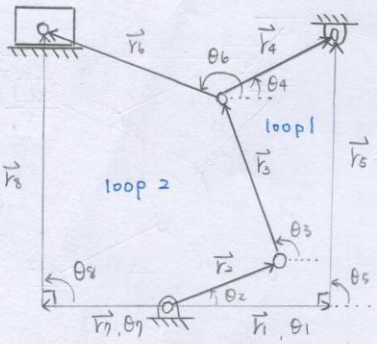
104-1 機構設計 Quiz#1 solution

Quiz 1

1. (a) $n=6,\ j=7.\quad F=3(6-1)-2\times 7=1$

(b)



loop - closure equations:

loop 1: $\vec{r_2}+\vec{r_3}+\vec{r_4}=\vec{r_1}+\vec{r_5}$

$$\begin{cases} r_2\cos\theta_2+r_3\cos\theta_3+r_4\cos\theta_4=r_1\cos\theta_1+r_5\cos\theta_5 \\ r_2\sin\theta_2+r_3\sin\theta_3+r_4\sin\theta_4=r_1\sin\theta_1+r_5\sin\theta_5 \end{cases}$$

known: $r_1, r_2, r_3, r_4, r_5, \theta_1, \theta_5, \theta_2$(input)

unknown: θ_3, θ_4.

loop 2: $\vec{r_2}+\vec{r_3}+\vec{r_6}=\vec{r_7}+\vec{r_8}$

$$\begin{cases} r_2\cos\theta_2+r_3\cos\theta_3+r_6\cos\theta_6=r_7\cos\theta_7+r_8\cos\theta_8 \\ r_2\sin\theta_2+r_3\sin\theta_3+r_6\sin\theta_6=r_7\sin\theta_7+r_8\sin\theta_8 \end{cases}$$

known: $r_2, r_3, r_6, r_8, \theta_7, \theta_8, \theta_2$(input), θ_3 (from sol. of loop 1)

unknown: r_7, θ_6

loop 1 and loop 2: 2 equations, 2 unknown, ⇒ Solvable!

velocity equations:

loop 1 對 t 微分, $r_1, r_2, r_3, r_4, r_5, \theta_1, \theta_5$ 為 constant.

$$\Rightarrow \begin{cases} -r_2\sin\theta_2\dot{\theta}_2-r_3\sin\theta_3\dot{\theta}_3-r_4\sin\theta_4\dot{\theta}_4=0 \\ r_2\cos\theta_2\dot{\theta}_2+r_3\cos\theta_3\dot{\theta}_3+r_4\cos\theta_4\dot{\theta}_4=0 \end{cases}$$

Matrix forms: $\begin{bmatrix}\dot{\theta}_3\\ \dot{\theta}_4\end{bmatrix}=\begin{bmatrix}-r_3\sin\theta_3 & -r_4\sin\theta_4\\ r_3\cos\theta_3 & r_4\cos\theta_4\end{bmatrix}^{-1}\cdot\begin{bmatrix}r_2\sin\theta_2\dot{\theta}_2\\ -r_2\cos\theta_2\dot{\theta}_2\end{bmatrix}$

θ_2、$\dot{\theta}_2$: input

θ_3, θ_4: 從 loop-closure eq. 得

↳ 只要得此解, equation is solvable!

loop 2 對 t 微分, $r_2, r_3, r_6, r_8, \theta_7, \theta_8$ 為 constant.

$$\Rightarrow \begin{cases} -r_2\sin\theta_2\dot{\theta}_2-r_3\sin\theta_3\dot{\theta}_3-r_6\sin\theta_6\dot{\theta}_6=\dot{r}_7\cos\theta_7 \\ r_2\cos\theta_2\dot{\theta}_2+r_3\cos\theta_3\dot{\theta}_3+r_6\cos\theta_6\dot{\theta}_6=\dot{r}_7\sin\theta_7 \end{cases}$$

