac{2.05^2 - 1.35^2 - 3^2}{-2 \times 1.35 \times 3}\right) = 35.19°$

$\theta_C = 360° - (180° - 154.74°) - 35.19° = 299.55°$

$\vec{r}_C = r_4(\cos 80.04° \, i + \sin 80.04° \, j) = \underline{0.40 \, i + 2.27 \, j}$ C

$\vec{r}_E = \vec{r}_C + \vec{r}_{E/C} = (0.4 \, i + 2.27 j) + 1.35 \times (\cos 299.55° i + \sin 299.55° \, j) = \underline{1.07 \, i + 1.10 \, j}$ E

②



$\phi = \alpha = 35.19°$

$\therefore \theta_C = 240.28° - 180° - 35.19° = 25.09°$

$\vec{r}_C = 2.3 (\cos 314.98° \, i + \sin 314.98° \, j) = \underline{1.63 \, i - 1.63 \, j}$ C

$\vec{r}_E = \vec{r}_C + \vec{r}_{E/C} = (1.63 \, i - 1.63 j) + 1.35 (\cos 25.09° \, i + \sin 25.09° \, j)$

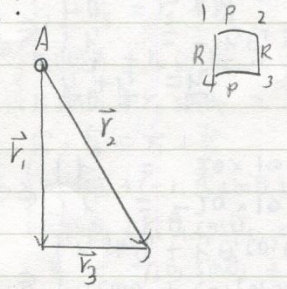
$= \underline{2.85 \, i + (-1.06) \, j}$ E

HW2：

(1)

Hw 2:

1.



Constant:

$r_1 = 6$, $\theta_1 = 270°$, $\theta_3 = 0°$, $\dot{r}_3 = 10$ in/min to right

Unknown:

r_2, r_3, θ_2

$$\vec{r}_1 + \vec{r}_3 = \vec{r}_2$$

$$\Rightarrow r_1(\cos\theta_1\, i + \sin\theta_1\, j) + r_3(\cos\theta_3\, i + \sin\theta_3\, j) = r_2(\cos\theta_2\, i + \sin\theta_2\, j)$$

$$\Rightarrow \begin{cases} r_1\cos\theta_1 + r_3\cos\theta_3 = r_2\cos\theta_2 & \text{——— ①} \\ r_1\sin\theta_1 + r_3\sin\theta_3 = r_2\sin\theta_2 & \text{——— ②} \end{cases}$$

$$\Rightarrow \begin{cases} r_3 = r_2\cos 300° \\ -6 = r_2\sin 300° \Rightarrow r_2 = 6.928 \end{cases}$$

$$\therefore r_3 = 6.928 \times \cos 300° = 3.464$$

①,② 微分 $\Rightarrow \begin{cases} \dot{r}_1\cos\theta_1 + (-r_1\dot{\theta}_1\sin\theta_1) + \dot{r}_3\cos\theta_3 + (-r_3\dot{\theta}_3\sin\theta_3) = \dot{r}_2\cos\theta_2 + (-r_2\dot{\theta}_2\sin\theta_2) & \text{— ③} \\ \dot{r}_1\sin\theta_1 + r_1\dot{\theta}_1\cos\theta_1 + \dot{r}_3\sin\theta_3 + r_3\dot{\theta}_3\cos\theta_3 = \dot{r}_2\sin\theta_2 + r_2\dot{\theta}_2\cos\theta_2 & \text{— ④} \end{cases}$

$$\Rightarrow \begin{cases} 10 = \dot{r}_2\cos 300° + (-6.928 \times \dot{\theta}_2 \times \sin 300°) \\ 0 = \dot{r}_2\sin 300° + 6.928 \times \dot{\theta}_2\cos 300° \end{cases}$$

$$\Rightarrow \begin{cases} \dot{r}_2 = 5 \text{ in/min} \\ \dot{\theta}_2 = 1.25 \text{ rad/min} \end{cases}$$