Matrix forms: $\begin{bmatrix}\dot{r}_7\\ \dot{\theta}_6\end{bmatrix}=\begin{bmatrix}\cos\theta_7 & r_6\sin\theta_6\\ \sin\theta_7 & -r_6\cos\theta_6\end{bmatrix}^{-1}\begin{bmatrix}-r_2\sin\theta_2\dot{\theta}_2-r_3\sin\theta_3\dot{\theta}_3\\ r_2\cos\theta_2\dot{\theta}_2+r_3\cos\theta_3\dot{\theta}_3\end{bmatrix}$

θ_2, $\dot{\theta}_2$: input

$\theta_3, \theta_7, \theta_6$: 從 loop-closure eq. 得

$\dot{\theta}_3$: 從 velocity eq. 得

→ Solvable!

acceleration equations:

velocity eq.'s loop 1 再對 t 微分.

$$\Rightarrow \begin{cases} -r_2\cos\theta_2\,\dot\theta_2^2 - r_2\sin\theta_2\,\ddot\theta_2 - r_3\cos\theta_3\cdot\dot\theta_3^2 - r_3\sin\theta_3\cdot\ddot\theta_3 - r_4\cos\theta_4\,\dot\theta_4^2 - r_4\sin\theta_4\cdot\ddot\theta_4 = 0 \\ -r_2\sin\theta_2\,\dot\theta_2^2 + r_2\cos\theta_2\,\ddot\theta_2 - r_3\sin\theta_3\cdot\dot\theta_3^2 + r_3\cos\theta_3\cdot\ddot\theta_3 - r_4\sin\theta_4\cdot\dot\theta_4^2 + r_4\cos\theta_4\cdot\ddot\theta_4 = 0 \end{cases}$$

Matrix forms: $$\begin{bmatrix}\ddot\theta_3\\ \ddot\theta_4\end{bmatrix} = \begin{bmatrix}-r_3\sin\theta_3 & -r_4\sin\theta_4\\ r_3\cos\theta_3 & r_4\cos\theta_4\end{bmatrix}^{-1}\cdot\begin{bmatrix} r_2\cos\theta_2\,\dot\theta_2^2 + r_2\sin\theta_2\,\ddot\theta_2 + r_3\cos\theta_3\,\dot\theta_3^2 + r_4\cos\theta_4\,\dot\theta_4^2 \\ r_2\sin\theta_2\,\dot\theta_2^2 - r_2\cos\theta_2\,\ddot\theta_2 + r_3\sin\theta_3\,\dot\theta_3^2 + r_4\sin\theta_4\,\dot\theta_4^2\end{bmatrix}$$

θ_2、$\dot\theta_2$、$\ddot\theta_2$: input

$\dot\theta_3$、$\dot\theta_4$: 從 velocity eq. 得. $\rightarrow$ equation is solvable!

velocity eq's loop 2 再對 t 微分.

$$\Rightarrow \begin{cases} -r_2\cos\theta_2\,\dot\theta_2^2 - r_2\sin\theta_2\,\ddot\theta_2 - r_3\cos\theta_3\,\dot\theta_3^2 - r_3\sin\theta_3\,\ddot\theta_3 - r_6\cos\theta_6\,\dot\theta_6^2 - r_6\sin\theta_6\,\ddot\theta_6 = \ddot r_7\cos\theta_7 \\ -r_2\sin\theta_2\,\dot\theta_2^2 + r_2\cos\theta_2\,\ddot\theta_2 - r_3\sin\theta_3\,\dot\theta_3^2 + r_3\cos\theta_3\,\ddot\theta_3 - r_6\sin\theta_6\,\dot\theta_6^2 + r_6\cos\theta_6\,\ddot\theta_6 = \ddot r_7\sin\theta_7 \end{cases}$$

Matrix forms: $$\begin{bmatrix}\ddot r_7\\ \ddot\theta_6\end{bmatrix} = \begin{bmatrix}\cos\theta_7 & r_6\sin\theta_6\\ \sin\theta_7 & r_6\cos\theta_6\end{bmatrix}^{-1}\cdot\begin{bmatrix} -r_2\cos\theta_2\,\dot\theta_2^2 - r_2\sin\theta_2\,\ddot\theta_2 - r_3\cos\theta_3\,\dot\theta_3^2 - r_3\sin\theta_3\,\ddot\theta_3 - r_6\cos\theta_6\,\dot\theta_6^2 \\ -r_2\sin\theta_2\,\dot\theta_2^2 + r_2\cos\theta_2\,\ddot\theta_2 - r_3\sin\theta_3\,\dot\theta_3^2 + r_3\cos\theta_3\,\ddot\theta_3 - r_6\sin\theta_6\,\dot\theta_6^2\end{bmatrix}$$

θ_2、$\dot\theta_2$、$\ddot\theta_2$: input

θ_3、θ_6 : 從 loop-closure eq. 得

$\dot\theta_3$、$\dot\theta_6$: 從 velocity eq. 得. $\rightarrow$ equation is solvable!

$\ddot\theta_3$: 從 acceleration eq's loop1 得

(c) the velocity matrix:

loop 1 : $$\begin{bmatrix}\dot\theta_3\\ \dot\theta_4\end{bmatrix} = \begin{bmatrix}-r_3\sin\theta_3 & -r_4\sin\theta_4\\ r_3\cos\theta_3 & r_4\cos\theta_4\end{bmatrix}^{-1}\cdot\begin{bmatrix} r_2\sin\theta_2\,\dot\theta_2\\ r_2\cos\theta_2\,\dot\theta_2\end{bmatrix}$$

$\hookrightarrow$ A, $\det(A)=0$ is singular.

loop 2 : $$\begin{bmatrix}\dot r_7\\ \dot\theta_6\end{bmatrix} = \begin{bmatrix}\cos\theta_7 & r_6\sin\theta_6\\ \sin\theta_7 & -r_6\cos\theta_6\end{bmatrix}^{-1}\begin{bmatrix} -r_2\sin\theta_2\,\dot\theta_2 & -r_3\sin\theta_3\,\dot\theta_3\\ r_2\cos\theta_2\,\dot\theta_2 & r_3\cos\theta_3\,\dot\theta_3\end{bmatrix}$$

$\hookrightarrow$ B, $\det(B)=0$ is singular.

case 1:

$\det(A) = -r_3 r_4\sin\theta_3\cdot\cos\theta_4 + r_3 r_4\cos\theta_3\cdot\sin\theta_4 = 0.$

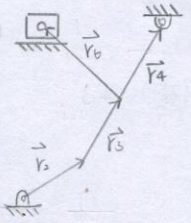
$r_3 r_4(\sin\theta_4\cos\theta_3 - \cos\theta_4\sin\theta_3) = 0. \rightarrow \sin(\theta_4-\theta_3)=0$

即 $\theta_4-\theta_3 = 0^\circ$ or π, $\underline{\theta_4=\theta_3}$ or $\underline{\theta_4=\theta_3+\pi}$

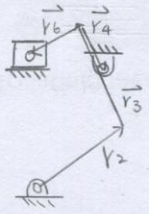
case 2:

$\det(B) = r_6\cos\theta_6 = 0$. $\cos\theta_6=0$. 即 $\underline{\theta_6=\frac{\pi}{2}}$ or $\underline{\theta_6=\frac{3}{2}\pi}$

case 1:

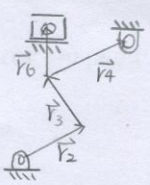


$\theta_3=\theta_4$

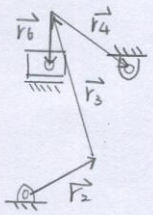


$\theta_4=\theta_3+\pi$

case 2:



$\theta_6=\frac{\pi}{2}$



$\theta_6=\frac{3}{2}\pi$

(d) link 4 is input :