③,④ 微分 $\Rightarrow \begin{cases} 0 = \ddot{r}_2\cos\theta_2 - \dot{r}_2\dot{\theta}_2\sin\theta_2 - \dot{r}_2\dot{\theta}_2\sin\theta_2 - r_2\ddot{\theta}_2\sin\theta_2 - r_2\dot{\theta}_2^2\cos\theta_2 \\ 0 = \ddot{r}_2\sin\theta_2 + \dot{r}_2\dot{\theta}_2\cos\theta_2 + \dot{r}_2\dot{\theta}_2\cos\theta_2 + r_2\ddot{\theta}_2\cos\theta_2 - r_2\dot{\theta}_2^2\sin\theta_2 \end{cases}$

$$\Rightarrow \begin{cases} 0 = \ddot{r}_2\cos 300° - 5\times1.25\times\sin 300° - 5\times1.25\times\sin 300° - 6.928\times\ddot{\theta}_2\sin 300° - 6.928\times1.25^2\times\cos 300° \\ 0 = \ddot{r}_2\sin 300° + 5\times1.25\times\cos 300° + 5\times1.25\times\cos 300° + 6.928\times\ddot{\theta}_2\times\cos 300° - 6.928\times1.25^2\times\sin 300° \end{cases}$$

$$\Rightarrow \begin{cases} -5.413 = \ddot{r}_2\cos 300° - 6.928\times\ddot{\theta}_2\sin 300° \\ -15.625 = \ddot{r}_2\sin 300° + 6.928\times\ddot{\theta}_2\cos 300° \end{cases}$$

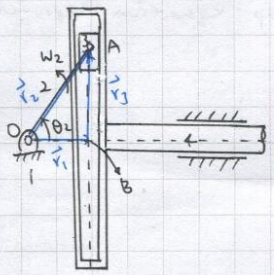
$$\Rightarrow \begin{cases} \ddot{r}_2 = 10.825 \\ \ddot{\theta}_2 = -1.804 \end{cases}$$

angular acceleration $\ddot{\theta}_2 = -1.804$ rad/min² CCW

(2)

(2)

5.18 In the Scotch yoke mechanism shown, $\omega_2 = 10$ rad/s, $\alpha_2 = 100$ rad/s^2, and $\theta_2 = 60°$. Also, length OA = 20 in. Determine v_{A4} and a_{A4} using loop equation.



position

$\vec{r}_2 = \vec{r}_1 + \vec{r}_3$

$$\begin{cases} r_2 \cos\theta_2 = r_1 \cos\theta_1 + \cancel{r_3 \cos\theta_3}^{\,0} \\ r_2 \sin\theta_2 = \cancel{r_1 \sin\theta_1}^{\,0} + r_3 \sin\theta_3 \end{cases} \Rightarrow \begin{cases} r_2 \cos\theta_2 = r_1 \\ r_2 \sin\theta_2 = r_3 \end{cases}$$

velocity $(\dot{r}_2 = 0)$

$$\begin{cases} r_2 (-\sin\theta_2) \cdot \omega_2 = \dot{r}_1 \\ r_2 (\cos\theta_2)\, \omega_2 = \dot{r}_3 \end{cases}$$

$\theta_1 = 0°$ $r_1 = 10$ in

$\theta_3 = 90°$ $r_3 = 10\sqrt{3}$ in

$\theta_2 = 60°$ $r_2 = 20$ in

$\dot{r}_1 = -173.2$ in/s $= V_{A4}$ #

$\dot{r}_3 = 100$ in/s

acceleration

$$\begin{cases} r_2 \left[(-\cos\theta_2) \cdot \omega_2^2 + (-\sin\theta_2) \cdot \alpha_2 \right] = \ddot{r}_1 \\ r_2 \left[(-\sin\theta_2)\, \omega_2^2 + (\cos\theta_2) \cdot \alpha_2 \right] = \ddot{r}_3 \end{cases}$$

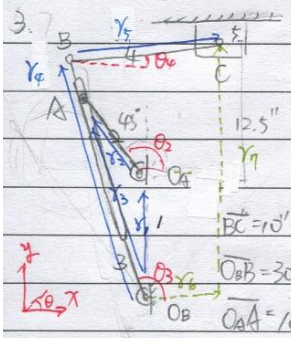
$\ddot{r}_1 = -2732$ in/s^2 $= a_{A4}$ #

$\ddot{r}_3 = -732$ in/s^2

(3)

HW2

3.