已知與未知改變

loop 1 : $\begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -r_2 \sin\theta_2 & -r_3 \sin\theta_3 \\ r_2 \cos\theta_2 & r_3 \cos\theta_3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} r_4 \sin\theta_4 \cdot \dot{\theta}_4 \\ -r_4 \cos\theta_4 \cdot \dot{\theta}_4 \end{bmatrix}$

θ_4, $\dot{\theta}_4$: input

θ_2, θ_3 : 從 loop-closure eq. 得

$\hookrightarrow$ A , $\det(A) = 0$ is singular.

loop 2 : 因 $\vec{r}_4$ 不在 loop 2 內，所以不影響已知與未知的位置.

$$\begin{bmatrix} \dot{r}_7 \\ \dot{\theta}_6 \end{bmatrix} = \begin{bmatrix} \cos\theta_7 & r_6 \sin\theta_6 \\ \sin\theta_7 & -r_6 \cos\theta_6 \end{bmatrix}^{-1} \begin{bmatrix} -r_2 \sin\theta_2 \dot{\theta}_2 & -r_3 \sin\theta_3 \dot{\theta}_3 \\ r_2 \cos\theta_2 \dot{\theta}_2 & r_3 \cos\theta_3 \dot{\theta}_3 \end{bmatrix}$$

$\dot{\theta}_2$, $\dot{\theta}_3$: 從 velocity eq. 得.

θ_2, θ_3, θ_6, θ_7 : 從 loop-closure eq. 得

$\hookrightarrow$ B , $\det(B) = 0$ is singular

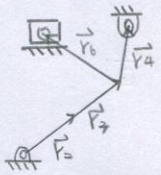
case 1 :

$\det(A) = -r_2 r_3 \sin\theta_2 \cos\theta_3 + r_2 r_3 \cos\theta_2 \sin\theta_3 = 0$

$r_2 r_3 (\sin\theta_3 \cos\theta_2 - \cos\theta_3 \sin\theta_2) = 0 \longrightarrow \sin(\theta_3 - \theta_2) = 0$

即 $\theta_3 - \theta_2 = 0^\circ$ or π , $\underline{\theta_3 = \theta_2}$ or $\underline{\theta_3 = \theta_2 + \pi}$

case 1 :

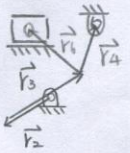


$\theta_3 = \theta_2$

case 2 :

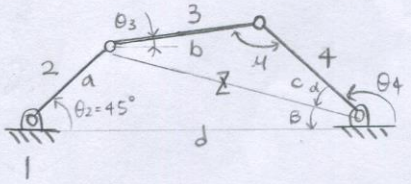
$\det(B) = r_6 \cos\theta_6 = 0$, $\cos\theta_6 = 0$, 即 $\underline{\theta_6 = \frac{\pi}{2}}$ or $\underline{\theta_6 = \frac{3}{2}\pi}$

* 與 link 2 為 input 時的 case 2 一樣.



$\theta_3 = \theta_2 + \pi$

2.



$a = 5''$

$b = 7''$

$c = 9''$

$d = 6''$

$z^2 = d^2 + a^2 - 2ad \cdot \cos\theta_2$

$= b^2 + c^2 - 2bc \cos\mu$

$\therefore$ $z = 4.31''$ $\cos\mu = 0.8843$, $\mu = 27.83^\circ$ (open)

or 332.17° (crossed)

$\alpha = \cos^{-1}\left(\frac{z^2 + c^2 - b^2}{2z \cdot c}\right)$, $\alpha = 49.31^\circ$ or 310.69°

$\beta = \cos^{-1}\left(\frac{z^2 + d^2 - a^2}{2zd}\right)$, $\beta = 55.12^\circ$ or 304.88°

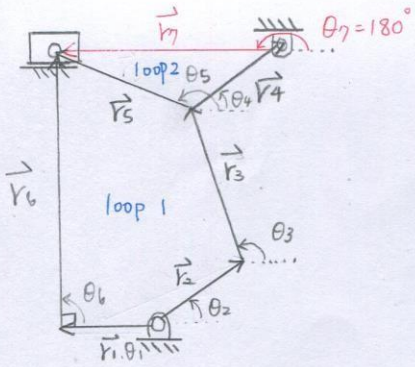
at upper branch when $\theta_2 = 45^\circ$,

$\rightarrow$ $\mu = \underline{\underline{27.83^\circ}}$. $\alpha = 49.31^\circ$, $\beta = 55.12^\circ$

$\theta_4 = 180^\circ - \alpha - \beta = \underline{\underline{75.57^\circ}}$

$\theta_3 = 180^\circ - \mu - \alpha - \beta = \underline{\underline{47.74^\circ}}$

1. (c) (d) Answer_v2



<u>loop-closure equations:</u>

loop 1: $\vec{r_1} + \vec{r_6} = \vec{r_2} + \vec{r_3} + \vec{r_5}$

$$\begin{cases} r_1\cos\theta_1 + r_6\cos\theta_6 = r_2\cos\theta_2 + r_3\cos\theta_3 + r_5\cos\theta_5 \\ r_1\sin\theta_1 + r_6\sin\theta_6 = r_2\sin\theta_2 + r_3\sin\theta_3 + r_5\sin\theta_5 \end{cases}$$

known: r_2, r_3, r_5, r_6, θ_1, θ_2 (input), θ_6

unknown: θ_3, θ_4, θ_5, r_1

合併 loop 1 and loop 2,

4 unknowns 4 equations.

→ solvable!

loop 2: $\vec{r_4} + \vec{r_7} = \vec{r_5}$

$$\begin{cases} r_4\cos\theta_4 + r_7\cos\theta_7 = r_5\cos\theta_5 \\ r_4\sin\theta_4 + r_7\sin\theta_7 = r_5\sin\theta_5 \end{cases}$$

known: r_4, r_5, θ_7

unknown: θ_4, θ_5, r_7 ($r_7 = r_1 + c$)

<u>velocity equations:</u>

loop 1 對 t 微分, r_2, r_3, r_5, r_6, θ_1, θ_6 為 constant

$$\Rightarrow \begin{cases} -r_2\sin\theta_2\,\dot{\theta}_2 - r_3\sin\theta_3\,\dot{\theta}_3 - r_5\sin\theta_5\,\dot{\theta}_5 = \dot{r}_1\cos\theta_1 \\ r_2\cos\theta_2\,\dot{\theta}_2 + r_3\cos\theta_3\,\dot{\theta}_3 + r_5\cos\theta_5\,\dot{\theta}_5 = \dot{r}_1\sin\theta_1 \end{cases}$$

Matrix forms:

$$\begin{bmatrix} \dot{r}_1 \\ \dot{\theta}_3 \\ \dot{\theta}_5 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & r_3\sin\theta_3 & r_5\sin\theta_5 \\ \sin\theta_1 & -r_3\cos\theta_3 & -r_5\cos\theta_5 \end{bmatrix}^{-1} \begin{bmatrix} -r_2\sin\theta_2\,\dot{\theta}_2 \\ r_2\cos\theta_2\,\dot{\theta}_2 \end{bmatrix}$$

θ_2, $\dot{\theta}_2$: input

θ_3, θ_5: 從 loop-closure eq. 得

loop 2 對 t 微分, r_4, r_5, θ_7 為 constant, $r_7 = r_1 + c$, $\therefore \dot{r}_7 = \dot{r}_1$

$$\Rightarrow \begin{cases} -r_4\sin\theta_4\,\dot{\theta}_4 + \dot{r}_7\cos\theta_7 = -r_5\sin\theta_5\,\dot{\theta}_5 \\ r_4\cos\theta_4\,\dot{\theta}_4 + \dot{r}_7\sin\theta_7 = r_5\cos\theta_5\,\dot{\theta}_5 \end{cases}$$