(1) $\begin{cases} \vec{r}_1 + \vec{r}_2 = \vec{r}_3 & \text{— loop 1} \\ \vec{r}_4 + \vec{r}_5 = \vec{r}_6 + \vec{r}_7 & \text{— loop 2} \end{cases}$

loop 1 : $\begin{cases} r_1 + r_2 \sin\theta_2 = r_3 \sin\theta_3 \\ r_2 \cos\theta_2 = r_3 \cos\theta_3 \end{cases}$

loop 2 : $\begin{cases} r_6 = r_4 \cos\theta_3 + r_5 \cos\theta_4 \\ r_7 = r_4 \sin\theta_3 + r_5 \sin\theta_4 \end{cases}$

$\Rightarrow \begin{cases} \dot{r}_2 \dot{\theta}_2 \cos\theta_2 = \dot{r}_3 \sin\theta_3 + r_3 \dot{\theta}_3 \cos\theta_3 \\ -r_2 \dot{\theta}_2 \sin\theta_2 = \dot{r}_3 \cos\theta_3 - r_3 \dot{\theta}_3 \sin\theta_3 \end{cases}$

$\Rightarrow \begin{cases} \dot{r}_6 = -r_4 \dot{\theta}_3 \sin\theta_3 + r_5 \dot{\theta}_4 \sin\theta_4 \\ 0 = r_4 \dot{\theta}_3 \cos\theta_3 + r_5 \dot{\theta}_4 \cos\theta_4 \end{cases}$

Matrix form : $\begin{bmatrix} \cos\theta_3 & -r_3 \sin\theta_3 \\ \sin\theta_3 & r_3 \cos\theta_3 \end{bmatrix} \begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -r_2 \dot{\theta}_2 \sin\theta_2 \\ r_2 \dot{\theta}_2 \cos\theta_2 \end{bmatrix} \Rightarrow$ loop 1

$$\begin{bmatrix} 1 & +r_4 \sin\theta_3 & +r_5 \sin\theta_4 \\ 0 & r_4 \cos\theta_3 & r_5 \cos\theta_4 \end{bmatrix} \begin{bmatrix} \dot{r}_6 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \text{loop 2}$$

Combine both Matrics

$$\underbrace{\begin{bmatrix} \cos\theta_3 & -r_3 \sin\theta_3 & 0 & 0 \\ \sin\theta_3 & r_3 \cos\theta_3 & 0 & 0 \\ 0 & +r_4 \sin\theta_3 & 1 & +r_5 \sin\theta_4 \\ 0 & r_4 \cos\theta_3 & 0 & r_5 \cos\theta_4 \end{bmatrix}}_{\underset{\sim}{A}} \begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \\ \dot{r}_6 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} -r_2 \dot{\theta}_2 \sin\theta_2 \\ r_2 \dot{\theta}_2 \cos\theta_2 \\ 0 \\ 0 \end{bmatrix}$$

from loop 1

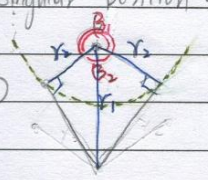
$\det \underset{\sim}{A} = \begin{vmatrix} \cos\theta_3 & -r_3 \sin\theta_3 \\ \sin\theta_3 & r_3 \cos\theta_3 \end{vmatrix} \begin{vmatrix} 1 & r_5 \sin\theta_4 \\ 0 & r_5 \cos\theta_4 \end{vmatrix} = r_3 (r_5 \cos\theta_4) \Rightarrow$ to have singular position

$\Rightarrow \det A = 0 \Rightarrow \cos\theta_4 = 0$ or $r_3 = 0$

$\Rightarrow \theta_4 = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\theta_2 = \frac{\pi}{2}, \frac{3\pi}{2}$

Singular position : 代表當機構處在此構形(configuration)下，不管施力如何皆不會作動。

(2)



$\cos\frac{B_2}{2} = \frac{r_2}{r_1} \Rightarrow B_2 = 2\cos^{-1}\left(\frac{r_2}{r_1}\right)$