Matrix forms:

$$\begin{bmatrix} \dot{r}_1 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \end{bmatrix} = \begin{bmatrix} \cos\theta_7 & -r_4\sin\theta_4 & r_5\sin\theta_5 \\ \sin\theta_7 & r_4\cos\theta_4 & -r_5\cos\theta_5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

θ_4, θ_5: 從 loop-closure eq. 得

(C) 合併 loop 1 、 loop 2 的 velocity matrix :

$$\begin{bmatrix} \dot{r}_1 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & r_3\sin\theta_3 & 0 & r_5\sin\theta_5 \\ \sin\theta_1 & -r_3\cos\theta_3 & 0 & -r_5\cos\theta_5 \\ \cos\theta_7 & 0 & -r_4\sin\theta_4 & r_5\sin\theta_5 \\ \sin\theta_7 & 0 & r_4\cos\theta_4 & -r_5\cos\theta_5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -r_2\sin\theta_2\,\dot{\theta}_2 \\ r_2\cos\theta_2\,\dot{\theta}_2 \\ 0 \\ 0 \end{bmatrix}$$

$\cos\theta_1 = -1$
$\sin\theta_1 = 0$
$\cos\theta_7 = -1$
$\sin\theta_7 = 0$

↳ A , $\det(A) = 0$ is singular.

$$\det(A) = -r_3 r_4 r_5 \cdot \left[\cos\theta_3 \cdot \sin(\theta_4 - \theta_5) + \cos\theta_4 \sin(\theta_5 - \theta_3)\right] = 0$$

∴ $\cos\theta_3 = \cos\theta_4 = 0$ or $\sin(\theta_4 - \theta_5) = \sin(\theta_5 - \theta_3) = 0$ is singular

即 $\theta_3 = \theta_4 = \frac{\pi}{2}$

$\theta_3 = \frac{\pi}{2}$, $\theta_4 = -\frac{\pi}{2}$

$\theta_3 = -\frac{\pi}{2}$, $\theta_4 = \frac{\pi}{2}$

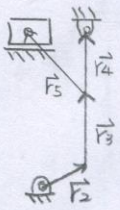
$\theta_3 = \theta_4 = -\frac{\pi}{2}$

or $\theta_4 - \theta_5 = \theta_5 - \theta_3 = 0 \rightarrow \theta_3 = \theta_4 = \theta_5$

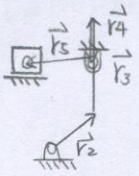
$\theta_4 - \theta_5 = 0$, $\theta_5 - \theta_3 = \pi$

$\theta_4 - \theta_5 = \pi$, $\theta_5 - \theta_3 = 0$

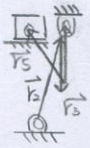
$\theta_4 - \theta_5 = \theta_5 - \theta_3 = \pi$.



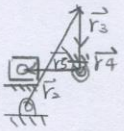
$\theta_3 = \theta_4 = \frac{\pi}{2}$



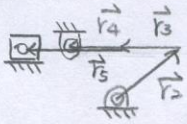
$\theta_3 = \frac{\pi}{2}$, $\theta_4 = -\frac{\pi}{2}$



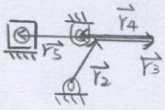
$\theta_3 = -\frac{\pi}{2}$, $\theta_4 = \frac{\pi}{2}$



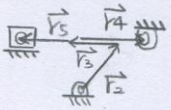
$\theta_3 = \theta_4 = -\frac{\pi}{2}$



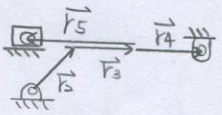
$\theta_3 = \theta_4 = \theta_5$.



$\theta_4 = \theta_5$. $\theta_5 = \theta_3 + \pi$



$\theta_3 = \theta_5$. $\theta_4 = \theta_5 + \pi$



$\theta_4 - \theta_5 = \theta_5 - \theta_3 = \pi$

(d) 已知與未知改變

$$\begin{bmatrix} \dot{r}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_5 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & r_2\sin\theta_2 & r_3\sin\theta_3 & r_5\sin\theta_5 \\ \sin\theta_1 & -r_2\cos\theta_2 & -r_3\cos\theta_3 & -r_5\cos\theta_5 \\ \cos\theta_7 & 0 & 0 & r_5\sin\theta_5 \\ \sin\theta_7 & 0 & 0 & -r_5\cos\theta_5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ r_4\sin\theta_4\cdot\dot{\theta}_4 \\ -r_4\cos\theta_4\,\dot{\theta}_4 \end{bmatrix}$$

$\cos\theta_1 = -1$

$\sin\theta_1 = 0$

$\cos\theta_7 = -1$

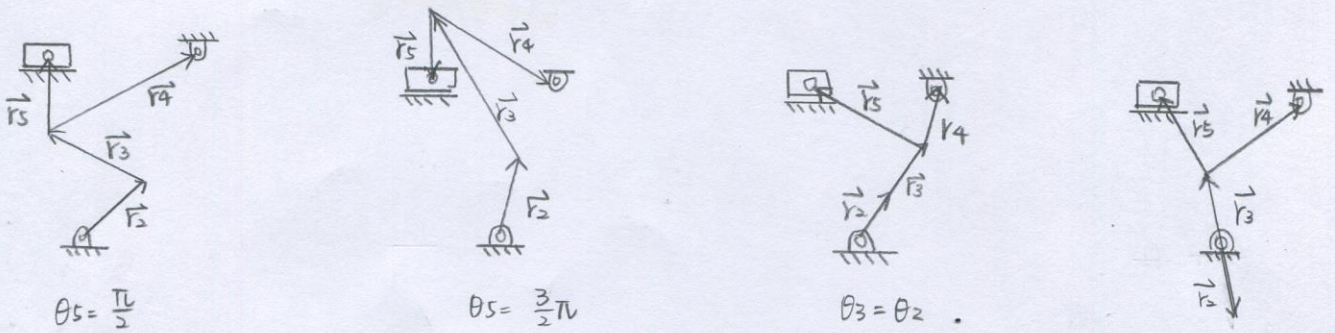
$\sin\theta_7 = 0.$

$\hookrightarrow$ B. det (B) = 0 is singular.

$\det(B) = r_2 r_3 r_5 \cos\theta_5 \cdot \sin(\theta_3-\theta_2) = 0$

$\therefore$ $\cos\theta_5 = 0$ or $\sin(\theta_3-\theta_2)=0$ is singular.

即 $\underline{\theta_5 = \frac{\pi}{2} \text{ or } \frac{3}{2}\pi}$ or $\underline{\theta_3-\theta_2 = 0 \text{ or } \pi}$



1. 70%

(a) 無算式不給分，答案錯就全錯 (10%)

(b) Loop-closure equations (8%)
Velocity equations (8%) + matrix forms (2%) + solvability (1%)
Acceleration equations (8%) + matrix forms (2%) + solvability (1%)

* 如果 loop 一開始就錯，理論上後面會整個都錯。 因此若是 loop-closure equations 一開始就寫錯，就會看你求 Velocity 跟 Acceleration 的過程，並依據完整性給部分分數

* loop 定義成滑塊與頂端地桿為同一水平:
因為定義關係，會影響 (c) 與 (d) 小題的 singular configurations，答案就會整個不一樣，此種情況會依照第二種答案來批改。

(c) velocity matrix(5%) + singular conditions (5%) + linkage configuration (5%)
* linkage configuration : 一個圖 1%，缺圖最多扣 5%

(d) velocity matrix(5%) + singular conditions (5%) + linkage configuration (5%)
* linkage configuration : 一個圖 1%，缺圖最多扣 5%

2. 30%

μ (10%)，θ_3 (5%)，θ_4 (5%)，α (5%)，β (5%)

* 若過程全對，但計算機按錯等計算錯誤之情況，整題扣 5%

* upper branch 判斷錯誤，整題只得 15%

* θ_3定義不同時，會根據你的定義，並看你是否算對，如果算對則不扣分